

Assignment #1

LINEAR PROGRAMMING
FORMULATION, GRAPHICAL METHOD AND EXCEL SOLVER

Problem #1

Decision Variables:

x_1 = Number of large space storage rooms to develop

x_2 = Number of small space storage rooms to develop

Objective Function:

Maximize monthly earnings $Z = 50x_1 + 20x_2$ (Profit)

Subject To:

$2x_1 + 4x_2 \leq 400$ (Advertising budget in \$)

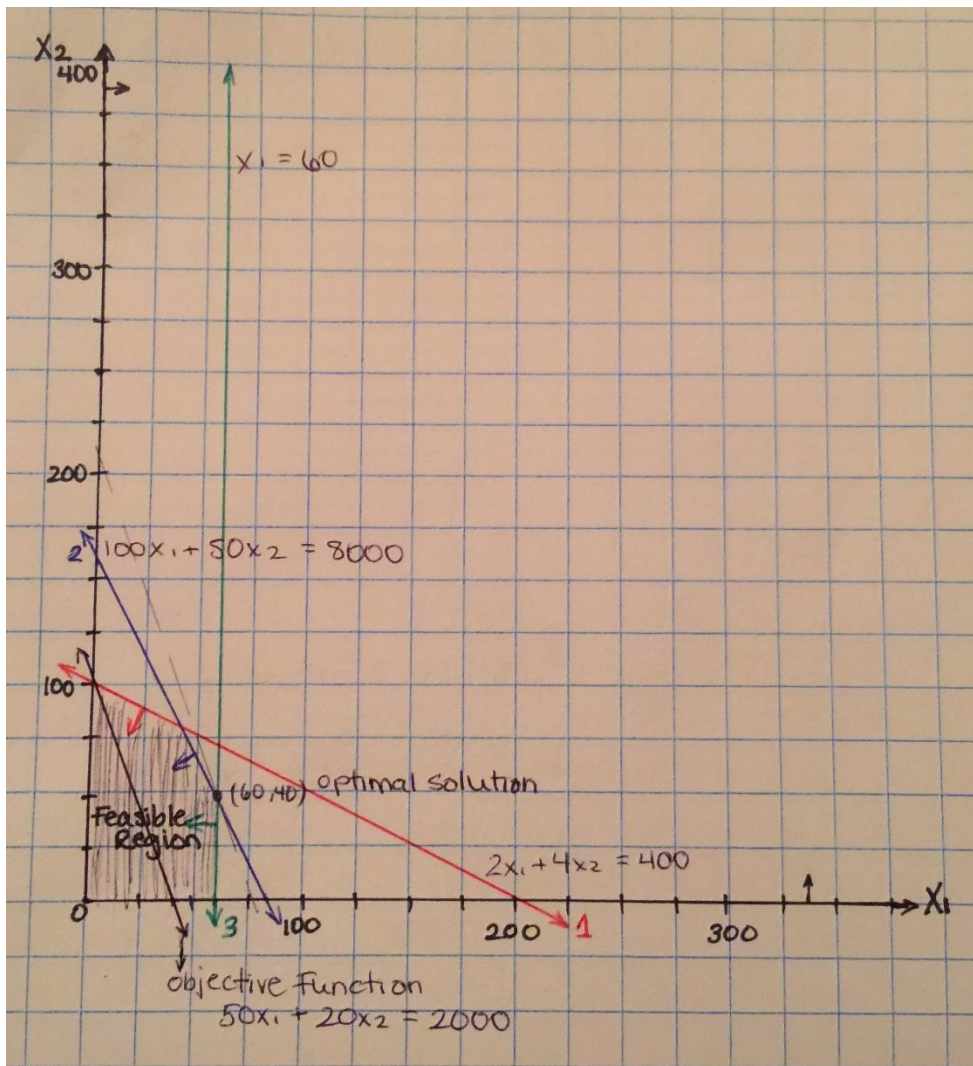
$100x_1 + 50x_2 \leq 8000$ (Space area in square meters)

$x_1 \leq 60$ (Rental limit expected)

