

# Assignment #3

TRANSPORTATION PROBLEM, INTEGER/BINARY PROGRAMMING AND GOAL  
PROGRAMMING

# Problem #1

Decision Variables:

Let  $x_{ij}$  be the number of sinks produced in  $i$ ; where  $i = 1$  for regular production,  $i = 2$  for overtime production,  $i = 3$  for purchasing from a competitor and  $i = 4$  for sinks in beginning inventory, and transported in  $j$ ; where  $j = 1-4$  for the four months.

Objective Function:

$$\text{Minimize } Z = 120(100)x_{11} + 160(100)x_{12} + 240(100)x_{13} + 100(100)x_{14} + 120(130)x_{21} + 160(130)x_{22} + 240(130)x_{23} + 100(130)x_{24} + 120(150)x_{31} + 160(150)x_{32} + 240(150)x_{33} + 100(150)x_{34} + 120(10)x_{41} + 160(10)x_{42} + 240(10)x_{43} + 100(10)x_{44}$$

$$= 12000x_{11} + 16000x_{12} + 24000x_{13} + 10000x_{14} + 15600x_{21} + 20800x_{22} + 31200x_{23} + 13000x_{24} + 18000x_{31} + 24000x_{32} + 36000x_{33} + 15000x_{34} + 1200x_{41} + 1600x_{42} + 2400x_{43} + 1000x_{44}$$

Subject To:

Regular Time Supply

$$x_{23} \leq 50$$

Non-Negativity

$$x_{11} \leq 100$$

$$x_{24} \leq 50$$

$$x_{ij} \geq 0$$

$$x_{12} \leq 100$$

Competitor Supply

Demand

$$x_{13} \leq 100$$

$$x_{31} + x_{32} + x_{33} + x_{34} \leq 450$$

$$x_{11} + x_{21} + x_{31} + x_{41} = 120$$

$$x_{14} \leq 100$$

Beginning Inventory

$$x_{12} + x_{22} + x_{32} + x_{42} = 160$$

Overtime Supply

Supply

$$x_{13} + x_{23} + x_{33} + x_{43} = 240$$

$$x_{21} \leq 50$$

$$x_{41} + x_{42} + x_{43} + x_{44} \leq 40$$

$$x_{14} + x_{24} + x_{34} + x_{44} = 100$$

$$x_{22} \leq 50$$

	Month 1	Month 2	Month 3	Month 4	Supply		
<b>1. Regular Time</b>	100	100	100	100	100		
<b>2. Overtime</b>	130	130	130	130	50		
<b>3. Competitor</b>	150	150	150	150	450		
<b>4. Beginning Inventory</b>	10	10	10	10	40		
<b>Demand</b>	120	160	240	100			
	Month 1	Month 2	Month 3	Month 4	Total Flow Out	Sign	Supply
<b>1. Regular Time</b>	0	0	40	60	100	<=	100
<b>2. Overtime</b>	0	0	50	0	50	<=	50
<b>3. Competitor</b>	120	160	150	0	430	<=	450
<b>4. Beginning Inventory</b>	0	0	0	40	40	<=	40
<b>Total Flow In</b>	120	160	240	100	81400		
	=	=	=	=			
<b>Demand</b>	120	160	240	100			

Microsoft Excel 15.0 Answer Report

Worksheet: [Book1]Sheet1

Report Created: 2016-11-28 2:10:26 PM

Result: Solver found a solution. All Constraints and optimality conditions are satisfied.

Solver Engine

Engine: Simplex LP  
Solution Time: 0.485 Seconds.  
Iterations: 11 Subproblems: 0

Solver Options

Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling  
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Min)

Cell	Name	Original Value	Final Value
\$F\$13	Total Flow In Total Flow Out	0	81400

