

Question 1

- a) Using Boolean Algebra, minimize the following function (step by step):  
**i)  $F = A' + AB$ ; ii)  $F = (A' + B)(A + B')$ ; iii)  $F = BC + AC + A'B'C$**  (3marks)
- b) List all possible **prime implicants** and **essential prime implicants** for the function  $F = \Sigma(0, 2, 4, 5, 8, 12, 13, 15)$  with don't care conditions  $d = \Sigma(1, 3, 14)$ . Show groupings on a **Karnaugh map** accompanied with their algebraic forms. (4 marks)
- c) Express P and Q in Figure 1 as function of A, B and C and then simplify them. (3 marks)

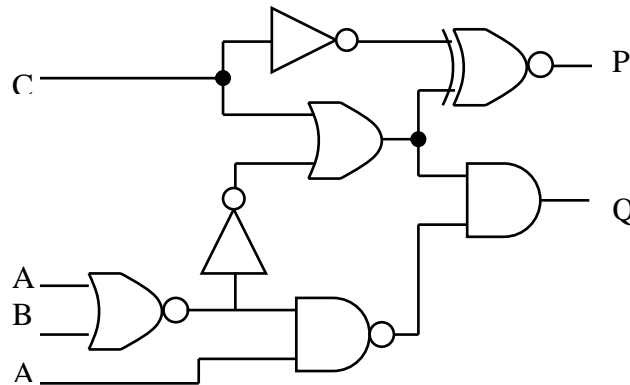
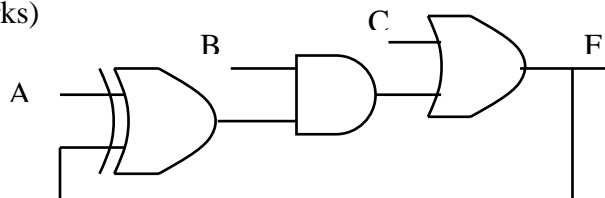


Figure 1

Question 2

- 2.1 Use DeMorgan's theorem to express the equivalence function  $F = xy + x'y'$  with:  
 i) NAND only. ii) NOR only  
 Show the different steps and the logic diagram. (4 marks)
- 2.2 Consider the circuit in Fig. 2. Draw the **timing diagram** of F when inputs A, B, and C go from 000 to 111 (binary counting) assuming each gate has a propagation delay of 10 ns. (4 marks). Assume F is initially 0.  
 Determine the minimum time between two input changes, for the circuit to operate properly. To help, you may add intermediate nodes to the timing diagram. (2 marks)



### Question 3

- A Function  $F$ , can be approximated by  $F = at^2 + b$  where  $a=2$ ,  $b=1$  and  $t$  is 2 bit binary input that can take a value of 0,1 or 2 only. Design and implement the circuit. (4 marks)
- Design a 4-bit Binary Squarer. (i.e, the circuit has one input  $A=a_3a_2a_1a_0$  and the output is the  $A^2$ ). (6 marks)

### Question 4

- What is the difference between ROM, PROM, PAL, and the PLA. (2 marks)
- A sequential circuit is to be implemented using **2 JK Flip Flops and a PLA**. The excitation equations are:  
$$J_A = Bx' \quad K_A = Bx$$
$$J_B = x \quad K_B = (A \oplus x)$$
Where  $A$  and  $B$  are the present states and  $x$  is an external input. Implement the circuit, giving PLA personality matrix (programming table) and showing your programming of the PLA clearly by placing an  $\bullet$  on the programmable nodes. (5 marks)
- Implement the following function  $F$ , using only 2 to 1 multiplexers:  
 $F = AB + CD$  (3marks)

### Question 5

As an engineer, you have been contracted by the ACME Candy Corporation to design the control circuitry for a 15-cent candy bar machine. The machine accepts only nickels (5-cents) and dimes (10-cents). There will be two inputs to your control system:  $N$  and  $D$ . when a nickel is put into the machine, the input signal  $N$  is asserted high. When a dime is inserted in the machine, the input signal  $D$  is asserted high. The control unit must be a synchronous sequential circuit. You may assume that the inputs ( $N$  and  $D$ ) will be logic high for exactly one clock period.

