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PART I: Multiple Choice Questions.

1. Let $A = \begin{bmatrix} 1 & 2 & -3 \\ -1 & 3 & 1 \\ 4 & 2 & -5 \end{bmatrix}$ and let $B = \begin{bmatrix} -1 & 2 & 5 & -3 \\ 4 & -2 & 1 & 6 \\ -2 & 1 & 3 & 2 \end{bmatrix}$.

   What is the third column of AB?

   a) $\begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}$ **b)** $\begin{bmatrix} -8 \\ 2 \\ 4 \end{bmatrix}$ **c)** $\begin{bmatrix} -2 \\ -8 \\ 4 \end{bmatrix}$ **d)** $\begin{bmatrix} 13 \\ -8 \\ -2 \\ 21 \end{bmatrix}$

2. Let $A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ -1 & 3 & 2 & -2 \\ 2 & -1 & 3 & 2 \end{bmatrix}$ and let $B = \begin{bmatrix} -1 & 2 & 5 & -3 \\ 4 & -2 & 1 & 6 \\ 3 & -1 & 3 & 2 \\ 2 & -3 & 2 & 1 \end{bmatrix}$.

   What is the second row of AB?

   a) $\begin{bmatrix} 10 & 10 & 8 & 8 \end{bmatrix}$ **b)** $\begin{bmatrix} 10 & -10 & 10 & 10 \end{bmatrix}$ **c)** $\begin{bmatrix} 15 & -4 & 0 & 23 \end{bmatrix}$ **d)** $\begin{bmatrix} 7 & -3 & 22 & -4 \end{bmatrix}$

3. Consider the following augmented matrix of a system of linear equations.

   $$\left[\begin{array}{cccc|c} 1 & 1 & -2 & 2 & 3 \\ -1 & 0 & 3 & -3 & -4 \\ 1 & 1 & -1 & k & 4 \end{array}\right]$$

   For what value(s) of k does the system have infinitely many solutions?

   a) There is no value of k for which the system has infinitely many solutions.
   b) For $k = 2$ the system has infinitely many solutions.
   c) For $k = -2$ the system has infinitely many solutions.
   d) For every real number k, the system has infinitely many solutions.

4. Consider the following augmented matrix of a system of linear equations.
   $\left[\begin{array}{ccc|c} 1 & 4 & 6 & 1 \\ 0 & 1 & 7 & -7 \\ 0 & 0 & k^2 - 64 & k+8 \end{array}\right]$. Which of the following statements is TRUE?

   a) There is a unique solution when $k = -8$.
   b) There are infinitely many solutions for $k = -8$.
   c) There is no value of k for which there is a unique solution.
   d) There is a unique solution when $k = 8$.

5. Find all the values of k for which the system with the following augmented matrix is consistent:
$$\left[\begin{array}{cc|c} 1 & -5 & 4 \\ 0 & (k+3)(k-8) & (k+3)(k-5) \end{array}\right].$$
a) $k \neq -3$ **b)** $k \neq 8$ **c)** $k = 8$ **d)** $k = -3, 8$

6. Consider the following augmented matrix of a system of linear equations:
$$\left[\begin{array}{cccc|c} 1 & 1 & -2 & 2 & 3 \\ 1 & -2 & -2 & 2 & 3 \\ 1 & 0 & -2 & 2 & 3 \\ 1 & 1 & -2 & -2 & 3 \end{array}\right].$$ Which of the following statements is TRUE?
a) The system has a unique solution
b) The system has infinitely many solutions with one parameter
c) The system has infinitely many solutions with two parameters
d) The system has no solutions

7. Consider the matrix equation $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 5 & 4 \\ 3 & 6 & k \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} h \\ 7 \\ 9 \end{bmatrix}$.
The above matrix equation has a UNIQUE solution if
a) $k = 6$ and $h \neq 3$ **b)** For all k in R and $h \neq 3$.
c) $k \neq 6$ and for all h in R. **d)** $k = 6$ and $h = 3$.