Variable Cells

Cell	Name	Original Value	Final Value	Integer
\$B\$9	1. Regular Time Month 1	0	0	Contin
\$C\$9	1. Regular Time Month 2	0	0	Contin
\$D\$9	1. Regular Time Month 3	0	40	Contin
\$E\$9	1. Regular Time Month 4	0	60	Contin
\$B\$10	2. Overtime Month 1	0	0	Contin
\$C\$10	2. Overtime Month 2	0	0	Contin
\$D\$10	2. Overtime Month 3	0	50	Contin
\$E\$10	2. Overtime Month 4	0	0	Contin
\$B\$11	3. Competitor Month 1	0	120	Contin
\$C\$11	3. Competitor Month 2	0	160	Contin
\$D\$11	3. Competitor Month 3	0	150	Contin
\$E\$11	3. Competitor Month 4	0	0	Contin
\$B\$12	4. Beginning Inventory Month 1	0	0	Contin
\$C\$12	4. Beginning Inventory Month 2	0	0	Contin
\$D\$12	4. Beginning Inventory Month 3	0	0	Contin
\$E\$12	4. Beginning Inventory Month 4	0	40	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
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\$B\$13	Total Flow In Month 1	120	\$B\$13=\$B\$15	Binding	0
\$C\$13	Total Flow In Month 2	160	\$C\$13=\$C\$15	Binding	0
\$D\$13	Total Flow In Month 3	240	\$D\$13=\$D\$15	Binding	0
\$E\$13	Total Flow In Month 4	100	\$E\$13=\$E\$15	Binding	0
\$F\$9	1. Regular Time Total Flow Out	100	\$F\$9<=\$H\$9	Binding	0
\$F\$10	2. Overtime Total Flow Out	50	\$F\$10<=\$H\$10	Binding	0
				Not	
\$F\$11	3. Competitor Total Flow Out	430	\$F\$11<=\$H\$11	Binding	20
\$F\$12	4. Beginning Inventory Total Flow Out	40	\$F\$12<=\$H\$12	Binding	0

After reviewing the answer report management will have a minimized cost of \$81,400 as their optimal solution. For month 1 they are looking at purchasing 120 sinks only from the competitor in order to satisfy demands. For month 2 they are looking at purchasing 160 sinks again only from the competitor. For month 3 management is looking at producing 40 sinks that were produced in regular time, 30 sinks from overtime and purchasing 170 from the competitor. Lastly, in month 4 they are producing 60 sinks in regular time and then using 40 sinks from their beginning inventory. For every month they are meeting the demands and are using their supplies fully. The only slack, which in this case is a good thing, is in the competitor supply. They aren't purchasing sinks from the competitors to the full extent and have 20 sinks left.

## Problem #2

Decision Variables:

Let  $x_{ij}$  be the number of units produced using machine  $i$ ; where  $i = 1-3$  for the three machines, in order to produce  $j$ ; where  $j = 1$  for product A,  $j = 2$  for product B and  $j = 3$  for product C.

Binary Decision Variables:

Let  $y_i = 1$  if machine  $i$  is selected and 0 otherwise.

Objective Function:

$$\text{Max } Z = 13(x_{11} + x_{21} + x_{31}) + 10(x_{12} + x_{22} + x_{32}) + 12(x_{13} + x_{23} + x_{33}) - 150y_1 - 120y_2 - 110y_3$$

Subject To:

$$4x_{11} + 3x_{12} + 5x_{13} - 1000y_1 \leq 0$$

$$3x_{21} + 2x_{22} + 4x_{23} - 800y_2 \leq 0$$

$$2x_{31} + 4x_{32} + 2x_{33} - 700y_3 \leq 0$$

$$x_{11} + x_{21} + x_{31} \geq 40$$

$$x_{12} + x_{22} + x_{32} \geq 40$$

$$x_{13} + x_{23} + x_{33} \geq 40$$

$$y_1 + y_2 + y_3 = 1$$

$$x_{ij} \geq 0 \text{ and integer}$$

$$y_1, y_2, y_3 = \text{binary}$$

	x11	x12	x13	x21	x22	x23	x31	x32	x33	Machine 1	Machine 2	Machine 3			
<b>Solution</b>	0	0	0	0	0	0	230	40	40	0	0	1		<b>Total</b>	
<b>Profit</b>	13	10	12	13	10	12	13	10	12	-150	-120	-110		3760	
<b>Constraints</b>													<b>LHS</b>	<b>Sign</b>	<b>RHS</b>
Machine 1	4	3	5							-1000			0	≤	0
Machine 2				3	2	4					-800		0	≤	0
Machine 3							2	4	2			-700	0	≤	0
Selection										1	1	1	1	=	1
Product A	1			1			1						230	≥	40
Product B		1			1			1					40	≥	40
Product C			1			1			1				40	≥	40

### Microsoft Excel 16.0 Answer Report

Worksheet: [Book1]Sheet1

Report Created: 2016-11-28 1:26:00 PM

Result: Solver found a solution. All Constraints and optimality conditions are satisfied.