The control unit will produce two outputs. When 15-cents or more is placed into the machine, an output  $CB$  (Candy Bar) will be set to logic 1 and the unit will return to idle state. When more than 15-cents (i.e. 20-cents,) has been deposited into the machine, a signal  $CH$  (Change) will be asserted to return (Change) to the vendor and the unit will return to idle state waiting for the next transaction.

The ACME Candy Corporation requires that you construct the control unit using only D-type flip-flops and any other combinational logic.

It is not physically possible for the  $N$  and  $D$  inputs to be '1' at the same time. However, if this ever occurs your system must return to the "initial" state. Furthermore, an active low reset should be incorporated into your system at the circuit level, resetting to the "initial" state. (10 marks)

- Draw the State Transition Diagram for the candy bar control unit.
- Derive the State Transition Table.
- Draw the sequential circuit diagram.

Question 6

Analyze the circuit shown in Figure 3. Derive the state transition table and diagram including all inputs and outputs. (10 marks)

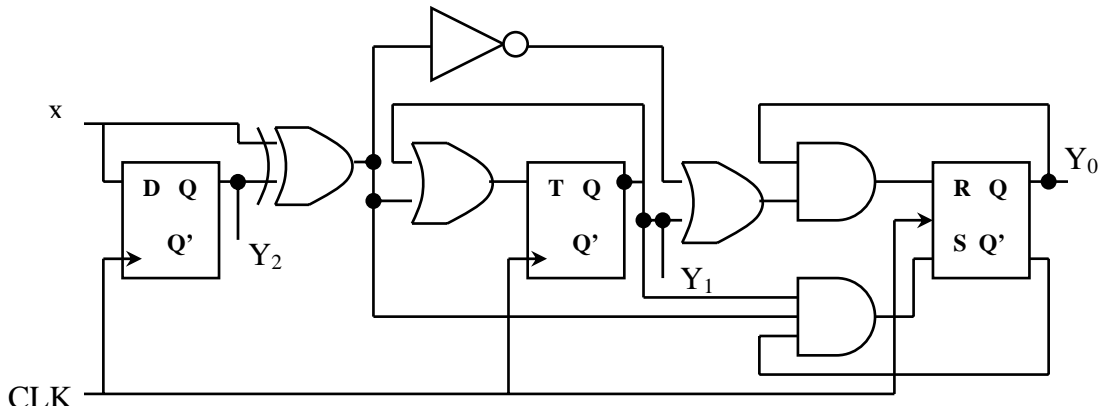
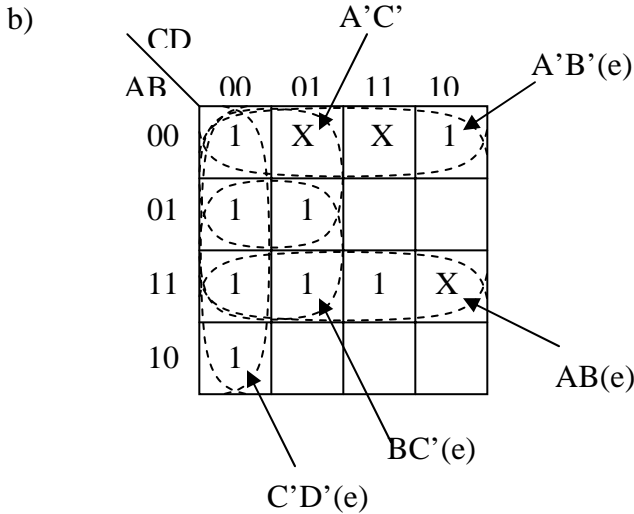


