

DIGITAL DESIGN COEN 312	Instructor: Asim J. Al-Khalili
Time Allowed 3:00 hrs. All questions carry equal marks	Final Summer, 2010
No materials are allowed	2 Pages

Question 1

- a) Design a network that adds either 1 or 2 to a 3-bit binary number N (n2, n1, n0). The network output is M (m3, m2, m1, m0). K is a control signal.
 When $K = 0, M = N + 1$
 When $K = 1, M = N + 2$
- b) Implement m_3 using a 4-to-1 multiplexer with minimum external logic.

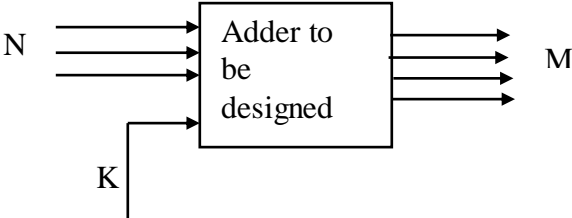


Fig 1 Adder to be designed of Question 1

Question 2

Design a 4-bit binary number to 7 segment decoder. The decoder should show if the number is a multiple of 2 or 3 as shown (use 2 when the number can be divided by 2 and 3) . Please refer to Fig. 2 below for the segment's destination. Assume that all other numbers will be displayed as E.

Give circuit Implementation.

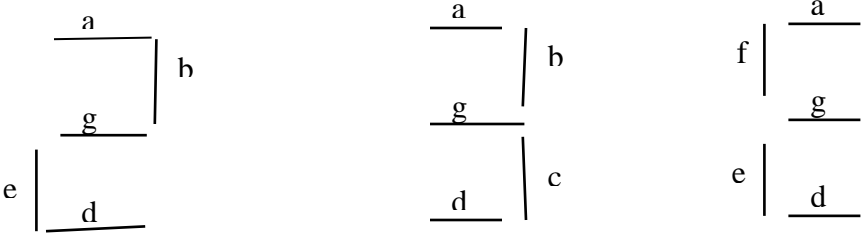


Fig. 2. 7-Segment Display of Question 2

Question 3

- Using K-Map minimize the following function F
- a) $F(A,B,C,D) = m(0,2,5,7,12,14) + d(8,10,13,15)$ Give results in SOP form.
 - b) Implement in NAND-NAND form
 - c) Implement in NOR_NOR form
 - d) Implement in AND_OR_NOT form

Question 4

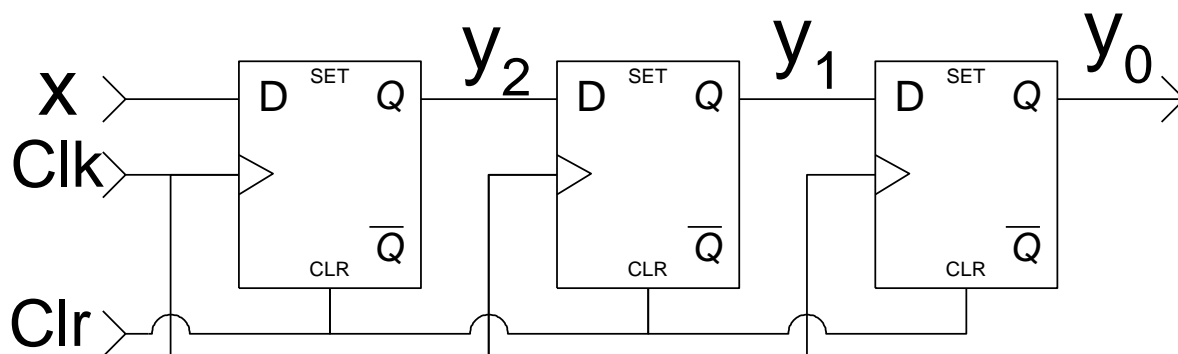
- a) Represent the following by a Boolean equation.
The outdoor light “L” should be turned on if there is nobody at home ”N” , the time is between 8 p.m. and 5 a.m “T”. and it is not a holiday “H”.
- b) Using Boolean Algebra, show that the two functions f_1 and f_2 are equal:
 $f_1(a,b,c) = a'c + b'c' + ab$
 $f_2(a,b,c) = a'b' + bc + ac'$
- c) Using Boolean Algebra, minimize the following function:
 $f(a,b,c,d) = ac' + ab'd + a'b'c + a'cd' + b'c'd'$
- d) Give the maxterm list equivalent of the following function:
 $f(a,b,c) = ab$

Question 5

Design a clocked sequential circuit with 4 outputs. The outputs are to drive colored lights: Red, R, Yellow, Y, Blue, B and Green, G consecutively and repeatedly in the same order ie: $R \rightarrow Y \rightarrow B \rightarrow G \rightarrow R \rightarrow Y \rightarrow \dots$. **Start the design with the state diagram and follow standard design procedure.** What is the frequency of operation if each light is to stay ON for 1 second. Neglect all logic and FlipFlop delays.

Question 6

Analyze the circuit given below fully. Determine Excitation equations, Transition table and the State diagram. Explain what function the circuit performs.