$x_1, x_2 \geq 0$

- a) The 50 in the objective function is used to portray the profit of large storage rooms. In this case it's \$50 in rent per large room.
- b) The $20x_2$ in the objection function is also used to portray profits, but in this case the 20 is portraying \$20 while the x_2 relates to the number of small storage rooms for rent. For each other small room built you are making an extra \$20 in profit.
- c) The second constraint is based on the space in square meters that's available. The 8000 in the constraint portrays the total amount of square meters available. In this case, they can have up to and including 8000 square meters, however, not anything above that.
- d) The 100 in the second constraint represents the area of a large room in square meters, meaning each large room consists of 100 square meters.
- e) The $2x_1$ in the advertising budget represents the cost of advertising needs per large room.
- f)

	x1	x2			
Solution	60	40			
Profit	50	20		Total	3800
Constraints			LHS	Sign	RHS
Advertising	2	4	280	<=	400
Sq Meters	100	50	8000	<=	8000
Rental limit	1		60	<=	60



g) The optimal solution is the point of intersection between constraint #1 and #2.

$$x_1 = 60 \text{ and } 100x_1 + 50x_2 = 8000$$

$$100(60) + 50x_2 = 8000$$

$$50x_2 = 2000$$

$$x_2 = 40$$

Point of intersection (60,40).

In order for Personal Mini Warehouses to maximize its monthly earnings and constrain their costs to maximize profits they must produce a certain amount of small and large rental rooms. In this case, they should be looking at producing 60 large rental rooms and 40 small rental rooms in order to maximize their monthly profits.

- h) As shown above in the solution chart, only the advertising constraint consists of slack. The slack in this case is $RHS - LHS$ which is 120.
- i) You would look to make the objective function parallel to constraint #2. If x_1 remains at 50 then in order to have the same multiple as constraint #2, which is 100, you'd multiply it by 2. Since x_2 in this case would already be a multiple, you look to divide the 50 by 2. Therefore to have an objective function parallel to a constraint, the objective function would have to be $50x_1 + 25x_2$ or any other multiple of that.

Problem #2

$$\text{Minimize } Z = 3x_1 + 6x_2$$

Subject To:

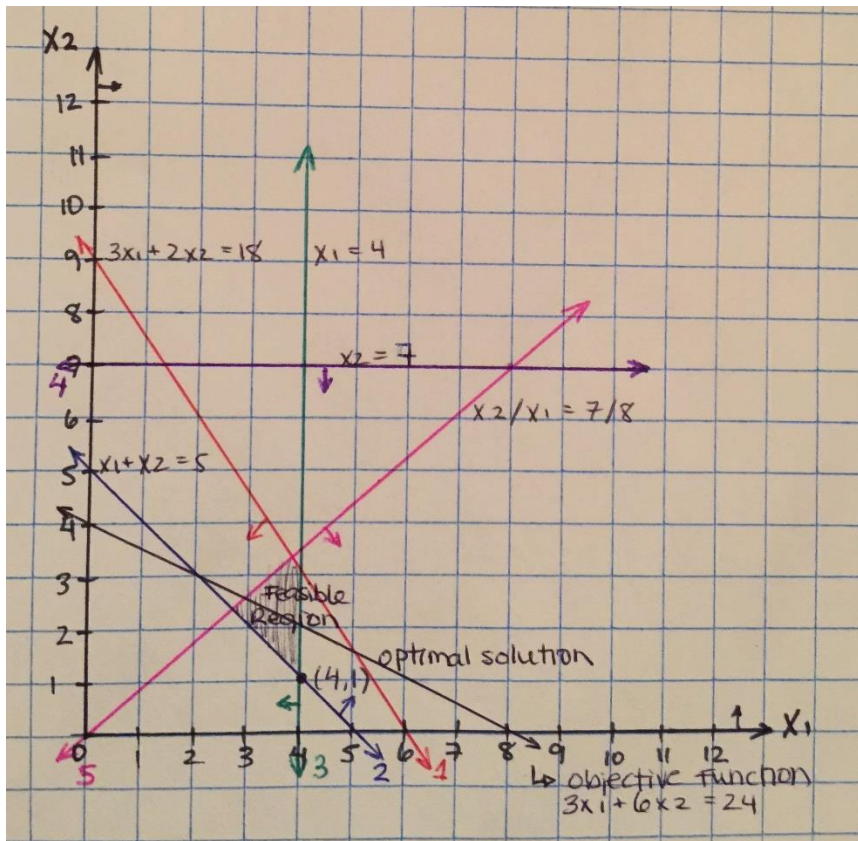
$$2x_1 + x_2 \geq 40$$

$$-6x_1 + 8x_2 \leq 120$$

$$70x_1 + 105x_2 \geq 2100$$

$$x_1, x_2 \geq 0$$

a)



b) The optimal solution is the point of intersection between constraint #2 and #3.

$$x_1 = 4 \text{ and } x_1 + x_2 = 5$$

$$4 + x_2 = 5$$

$$x_2 = 1$$

Point of intersection (4,1).

c) The effect on the solution if constraint #4 is changed from $x_2 \leq 7$ to $x_2 \geq 7$ is that there wouldn't be any optimal solution. It would contradict with the other constraints making it impossible to solve.