8. Let $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, $v_4 = \begin{bmatrix} 5 \\ -4 \\ -8 \end{bmatrix}$.
Which of the following statements is TRUE?
a) v_3 is in Span$\{v_1, v_2\}$ **b)** v_4 is in Span$\{v_1, v_2\}$
c) v_1 is in Span$\{v_2, v_3\}$ **d)** v_4 is in Span$\{v_1, v_3\}$

9. Let $v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 3 \\ 2 \\ 4 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix}$, $v_4 = \begin{bmatrix} 5 \\ 1 \\ -3 \\ h \end{bmatrix}$.
For what value(s) of h is the vector v_4 in Span$\{v_1, v_2, v_3\}$?
a) $h = -10$.
b) $h = 10$.
c)There is no value of h for which the vector v_4 is in Span$\{v_1, v_2, v_3\}$.
d) For every real number h, the vector v_4 is in Span$\{v_1, v_2, v_3\}$.

10. What is the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & -2 \end{bmatrix}$?

a) $\begin{bmatrix} -2 & -1 & -1 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ **b)** $\begin{bmatrix} 2 & -1 & 1 \\ -2 & 2 & -1 \\ -1 & 1 & -1 \end{bmatrix}$ **c)** $\begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ **d)** $\begin{bmatrix} -2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & 1 & -1 \end{bmatrix}$

11. Let A be a 5×4 matrix such that its reduced row echelon form has 4 pivot positions (leading entries). Which of the following statements is TRUE?

a) The linear transformation T defined by $T(x) = Ax$ is onto.
b) $Ax = 0$ has a unique solution.
c) Columns of A are linearly dependent.
d) $Ax = b$ is consistent for every vector b in R^5.

12. If $\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}^{-1} \cdot A = \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix}$, what is A?

a) $\begin{bmatrix} 5 & 17 \\ 1 & 3 \end{bmatrix}$ **b)** $\begin{bmatrix} 0 & 2 \\ 1 & 2 \end{bmatrix}$ **c)** $\begin{bmatrix} 5 & 1 \\ 17 & 3 \end{bmatrix}$ **d)** $\begin{bmatrix} 1 & 2 \\ 3 & 17 \end{bmatrix}$

13. If $A \cdot \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ -1 & 3 \end{bmatrix}$, what is A?

a) $\begin{bmatrix} -4 & -3 \\ 5 & 6 \end{bmatrix}$ **b)** $\begin{bmatrix} 4 & -3 \\ 5 & -6 \end{bmatrix}$ **c)** $\begin{bmatrix} -4 & 3 \\ -5 & 6 \end{bmatrix}$ **d)** $\begin{bmatrix} 4 & 3 \\ -5 & -6 \end{bmatrix}$

14. Let $T : R^2 \to R^2$ be the linear transformation which rotates vectors $\pi/3$ radians counter-clockwise. Which of the following matrices is the standard matrix of T?

a) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ **b)** $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ **c)** $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ **d)** $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

15. Let $A = \begin{bmatrix} 1 & 1 & 2 & 2 & 3 \\ 2 & 1 & 4 & 3 & 2 \\ 1 & 0 & 2 & 1 & -1 \\ 1 & 1 & 2 & 2 & 2 \end{bmatrix}$.

What is the general solution x of the matrix equation $Ax = 0$?

a) $\begin{bmatrix} t \\ -2s-t \\ s \\ t \\ 0 \end{bmatrix}$ **b)** $\begin{bmatrix} 2s+t \\ -t \\ s \\ 0 \\ t \end{bmatrix}$ **c)** $\begin{bmatrix} -2s-t \\ t \\ -s \\ 0 \\ t \end{bmatrix}$ **d)** $\begin{bmatrix} -2s-t \\ -t \\ s \\ t \\ 0 \end{bmatrix}$ $(s,\ t \in R)$

16. Let $s\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} + t\begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix}$ be the general solution of a homogeneous system $Ax = 0$, and let $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ be a particular solution of a non-homogeneous system $Ax = b$ with the same coefficient matrix A. Which of the following is a solution of $Ax = b$?