#### Solver Engine

Engine: Simplex LP

Solution Time: 0.046 Seconds.

Iterations: 2 Subproblems: 4

#### Solver Options

Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use AutomaticScaling

Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Max)

Cell	Name	Original Value	Final Value
\$O\$4	Profit	0	3760

Variable Cells

Cell	Name	Original Value	Final Value	Integer
\$B\$3	Solution X11	0	0	Integer
\$C\$3	Solution X12	0	0	Integer
\$D\$3	Solution X13	0	0	Integer
\$E\$3	Solution X21	0	0	Integer
\$F\$3	Solution X22	0	0	Integer
\$G\$3	Solution X23	0	0	Integer
\$H\$3	Solution X31	0	230	Integer
\$I\$3	Solution X32	0	40	Integer
\$J\$3	Solution X33	0	40	Integer
\$K\$3	Solution Machine 1	0	0	Binary
\$L\$3	Solution Machine 2	0	0	Binary
\$M\$3	Solution Machine 3	0	1	Binary

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$N\$10	Selection LHS	1	\$N\$10=\$P\$10	Binding	0
\$N\$11	Product A LHS	230	\$N\$11>=\$P\$11	Not Binding	190
\$N\$12	Product B LHS	40	\$N\$12>=\$P\$12	Binding	0
\$N\$13	Product C LHS	40	\$N\$13>=\$P\$13	Binding	0
\$N\$7	Machine 1 LHS	0	\$N\$7<=\$P\$7	Binding	0
\$N\$8	Machine 2 LHS	0	\$N\$8<=\$P\$8	Binding	0
\$N\$9	Machine 3 LHS	0	\$N\$9<=\$P\$9	Binding	0
\$B\$3:\$J\$3=Integer					
\$K\$3:\$M\$3=Binary					

After reviewing the answer report, the conclusion in order to maximize profit management should consider using only machine 3. In doing so they would be making 230 units of product A, 40 units of product B and another 40 units of product C. Management will ultimately be using the machines entire capacity of 700, making a profit of \$3,760.

## Problem #3

Decision Variables:

Let  $x_1$  be the tour bus going to San Francisco

Let  $x_2$  be the tour bus going to Oakland

Let  $x_3$  be the tour bus going to Palo Alto

Let  $x_4$  be the tour bus going to San Jose

Let  $x_5$  be the tour bus going to San Mateo

Let  $x_6$  be the tour bus going to Concord

Let  $x_7$  be the tour bus going to Santa Cruz

Let  $x_8$  be the tour bus going to Monterey

Deviation Variables:

$d_1^-$  = Underachievement of spending goal

$d_1^+$  = Overachievement of spending goal

$d_2^-$  = Underachievement of seeing at least 5 cities

$d_2^+$  = Overachievement of seeing at least 5 cities

$d_3^-$  = Overachievement of going to San Mateo

$d_3^+$  = Overachievement of going to San Mateo

Objective Function:

$$\text{Min } Z = p_1d_1^+ + p_2d_2^- + p_3d_3^- - p_3d_3^+$$

Subject To:

$$5000x_1 + 4500x_2 + 3600x_3 + 4100x_4 + 3500x_5 + 2500x_6 + 3200x_7 + 4000x_8 + d_1^- - d_1^+ = 15000$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + d_2^- - d_2^+ = 5$$

$$x_5 + d_3^- - d_3^+ = 1$$

$$x_i \geq 0$$

$$d_i^-, d_i^+ \geq 0$$

### Priority of Goal 1

	X1	X2	X3	X4	X5	X6	X7	X8	d1-	d1+	d2-	d2+	d3-	d3+			
	SF	OK	PA	SJ	SM	CO	SC	MN	Spending Under	Spending Over	5 Cities Under	5 Cities Over	San Mateo Under	San Mateo Over			
Solution	0	0	1	0	1	0	1	1	700	0	1	0	0	0			
Goals											1				0		
<b>Constraints:</b>															LHS		RHS
Goal 1: Spending	5000	4500	3600	4100	3500	2500	3200	4000	1	-1					15000 =		15000
Goal 2: 5 Cities	1	1	1	1	1	1	1	1			1	-1			5 =		5
Goal 3: San Mateo					1								1	-1	1 =		1