Problem #3

Maximize $Z = 110x_1 + 75x_2$

Subject To:

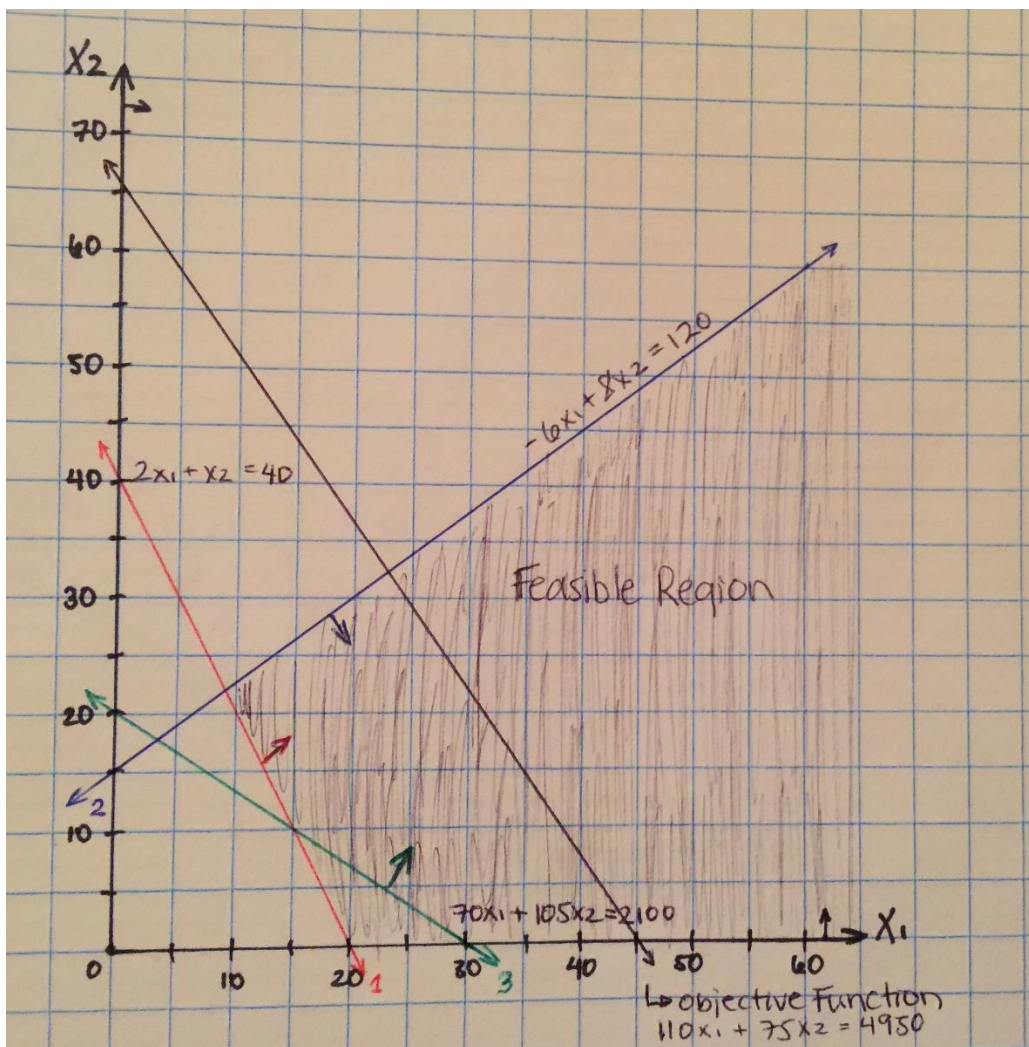
$$2x_1 + x_2 \geq 40$$

$$-6x_1 + 8x_2 \leq 120$$

$$70x_1 + 105x_2 \geq 2100$$

$$x_1, x_2 \geq 0$$

a)



- b) As shown in the graph, there's no constraint to close off the feasible region meaning nothing prevents the solution from being infinitely large. This would be considered an unbounded solution.