    (I) $\begin{bmatrix} 3 \\ 1 \\ 6 \end{bmatrix}$ (II) $\begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix}$

    a) Both (I) and (II) **b)** (I), but not (II) **c)** (II), but not (I) **d)** Neither (I) nor (II)

17. Let $T : R^2 \longrightarrow R^3$ be a linear transformation such that

    $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 0 \\ -3 \end{bmatrix}$ and $T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ 2 \\ -5 \end{bmatrix}$. What is $T\left(\begin{bmatrix} 3 \\ 1 \end{bmatrix}\right)$?

    a) $\begin{bmatrix} 3 \\ 2 \\ 11 \end{bmatrix}$ **b)** $\begin{bmatrix} -3 \\ -2 \\ 11 \end{bmatrix}$ **c)** $\begin{bmatrix} 3 \\ 2 \\ -11 \end{bmatrix}$ **d)** $\begin{bmatrix} 2 \\ -11 \\ 3 \end{bmatrix}$

18. Let $T : R^4 \longrightarrow R^3$ the linear transformations defined by $T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} x - y + z + 2w \\ 2x + y + 5z - w \\ x + 2y + 3z + w \end{bmatrix}$.

    Which of the following statements about T is TRUE?

    a) T is onto but not one-to-one.
    b) T is one-to-one but not onto.
    c) T is one-to-one and onto.
    d) T neither one-to-one nor onto.

19. Let A be a 4×4 matrix such that $\det A = 0$.
    Exactly one of the following statements is TRUE. Which one?

    a) The linear transformation $T(x) = Ax$ is onto but not one-to-one.
    b) Each column of A is a linear combination of the other columns of A.
    c) Rank of A is 3.
    d) Rank$A \leq 3$.

20. Suppose that A is a 5×5 matrix, $\det A = 0$, and b is a vector in R^5.

    Which of the following statements is TRUE?

    a) $Ax = b$ has exactly one solution.
    b) $Ax = b$ has no solution.
    c) $Ax = b$ has infinitely many solution.
    d) $Ax = b$ has either no solution or infinitely many solutions.

21. Let $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 0 & 4 \\ 3 & 2 & 1 \end{bmatrix}$. What is $\det A$?

    a) 32 **b)** -32 **c)** 16 **d)** -16

22. Let A be a 4×4 matrix such that $\det A = 9$. What is $\det(3A)$?

    a) 3^2 **b)** 3^4 **c)** 3^6 **d)** 3^8

23. Let A and B be two 4×4 matrices such that $\det A = 9$ and $\det B = 3$.

    What is $\det(3AB^{-1}A^T)$?

    a) 3^7 **b)** 3^6 **c)** 3^5 **d)** 3^4

24. Let $A = \begin{bmatrix} 1 & 2 & k \\ 2 & k & 8 \\ k & 0 & 0 \end{bmatrix}$. Find all the values of k such that $\det A = 0$.

    a) $k = -4, 0, 1$ **b)** $k = -4,\ 0,\ 4$ **c)** $k = -2,\ 1, 4$ **d)** $k = -2, 0, 2$

25. Let $A = \begin{bmatrix} k & 6 & 7 \\ 0 & k & 4 \\ 0 & 9 & k \end{bmatrix}$. Find all the values of k such that $\det A = 0$.

    a) $k = -6, 0$ **b)** $k = 6,\ 0$ **c)** $k = -6,\ 6$ **d)** $k = -6, 0, 6$

26. Let A and B be 3×3 matrices such that $\det A = 3$, and $\det B = -4$.

    What is $\det\left((2A)^{-1}B^2B^T\right)$?

    a) 8 **b)** 24 **c)** $-8/3$ **d)** $8/3$

27. Let $u_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$, $u_3 = \begin{bmatrix} 2 \\ 4 \\ 2 \\ 2 \end{bmatrix}$, $u_4 = \begin{bmatrix} 2 \\ 3 \\ 1 \\ 2 \end{bmatrix}$ and $H = \text{Span}\{u_1, u_2, u_3, u_4\}$.

    What is the dimension of the subspace H?

    a) 1 **b)** 2 **c)** 3 **d)** 4

28. Which of the followings are a subspace of R^3?

(I) $\left\{ \begin{bmatrix} a+b \\ 2a-3c \\ b+5c \end{bmatrix} \middle| a,b,c \in R \right\}$ (II) $\left\{ \begin{bmatrix} a-b \\ 3 \\ b+3c \end{bmatrix} \middle| a,b,c \in R \right\}$ (III) $\left\{ \begin{bmatrix} a-2b \\ 5b+c \\ 0 \end{bmatrix} \middle| a,b,c \in R \right\}$

a) (III) only **b)** (II) and (III) only **c)** (I) and (III) only **d)** (I) and (II) only

29. Let $H = \left\{ \begin{bmatrix} a-2b+7c \\ 3a+b+7c \\ 2a-3b+12c \\ 4a+2b+8c \end{bmatrix} \middle| a,b,c \in R \right\}$.