Microsoft Excel 16.0 Answer Report

Worksheet: [Book1]Sheet 1

Report Created: 2016-11-28 1:21:02 PM

Result: Solver found an integer solution within tolerance. All Constraints are satisfied.

Solver Engine

Engine: Simplex LP  
Solution Time: 0.047 Seconds.  
Iterations: 1 Subproblems: 8

Solver Options

Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling  
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Min)

Cell	Name	Original Value	Final Value
\$P\$6	Goals	0	0

Variable Cells

Cell	Name	Original Value	Final Value	Integer
\$B\$5	Solution SF	0	0	Binary
\$C\$5	Solution OK	0	0	Binary
\$D\$5	Solution PA	0	1	Binary
\$E\$5	Solution SJ	0	0	Binary
\$F\$5	Solution SM	0	1	Binary
\$G\$5	Solution CO	0	0	Binary
\$H\$5	Solution SC	0	1	Binary
\$I\$5	Solution MN	0	1	Binary
\$J\$5	Solution Spending Under	0	700	Contin
\$K\$5	Solution Spending Over	0	0	Contin
\$L\$5	Solution 5 Cities Under	0	1	Contin
\$M\$5	Solution 5 Cities Over	0	0	Contin
\$N\$5	Solution San Mateo Under	0	0	Contin
\$O\$5	Solution San Mateo Over	0	0	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
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\$P\$9	Goal 1: Spending LHS	15000	\$P\$9=\$R\$9	Binding	0
\$P\$10	Goal 2: 5 Cities LHS	5	\$P\$10=\$R\$10	Binding	0
\$P\$11	Goal 3: San Mateo LHS	1	\$P\$11=\$R\$11	Binding	0
\$B\$5:\$I\$5=Binary					

Priority of Goal 2

	X1	X2	X3	X4	X5	X6	X7	X8	d1-	d1+	d2-	d2+	d3-	d3+		
	SF	OK	PA	SJ	SM	CO	SC	MN	Spending Under	Spending Over	5 Cities Under	5 Cities Over	San Mateo Under	San Mateo Over		
Solution	0	0	1	1	0	0	1	1	100	0	1	0	1	0		
Goals											1				1	
<b>Constraints:</b>															LHS	RHS
Goal 1: Spending	5000	4500	3600	4100	3500	2500	3200	4000	1	-1					15000 =	15000
Goal 2: 5 Cities	1	1	1	1	1	1	1	1			1	-1			5 =	5
Goal 3: San Mateo					1								1	-1	1 =	1

Microsoft Excel 16.0 Answer Report

Worksheet: [Book1]Sheet 2

Report Created: 2016-11-28 1:49:06 PM

Result: Solver found a solution. All Constraints and optimality conditions are satisfied.

Solver Engine

Engine: Simplex LP

Solution Time: 0.172 Seconds.

Iterations: 1 Subproblems: 144

Solver Options

Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling

Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Min)

Cell	Name	Original Value	Final Value
\$P\$6	Goals	1	1

Variable Cells

Cell	Name	Original Value	Final Value	Integer
\$B\$5	Solution SF	0	0	Binary
\$C\$5	Solution OK	0	0	Binary
\$D\$5	Solution PA	1	1	Binary
\$E\$5	Solution SJ	1	1	Binary