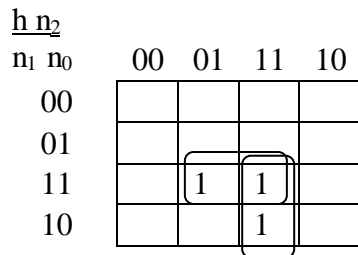


Solution:

Q1

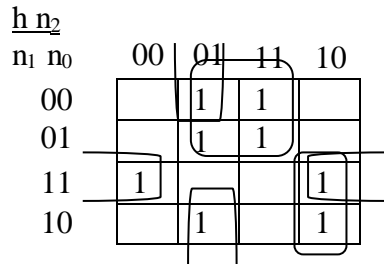
#1 a)

	h	n ₂	n ₁	n ₀	m ₃	m ₂	m ₁	m ₀
0	0	0	0	0	0	0	0	1
1	0	0	0	1	0	0	1	0
2	0	0	1	0	0	0	1	1
3	0	0	1	1	0	1	0	0
4	0	1	0	0	0	1	0	1
5	0	1	0	1	0	1	1	0
6	0	1	1	0	0	1	1	1
7	0	1	1	1	1	0	0	0
8	1	0	0	0	0	0	1	0
9	1	0	0	1	0	0	1	1
10	1	0	1	0	0	1	0	0
11	1	0	1	1	0	1	0	1
12	1	1	0	0	0	1	1	0
13	1	1	0	1	0	1	1	1
14	1	1	1	0	1	0	0	0
15	1	1	1	1	1	0	0	1