Which of the following sets is a basis for H?

a) $\left\{ \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -3 \\ 1 \end{bmatrix} \right\}$ **b)** $\left\{ \begin{bmatrix} 1 \\ 3 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ -3 \\ 2 \end{bmatrix} \right\}$ **c)** $\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 2 \\ 1 \end{bmatrix} \right\}$ **d)** $\left\{ \begin{bmatrix} 3 \\ 1 \\ -2 \\ 1 \end{bmatrix} \right\}$

30. What is the coordinate vector of $x = \begin{bmatrix} -7 \\ 4 \\ -3 \end{bmatrix}$ relative to basis $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \right\}$?

a) $\begin{bmatrix} -4 \\ 3 \\ 2 \end{bmatrix}$ **b)** $\begin{bmatrix} 2 \\ 3 \\ -4 \end{bmatrix}$ **c)** $\begin{bmatrix} -3 \\ 2 \\ -4 \end{bmatrix}$ **d)** $\begin{bmatrix} 3 \\ -2 \\ -4 \end{bmatrix}$

31. Suppose A is a 3×3 matrix that is **NOT** invertible. Which of the following could not be the characteristic polynomial of A?

a) $-\lambda(\lambda-1)^2$ **b)** $-\lambda^2(\lambda+2)$ **c)** $-\lambda(\lambda^2-4)$ **d)** $-(\lambda-2)^3$

32. Let $A = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 4 \end{bmatrix}$. What is the characteristic equation of the matrix A?

a) $\lambda^3 - 5\lambda^2 - 6\lambda - 4 = 0$ **b)** $\lambda^3 + 5\lambda^2 - 5\lambda + 4 = 0$

c) $\lambda^3 - 6\lambda^2 + 6\lambda + 4 = 0$ **d)** $\lambda^3 + 7\lambda^2 - 5\lambda - 4 = 0$

33. Let $A = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{bmatrix}$. What is the characteristic polynomial of the matrix A?

a) $\lambda^3 - 6\lambda^2 - 3\lambda - 10$ **b)** $-\lambda^3 + 6\lambda^2 - 3\lambda - 10$

c) $\lambda^3 + 6\lambda^2 - 3\lambda$ **d)** $-\lambda^3 + 6\lambda^2 + 3\lambda$

34. Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 2 & k \\ 2 & 0 & 4 \end{bmatrix}$. For what value of k is the rank of A equal to 2?

a) 2   **b)** 3   **c)** 4   **d)** 5

35. Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$. What are the eigenvalues of A?

a) 3 and 1   **b)** 3 and -1 **c)** -3 and 1   **d)** -3 and -1

36. Let $A = \begin{bmatrix} 1 & -3 \\ 2 & 1 \end{bmatrix}$. What are the eigenvalues of A?

a) 1 and 6   **b)** $1 \pm 3i$ **c)** $1 \pm \sqrt{6}i$ **d)** 1 and -3

37. Let $A = \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}$. What are the eigenvalues of A?

a) $-3 \pm 4i$ **b)** $4 \pm 3i$ **c)** $-4 \pm 3i$ **d)** $3 \pm 4i$

38. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 5 \\ 0 & 1 & 8 \end{bmatrix}$. What are the eigenvalues of A?

a) $1,\ 4,\ 8$ **b)** $1,\ 4, 6$ **c)** $1,\ 3,\ 9$ **d)** $2,\ 5,\ 8$

39. Let A be a 2×2 matrix with eigenvectors $v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 3 \\ 9 \end{bmatrix}$ corresponding to eigenvalues $\lambda_1 = -3$ and $\lambda_2 = 3$, respectively.