Problem #4

a) Decision Variables:

x_1 = Number of permanent operators needed to hire

x_2 = Number of temporary operators needed to hire

Objective Function:

Minimize $Z = 64x_1 + 42x_2$ (Cost)

Subject To:

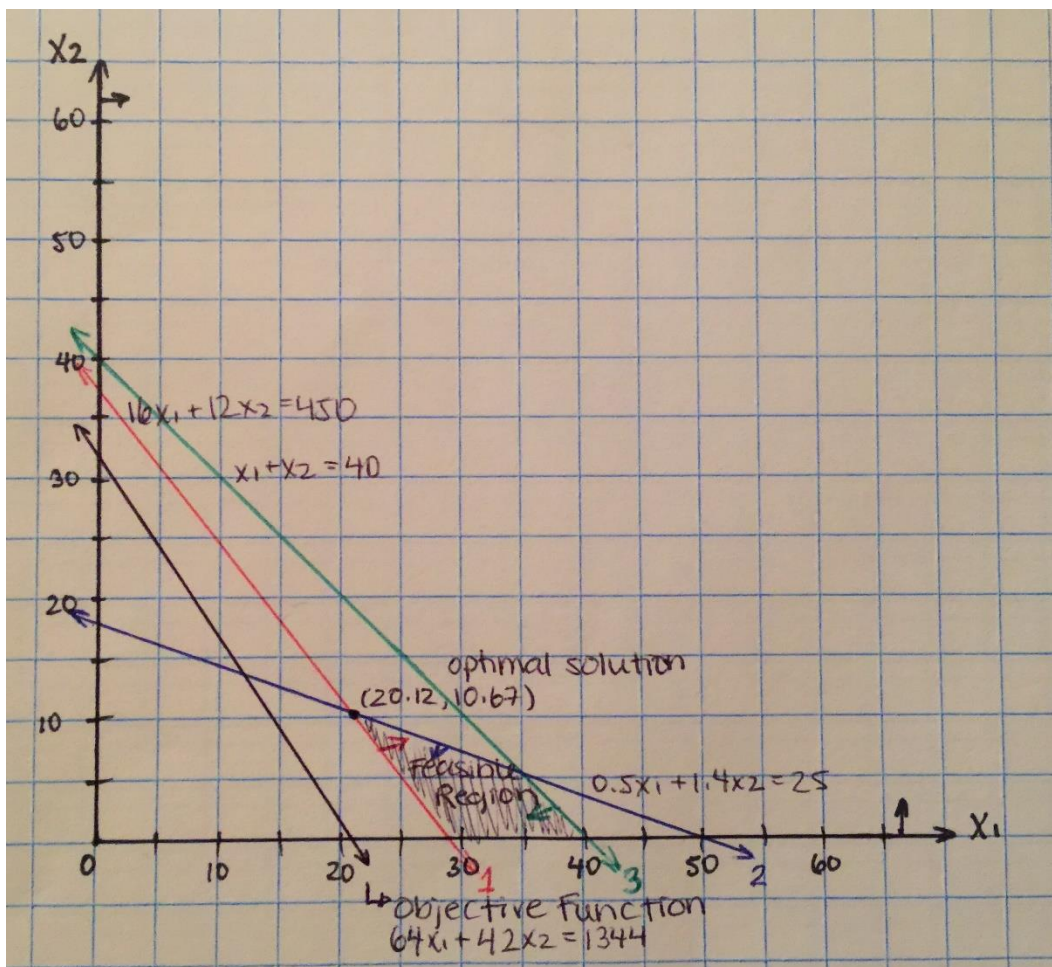
$16x_1 + 12x_2 \geq 450$ (Processes made per day)

$0.5x_1 + 1.4x_2 \leq 25$ (Errors made per day)

$x_1 + x_2 \geq 40$ (Work stations available)

$x_1, x_2 \geq 0$ (Non-negativity)

b)



	x1	x2			
Solution	20.12195	10.67073			
				Total	1735.976
Profit	64	42			
Constraints			LHS	Sign	RHS
Process	16	12	450	>=	450
Errors	0.5	1.4	25	<=	25
Stations	1	1	30.79268	<=	40

Microsoft Excel 15.0 Answer Report

Worksheet: [Book1]Sheet2

Report Created: 2016-10-07 7:47:14 PM

Result: Solver found a solution. All Constraints and optimality conditions are satisfied.

Solver Engine

Engine: Simplex LP

Solution Time: 0.079 Seconds.