Figure 3

## Solutions

### Question 1

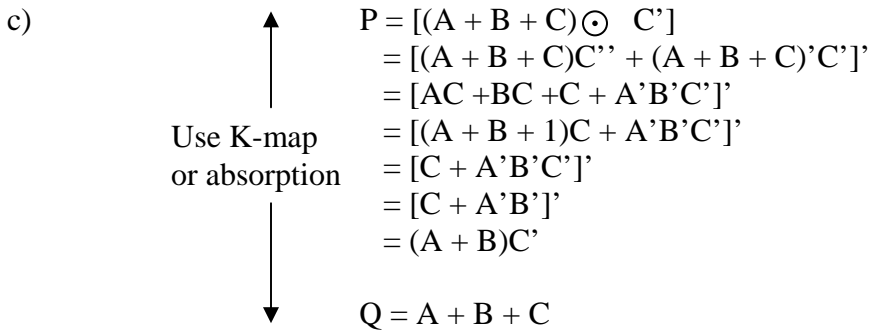
- a) i)  $F = A' + AB = (A' + A)(A' + B) = 1 \bullet (A' + B) = A' + B$  (1/1)  
 ii)  $F = (A' + B)(A + B') = A'A + BA + A'A' + BB' = AB + A'B'$  (1/1)  
 iii)  $F = BC + AC + A'B'C = C(B + A + A'B') = C(B + A + B')$   
 $= C(A + (B + B')) = C$  (1/1)



Map: (1/1) (minterms & X correct)  
 Grouping: (1/1) (graphical)  
 Algebraic P.I.: (1/1)  
 Essentials: (1/1)

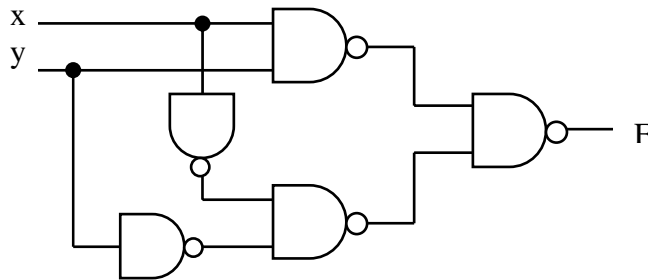
5 possible P.I. + 2 possible P.I. if X = 0  
 4 essential P. I. (AB, A'B', C'D', BC')

May be more essentials depending on function coverage of P.I.



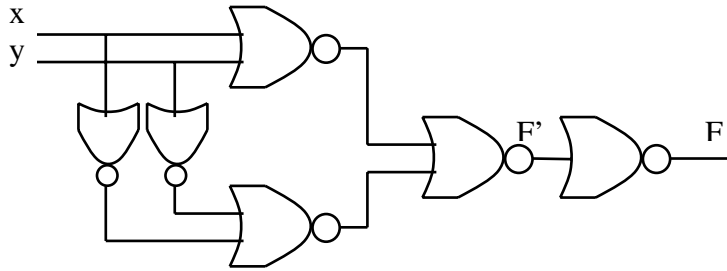
### Question 2

2.1  $F = xy + x'y' = [xy + x'y']'' = [(xy)'(x'y')']'$  (1/1)

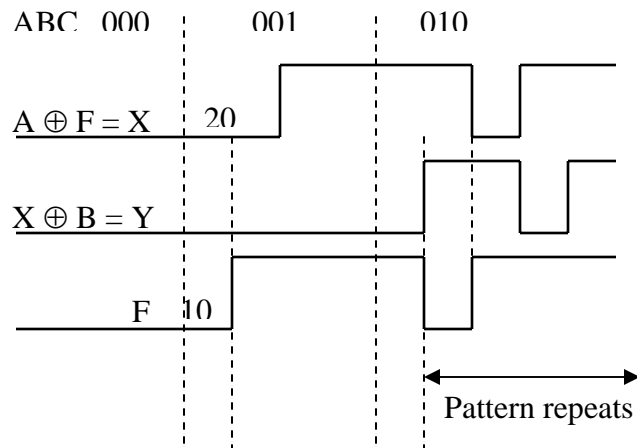


$$\begin{aligned}
 F &= xy + x'y' = [(x' + y')(x + y)]' \\
 &= (x' + y')' + (x + y)'
 \end{aligned}$$

$$F' = [(x' + y)'] + (x + y)']'$$



2.2 (1/1)



F unknown for rest of combination. F unstable.  $t_{\min} = \infty$ . (2/2)