What is the matrix A?

a) $\begin{bmatrix} -15 & 6 \\ -36 & 15 \end{bmatrix}$ **b)** $\begin{bmatrix} -15 & -36 \\ 6 & 15 \end{bmatrix}$ **c)** $\begin{bmatrix} -45 & 18 \\ -108 & 45 \end{bmatrix}$ **d)** $\begin{bmatrix} -15 & -12 \\ 18 & 15 \end{bmatrix}$

40. Let $\lambda_1 = 2$, $\lambda_2 = 4$ and $\lambda_3 = 5$ be three distinct eigenvalues of a 3×3 matrix A.

Which of the following statements is FALSE?

a) Each eigenspace of the matrix A is one-dimensional.
b) $\det A = 40$
c) The matrix equation $(A - 4I)x = 0$ has a unique solution.
d) The matrix A is diagonalizable.

41. Let $z = 2 - 2i$. What is z^{20} ?

a) -2^{15} **b)** 2^{30} **c)** $2^{15}\ i$ **d)** -2^{30}

42. Let $A = \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

Which of the following sets is a basis for the null space of A?

a) $\left\{ \begin{bmatrix} 2 \\ -1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 0 \\ 1 \end{bmatrix} \right\}$ **b)** $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ **c)** $\left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 0 \\ 1 \end{bmatrix} \right\}$ **d)** $\left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix} \right\}$

43. $\lambda = 3$ is an eigenvalue for the matrix $A = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 5 & -10 \\ 1 & 0 & 3 & 0 \\ 1 & 0 & 0 & 3 \end{bmatrix}$.

What is the dimension of the eigenspace for $\lambda = 3$?

a) 1   **b)** 2   **c)** 3   **d)** 4

44. $\lambda = 2$ is an eigenvalue for the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & -1 & 3 \\ 0 & 0 & 2 \end{bmatrix}$.

Which of the following is the eigenspace of A corresponding to $\lambda = 2$?

a) $\text{Span}\left\{ \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix} \right\}$ **b)** $\text{Span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} \right\}$ **c)** $\text{Span}\left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} \right\}$ **d)** $\text{Span}\left\{ \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \right\}$

45. You are given that $\lambda = 3$ is an eigenvalue of the matrix $A = \begin{bmatrix} 5 & 2 & -4 \\ 2 & 5 & -4 \\ 2 & 2 & -1 \end{bmatrix}$.

Which of the following sets is a basis for E_3?

a) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \right\}$ **b)** $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\}$ **c)** $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ **d)** $\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$

46. Which of the following statement is TRUE?

a) Every invertible matrix is diagonalizable.
b) Every diagonalixable matrix is invertible.
c) If an $n \times n$ matrix A is diagonalizable, then it must have n distinct eigenvalues.
d) If A is an $n \times n$ diagonalizable matrix, then it has n linearly independent eigenvectors.

47. Let $u_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$, $x = \begin{bmatrix} 3 \\ 0 \\ -9 \\ 6 \end{bmatrix}$ and $W = \text{Span}\{u_1, u_2\}$.

    What is the orthogonal projection of x on W?

    a) $\begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix}$ **b)** $\begin{bmatrix} 2 \\ 3 \\ -5 \\ 1 \end{bmatrix}$ **c)** $\begin{bmatrix} 3 \\ 2 \\ 1 \\ 5 \end{bmatrix}$ **d)** $\begin{bmatrix} 1 \\ -5 \\ -2 \\ 3 \end{bmatrix}$

48. Let $u_1 = \begin{bmatrix} 0 \\ 1 \\ -4 \\ -1 \end{bmatrix}$ $u_2 = \begin{bmatrix} 3 \\ 5 \\ 1 \\ 1 \end{bmatrix}$, $u_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ -4 \end{bmatrix}$, $u_4 = \begin{bmatrix} 5 \\ -3 \\ -1 \\ 1 \end{bmatrix}$ and $x = \begin{bmatrix} 10 \\ -8 \\ 2 \\ 0 \end{bmatrix}$.

    You are given that $B = \{u_1,\ u_2,\ u_3,\ u_4\}$ is an orthogonal basis for R^4.