\$F\$5	Solution SM	0	0	Binary
\$G\$5	Solution CO	0	0	Binary
\$H\$5	Solution SC	1	1	Binary
\$I\$5	Solution MN	1	1	Binary
\$J\$5	Solution Spending Under	100	100	Contin
\$K\$5	Solution Spending Over	0	0	Contin
\$L\$5	Solution 5 Cities Under	1	1	Contin
\$M\$5	Solution 5 Cities Over	0	0	Contin
\$N\$5	Solution San Mateo Under	1	1	Contin
\$O\$5	Solution San Mateo Over	0	0	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$P\$9	Goal 1: Spending LHS	15000	\$P\$9=\$R\$9	Binding	0
\$P\$10	Goal 2: 5 Cities LHS	5	\$P\$10=\$R\$10	Binding	0
\$P\$11	Goal 3: San Mateo LHS	1	\$P\$11=\$R\$11	Binding	0
\$K\$5	Solution Spending Over	0	\$K\$5=0	Binding	0
\$B\$5:\$I\$5=Binary					

Priority of Goal 3

	X1	X2	X3	X4	X5	X6	X7	X8	d1-	d1+	d2-	d2+	d3-	d3+		
	SF	OK	PA	SJ	SM	CO	SC	MN	Spending Under	Spending Over	5 Cities Under	5 Cities Over	San Mateo Under	San Mateo Over		
Solution	1	0	0	0	1	1	0	1	0	0	1	0	0	0	<b>Total</b>	
Goals													1	1	0	
<b>Constraints:</b>															LHS	RHS
Goal 1: Spending	5000	4500	3600	4100	3500	2500	3200	4000	1	-1					15000 =	15000
Goal 2: 5 Cities	1	1	1	1	1	1	1	1			1	-1			5 =	5
Goal 3: San Mateo					1								1	-1	1 =	1

Microsoft Excel 16.0 Answer Report

Worksheet: [Book1]Sheet 3

Report Created: 2016-11-28 1:56:43 PM

Result: Solver found an integer solution within tolerance. All Constraints are satisfied.

Solver Engine

Engine: Simplex LP

Solution Time: 0.031 Seconds.

Iterations: 1 Subproblems: 2

Solver Options

Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling

Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Min)

Cell	Name	Original Value	Final Value
\$P\$6	Goals	0	0

Variable Cells

Cell	Name	Original Value	Final Value	Integer
\$B\$5	Solution SF	1	1	Binary
\$C\$5	Solution OK	0	0	Binary
\$D\$5	Solution PA	0	0	Binary
\$E\$5	Solution SJ	0	0	Binary
\$F\$5	Solution SM	1	1	Binary
\$G\$5	Solution CO	1	1	Binary
\$H\$5	Solution SC	0	0	Binary
\$I\$5	Solution MN	1	1	Binary
\$J\$5	Solution Spending Under	0	0	Contin
\$K\$5	Solution Spending Over	0	0	Contin
\$L\$5	Solution 5 Cities Under	1	1	Contin
\$M\$5	Solution 5 Cities Over	0	0	Contin
\$N\$5	Solution San Mateo Under	0	0	Contin
\$O\$5	Solution San Mateo Over	0	0	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$P\$9	Goal 1: Spending LHS	15000	\$P\$9=\$R\$9	Binding	0
\$P\$10	Goal 2: 5 Cities LHS	5	\$P\$10=\$R\$10	Binding	0
\$P\$11	Goal 3: San Mateo LHS	1	\$P\$11=\$R\$11	Binding	0
\$K\$5	Solution Spending Over	0	\$K\$5=0	Binding	0
\$L\$5	Solution 5 Cities Under	1	\$L\$5=1	Binding	0
\$B\$5:\$I\$5=Binary					

After reviewing the answer report, the conclusion in order to succeed reaching the goals as much as possible is as following; Management should plan to take a bus tour seeing San Francisco, San Mateo, Concord and Monterey. Unfortunately they won't be able to reach all three goals but will come very close. They reached their first goal of staying within the budget of \$15,000 and was able to see San Mateo which was their third goal, however, in doing so they were only able to tour 4 cities instead of 5.

# Personal Ethics Statement

## Individual Assignment:

By signing this Statement, I am attesting to the fact that I have reviewed the entirety of my attached work and that I have applied all the appropriate rules of quotation and referencing in use at the Telfer School of Management at the University of Ottawa, as well as adhered to the fraud policies outlined in the Academic Regulations in the University's Undergraduate Studies Calendar. [Academic Fraud Webpage](#)

Bianca Chitiu, BC                      Monday, October 28, 2016

Signature

Date

Chitiu Bianca

7829013

Last Name (print), First Name (print)

Student Number