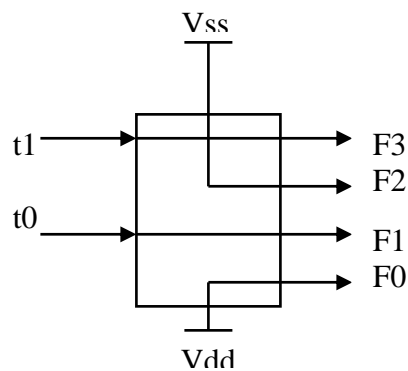
### Question 3

a)

t		F			
t1	t0	F3	F2	F1	F0
0	0	0	0	0	1
0	1	0	0	1	1
1	0	1	0	0	1
1	1	X	X	X	X

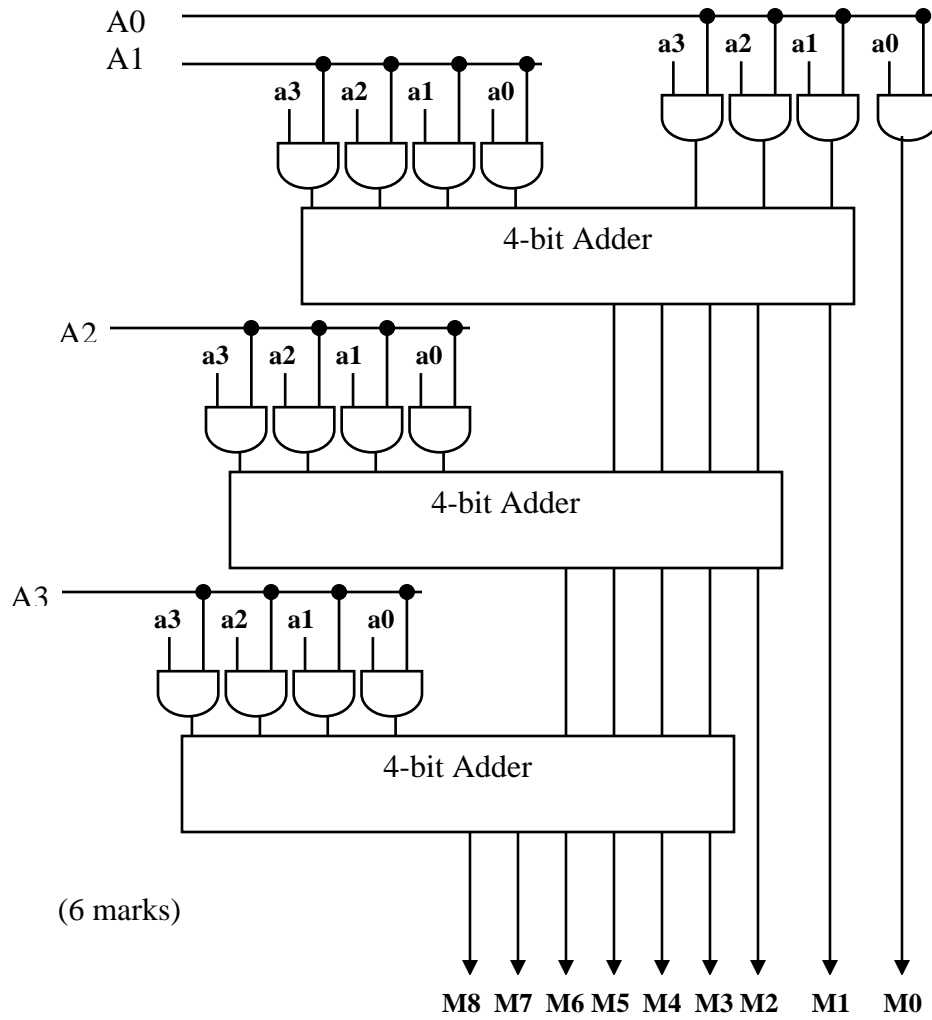
$F0 = 1$   
 $F1 = t0$   
 $F2 = 0$   
 $F3 = t1$

$F_{\text{minimum value}} = 1$   
 $F_{\text{maximum value}} = 9$  4 bit binary bits



(4 marks)

c) We can use a 4-bit multiplier with both input connected as A



**Question 4**

a) (2 marks)