Iterations: 2 Subproblems: 0

Solver Options

Max Time Unlimited, Iterations Unlimited, Precision 0.000001

Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%,

Assume NonNegative

Objective Cell (Min)

Cell	Name	Original Value	Final Value
\$F\$4	Total	2437.777778	1735.97561

Variable Cells

Cell	Name	Original Value	Final Value	Integer
\$B\$3	Solution x1	34.44444444	20.12195122	Contin
\$C\$3	Solution x2	5.555555556	10.67073171	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$D\$10	Stations LHS	30.79268293	\$D\$10<=\$F\$10	Not Binding	9.207317073
\$D\$8	Process LHS	450	\$D\$8>=\$F\$8	Binding	0
\$D\$9	Errors LHS	25	\$D\$9<=\$F\$9	Binding	0

- c) The optimal solution is the point of intersection between constraint #1 and #2.

$$16x_1 + 12x_2 = 450 \text{ and } 0.5x_1 + 1.4x_2 = 25$$

Multiply the second constraint by 32 and do the process of elimination.

$$16x_1 + 12x_2 = 450 -$$

$$\underline{16x_1 + 44.8x_2 = 800}$$

$$- 32.8x_2 = - 350$$

$$x_2 = 10.67 \quad \text{sub into constraint \#1}$$

$$16x_1 + 12(10.67) = 450$$

$$16x_1 = 321.95$$

$$x_1 = 20.12$$

Point of intersection (20.12, 10.67)

Universal Claims Processors processes insurance claims for large national insurance companies and is looking to determine the number of permanent and temporary operators to hire in order to minimize company costs. After calculating to find their optimal solution, which in this case would be the amount of workers to hire to gain profit but still obtain a low cost, the conclusion would be to hire 21 permanent workers and 11 temporary workers. The company wouldn't be able to fractions of a person and hiring less workers would mean that not all the constraints are justified or that there will be a lack of workers. For this reason alone the company would need to round up their numbers.

- d) If the pay for permanent workers is to change from \$64 to \$54 there wouldn't be a change in the optimal solution. The company would still have to hire 21 permanent workers and 11 temporary works. However, the optimal value or cost the company will have decreased since you're paying the same amount of employees just less money. The cost would change from \$1,735.976 to \$1,534.756.
- e) If the pay for temporary workers is to change from \$42 to \$36 there wouldn't be a change in the optimal solution. The company would still have to hire 21 permanent workers and 11 temporary works. However, the optimal value or cost the company will have decreased since you're paying the same amount of employees just less money. The cost would change from \$1,735.976 to \$1,671.951.
- f) After reviewing the linear programming model and using excel solver, as shown below, the conclusion is that the company shouldn't look at hiring any permanent workers. There's no benefit and in order to achieve the optimal solution which in this case is \$1,575 they should look to only hire 38 temporary workers. Again as stated before the company should always round up their numbers as you can't hire a fraction of a person.

	x1	x2			
Solution	0	37.5			
				Total	1575
Profit	64	42			
Constraints			LHS	Sign	RHS
Process	16	12	450	>=	450
Stations	1	1	37.5	<=	40

Microsoft Excel 15.0 Answer Report

Worksheet: [Book1]Sheet5

Report Created: 2016-10-07 7:53:54 PM

Result: Solver found a solution. All Constraints and optimality conditions are satisfied.

Solver Engine

Engine: Simplex LP

Solution Time: 0.062 Seconds.

Iterations: 2 Subproblems: 0

Solver Options

Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling

Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%,

Assume NonNegative

Objective Cell (Min)

Cell	Name	Original Value	Final Value
\$F\$3	Total	0	1575

Variable Cells

Cell	Name	Original Value	Final Value	Integer
\$B\$2	Solution x1	0	0	Contin
\$C\$2	Solution x2	0	37.5	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$D\$7	Process LHS	450	\$D\$7>=\$F\$7	Binding	0
\$D\$8	Stations LHS	37.5	\$D\$8<=\$F\$8	Not Binding	2.5

Personal Ethics Statement

Individual Assignment:

By signing this Statement, I am attesting to the fact that I have reviewed the entirety of my attached work and that I have applied all the appropriate rules of quotation and referencing in use at the Telfer School of Management at the University of Ottawa, as well as adhered to the fraud policies outlined in the Academic Regulations in the University's Undergraduate Studies Calendar. [Academic Fraud Webpage](#)

<u>Bianca Chitiu, BC</u>	<u>Sunday, October 9, 2016</u>
Signature	Date
<u>Chitiu Bianca</u>	<u>7829013</u>
Last Name (print), First Name (print)	Student Number