    What is the value of c_2 in the equation $x = c_1u_1 + c_2u_2 + c_3u_3 + c_4u_4$?

    a) $-8/9$ **b)** $-2/9$ **c)** $2/3$ **d)** 2

49. Let $W = \text{Span}\left\{\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}\right\}$ and $x = \begin{bmatrix} 1 \\ -4 \\ 9 \end{bmatrix}$. What is the distance from x to W?

    a) $\sqrt{68}$ **b)** $\sqrt{24}$ **c)** $\sqrt{96}$ **d)** $\sqrt{44}$

50. Let $y = \begin{bmatrix} 9 \\ 7 \end{bmatrix}$ and $u = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

    What is the distance from y to the line through u and the origin?

    a) 20 **b)** $\sqrt{40}$ **c)** $\sqrt{116}$ **d)** 4

Answer Key

1.a, 2.c, 3.d, 4.b, 5.b, 6.b, 7.c, 8.d, 9.b, 10.b,
11.b, 12.a, 13.b, 14.c, 15.d,16.c, 17.c, 18.a, 19.d, 20.d,
21.b, 22.c, 23.a, 24.b, 25.d, 26.c, 27.b, 28.c, 29.b, 30.d,
31.d, 32.c, 33.b, 34.c, 35.b, 36.c, 37.d, 38.c, 39.a, 40.c,
41.d, 42.c, 43.b, 44.c, 45.b, 46.d, 47.d, 48.b, 49.d, 50.b.

PART II: Long Answer Questions.

1. Find the general solution of the following system of linear equations. Write the solution in parametric vector form.

$$\begin{aligned} x_1 - x_2 + x_3 + 2x_4 &= 1 \\ 2x_1 - 2x_2 + 3x_3 + 7x_4 &= 4 \\ 3x_1 - 3x_2 + 4x_3 + 9x_4 &= 5 \\ -2x_1 + 2x_2 - x_3 - x_4 &= 0 \end{aligned}$$

2. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$.

3. Let $A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ -1 & 2 & 4 & 9 \\ 4 & 8 & 0 & -4 \end{bmatrix}$.

   (a) Find a basis for Col$A$ and a basis for Nul$A$.

   (b) What are dimCol$A$ and dimNul$A$?

   (c) Verify that the Rank Theorem holds for the matrix A.

4. Let $A = \begin{bmatrix} 1 & 0 & -2 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & -4 & 5 \end{bmatrix}$. You are given that $\det(A - \lambda I) = (\lambda - 1)^3(\lambda - 3)$.

   (a) Find all the eigenvalues of the matrix A.

   (b) For each eigenvalue, find a basis for the corresponding eigenspace.

   (c) Find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

5. Let $W = \text{Span}\left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \right\}$.

   (a) Find a basis for W. What is the dimension of W?

   (b) Write $x = \begin{bmatrix} 7 \\ -4 \\ 13 \end{bmatrix}$ as a linear combination of the basis vectors of W, which you found in part (a).

6. Let $u_1 = \begin{bmatrix} -1 \\ 2 \\ 0 \\ 2 \end{bmatrix}$, $u_2 = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}$, $u_3 = \begin{bmatrix} -2 \\ -1 \\ 2 \\ 0 \end{bmatrix}$, $W = \text{Span}\{u_1,\ u_2,\ u_3\}$ and $x = \begin{bmatrix} -6 \\ 3 \\ 9 \\ 3 \end{bmatrix}$.

   (a) Show that $\{u_1,\ u_2,\ u_3\}$ is an orthogonal basis for the subspace W.

   (a) Find the orthogonal projection of the vector x onto W.

   (b) Write x as the sum of a vector in W and a vector orthogonal to W.

   (c) Find the distance from x to W.

7. Suppose A, B and C are $n \times n$ invertible matrices.
   Solve the equation $C^{-1}(A + X^T)B^{-1} = I$ for X, in other words, express X in terms of A, B, C, A^T, B^T and C^T.

8. Let $A = \begin{bmatrix} 5 & 3 & 1 & 1 \\ 0 & 2 & 2 & 3 \\ 4 & 0 & 0 & 1 \\ 0 & 2 & 1 & 8 \end{bmatrix}$, $b = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 8 \end{bmatrix}$ and $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$.

   Use Cramer's Rule to solve for x_4 (without solving for x_1, x_2 and x_3)

   in the matrix equation $Ax = b$.

9. Find the angle between the vectors $u = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ and $v = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$.

If you notice any typos or errors in the questions, please let me know.