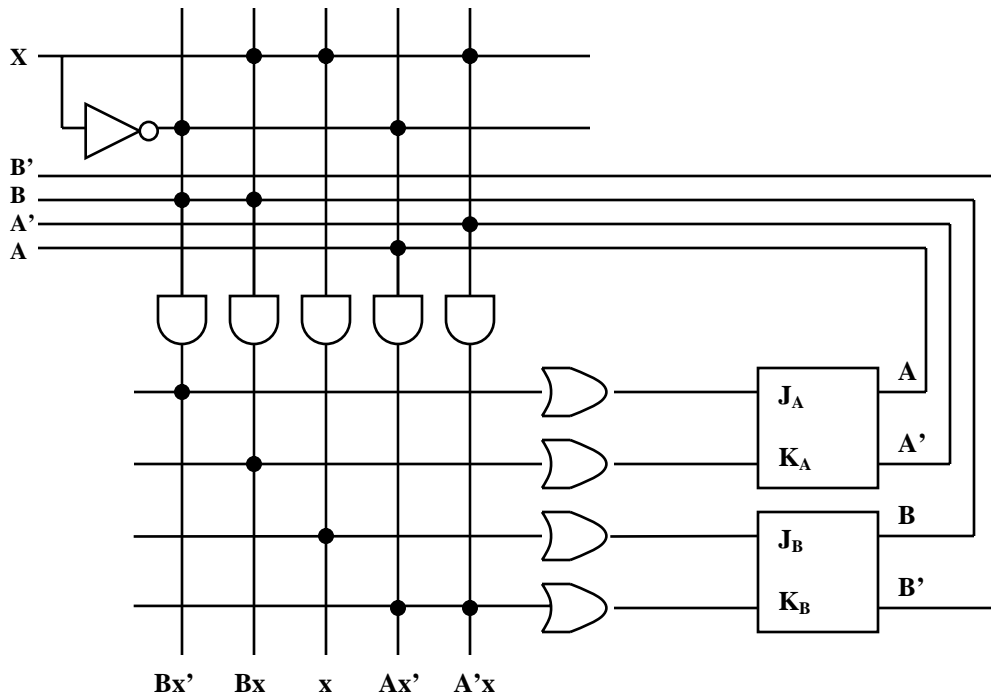
	AND gate	OR gate
ROM	Fixed	Fixed
PROM	Fixed	Programmable
PAL	Programmable	Fixed
PLA	Programmable	Programmable

b) (5 marks)

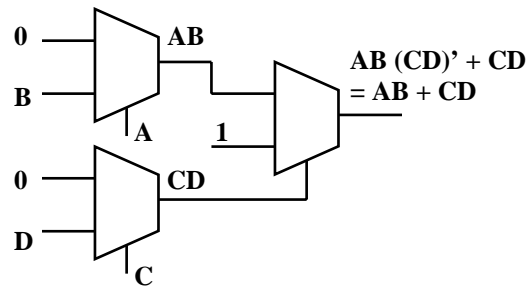
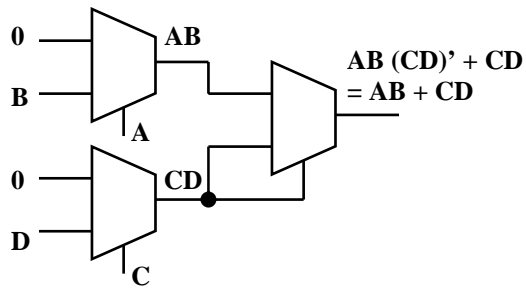
A	B	x	$J_A$	$K_A$	$J_B$	$K_B$
1	0	1	0	0	0	0
1	1	0	1	0	0	0
1	1	1	0	0	1	0
1	0	0	0	0	0	1
0	1	0	0	0	0	1

$$J_A = Bx' \quad K_A = Bx$$

$$J_B = x \quad K_B = (A \oplus x)$$

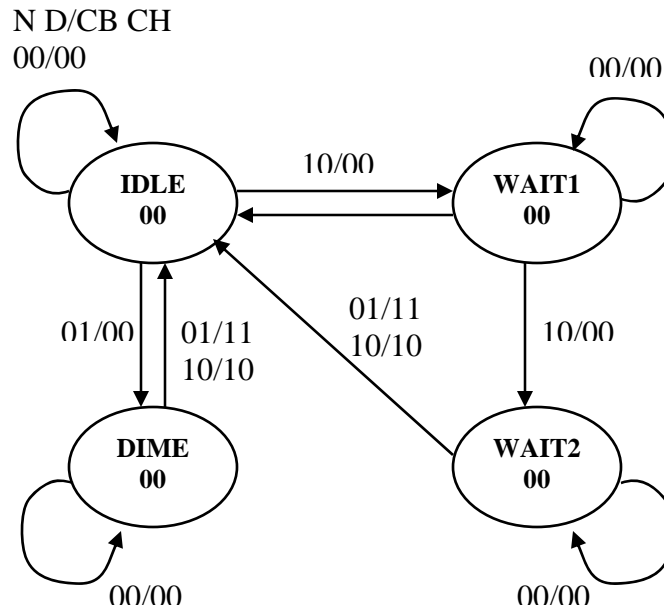


c) (3 marks)



Question 4

1)



Present State		FF Inputs		Next State		FF Inputs				Outputs	
A	B	N	D	A <sup>+</sup>	B <sup>+</sup>	J <sub>A</sub>	K <sub>A</sub>	J <sub>B</sub>	K <sub>B</sub>	CB	CH
0	0	0	0	0	0	0	X	0	X	0	0
0	0	0	1	1	1	1	X	1	X	0	0
0	0	1	0	0	1	0	X	1	X	0	0
0	0	1	1	0	0	0	X	0	X	0	0
-----											
0	1	0	0	0	1	0	X	X	0	0	0
0	1	0	1	0	0	0	X	X	1	1	0
0	1	1	0	1	0	1	X	X	1	0	0
0	1	1	1	0	0	0	X	X	1	0	0
-----											
1	0	0	0	1	0	X	0	0	X	0	0
1	0	0	1	0	0	X	1	0	X	1	1
1	0	1	0	0	0	X	1	0	X	1	0
1	0	1	1	0	0	X	1	0	X	0	0
-----											
1	1	0	0	1	1	X	0	X	0	0	0
1	1	0	1	0	0	X	1	X	1	1	1
1	1	1	0	0	0	X	1	X	1	1	0
1	1	1	1	0	0	X	1	X	1	0	0

ND \ AB	00	01	11	10
00		1		
01				1
11	X	X	X	X
10	X	X	X	X

$$J_A = B'N'D + BND'$$

ND \ AB	00	01	11	10
00		1		1
01	X	X	X	X
11	X	X	X	X
10				

$$J_B = A'N'D + A'ND' \\ = A'(N \oplus D)$$

ND \ AB	00	01	11	10
00	X	X	X	X
01	X	X	X	X
11		1	1	1
10		1	1	1

$$K_A = N + D$$

ND \ AB	00	01	11	10
00	X	X	X	X
01		1	1	1
11		1	1	1
10	X	X	X	X

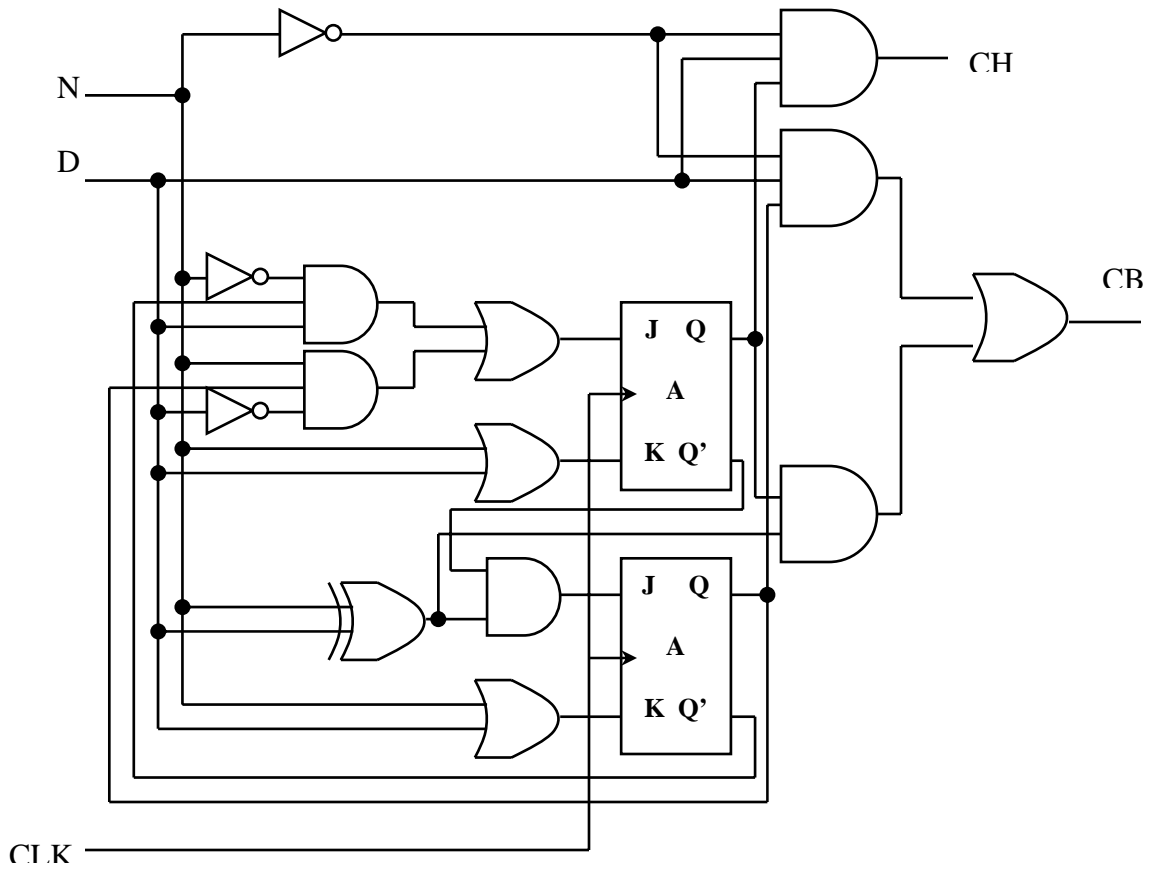
$$K_B = N + D$$

ND \ AB	00	01	11	10
00				
01		1		
11		2		1
10		1		1

$$CB = BN'D + BN'D + AND' \\ = BN'D + A(N \oplus D)$$

ND \ AB	00	01	11	10
00				
01				
11		1		
10		1		

$$CH = AN'D$$



Question 6

$$T = Y_1 + (Y_2 \oplus X) \quad R = Y_0[Y_1 + (Y_2 \oplus X)'] \quad S = Y_1Y_0'(Y_2 \oplus X)$$

Present State Y <sub>2</sub> Y <sub>1</sub> Y <sub>0</sub>	X D Y <sub>2</sub> ⊕ X	T	(Y <sub>2</sub> + X)'	Y <sub>1</sub> + (Y <sub>2</sub> ⊕ X)'	R S	Next State Y <sub>2</sub> <sup>+</sup> Y <sub>1</sub> <sup>+</sup> Y <sub>0</sub> <sup>+</sup>
0 0 0	0 0 0	0	1	1	0 0	0 0 0
0 0 0	1 1 1	1	0	0	0 0	1 1 0
0 0 1	0 0 0	0	1	1	1 0	0 0 0
0 0 1	1 1 1	1	0	0	0 0	1 1 1
0 1 0	0 0 0	1	1	1	0 0	0 0 0
0 1 0	1 1 1	1	0	1	0 1	1 0 1
0 1 1	0 0 0	1	1	1	1 0	0 0 0
0 1 1	1 1 1	1	0	1	1 0	1 0 0
1 0 0	0 0 1	1	0	0	0 0	0 1 0
1 0 0	1 1 0	0	1	1	0 0	1 0 0
1 0 1	0 0 1	1	0	0	0 0	0 1 1
1 0 1	1 1 0	0	1	1	1 0	1 0 0
1 1 0	0 0 1	1	0	1	0 1	0 0 1
1 1 0	1 1 0	1	1	1	0 0	1 0 0
1 1 1	0 0 1	1	0	1	1 0	0 0 0
1 1 1	1 1 0	1	1	1	1 0	1 0 0

Outputs same as present state