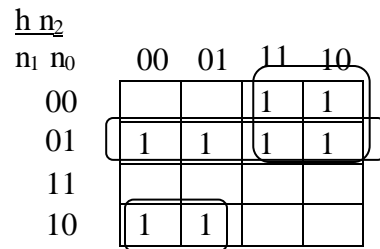
$$m_3 = \sum_m(7,14,15)$$

$$m_3 = n_2 n_1 n_0 + h n_2 n_1$$



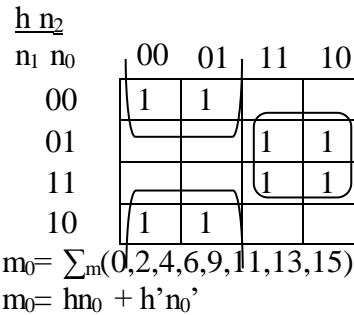
$$m_2 = \sum_m(3,4,5,6,10,11,12,13)$$

$$m_2 = n_2 n_1' + h n_2' n_1 + n_2' n_1 n_0 + h' n_2 n_1'$$



$$m_1 = \sum_m(1,2,5,6,8,9,12,13)$$

$$m_1 = h n_1' + n_1' n_0 + h' n_1 n_0'$$



$$m_0 = \sum_m(0,2,4,6,9,11,13,15)$$

$$m_0 = h n_0 + h' n_0'$$

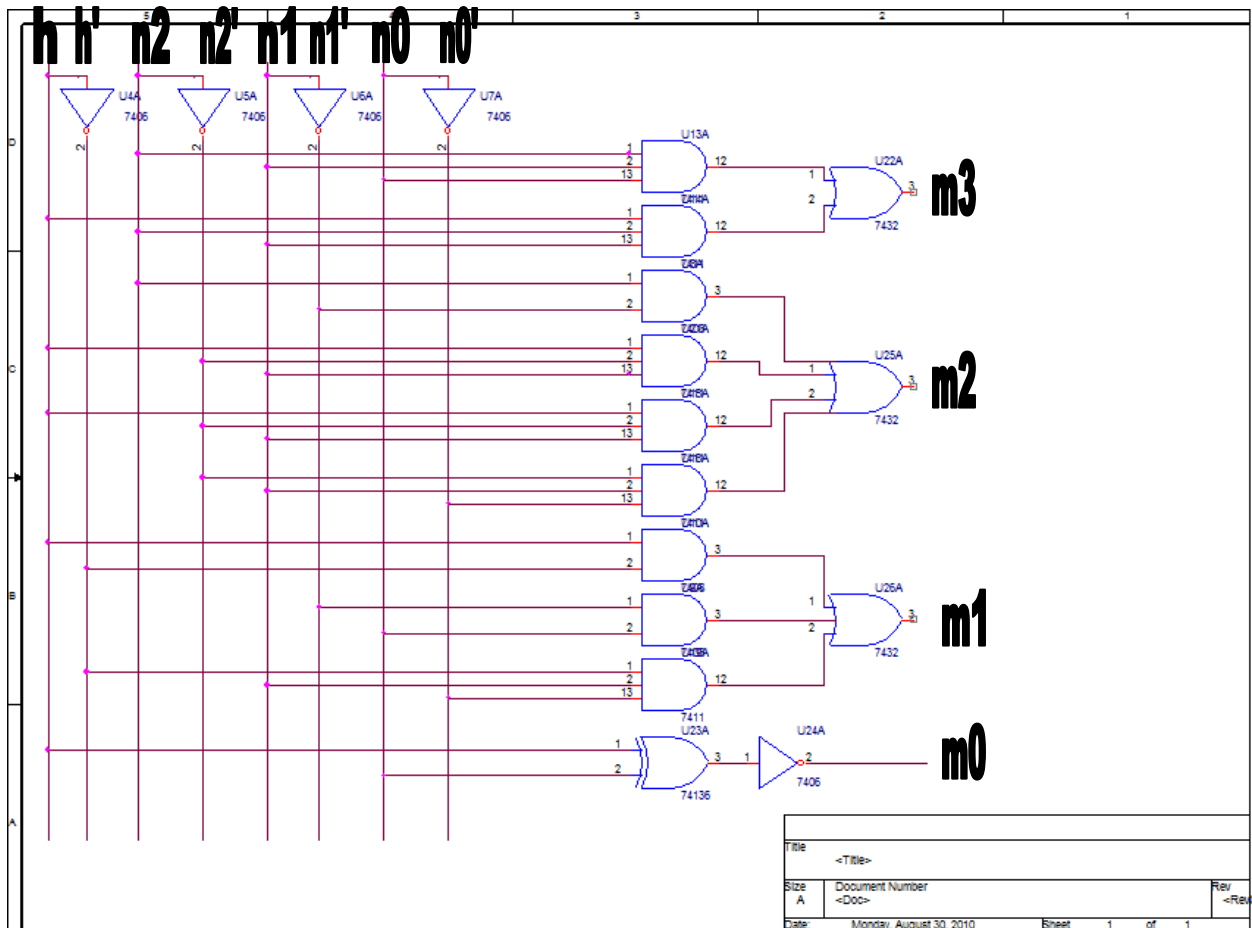
$$m_3 = n_2 n_1 n_0 + h n_2 n_1$$

$$m_2 = n_2 n_1' + h n_2' n_1 + n_2' n_1 n_0 + h' n_2 n_1'$$

$$m_1 = h n_1' + n_1' n_0 + h' n_1 n_0'$$

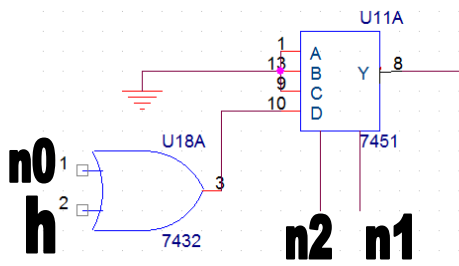
$$m_0 = (h \oplus n_0)'$$

Assuming double rail



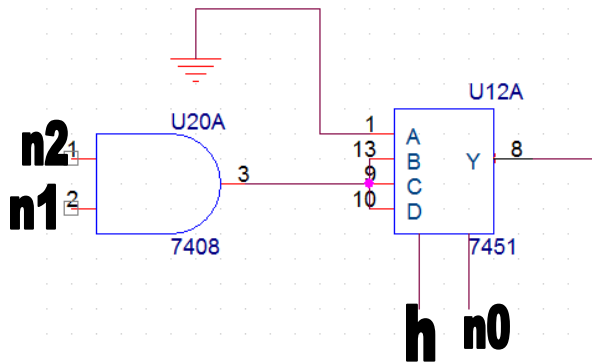
b)

$$m_3 = n_2 n_1 n_0 + h n_2 n_1 = n_2 n_1 (n_0 + h)$$



OR

$$m_3 = h'n_2n_1n_0 + hn_2n_1n_0 + hn_2n_1n_0' = n_2n_1(h'n_0 + hn_0 + hn_0')$$



Q2

	A3	A2	A1	A0	7 Segment	a	b	c	d	e	f	g
0	0	0	0	0	E	1	0	0	1	1	1	1
1	0	0	0	1	E	1	0	0	1	1	1	1
2	0	0	1	0	2	1	1	0	1	1	0	1
3	0	0	1	1	3	1	1	1	1	0	0	1
4	0	1	0	0	2	1	1	0	1	1	0	1
5	0	1	0	1	E	1	0	0	1	1	1	1
6	0	1	1	0	2	1	1	0	1	1	0	1
7	0	1	1	1	E	1	0	0	1	1	1	1
8	1	0	0	0	2	1	1	0	1	1	0	1
9	1	0	0	1	3	1	1	1	1	0	0	1
10	1	0	1	0	2	1	1	0	1	1	0	1
11	1	0	1	1	E	1	0	0	1	1	1	1
12	1	1	0	0	2	1	1	0	1	1	0	1
13	1	1	0	1	E	1	0	0	1	1	1	1
14	1	1	1	0	2	1	1	0	1	1	0	1
15	1	1	1	1	3	1	1	1	1	0	0	1

a=d=g= "1" and b=f' and c=e'

A3 A2

A1 A0	00	01	11	10
00		1	1	1
01			1	
11	1			1
10	1	1	1	1

$$b = A_2'A_1 + A_0'A_2 + A_0'A_3 + A_3A_2A_1'$$

A3 A2

A1 A0	00	01	11	10
00				
01				1
11	1		1	
10				

$$c = A_3'A_2'A_1A_0 + A_3A_2'A_1'A_0 + A_3A_2A_1A_0$$

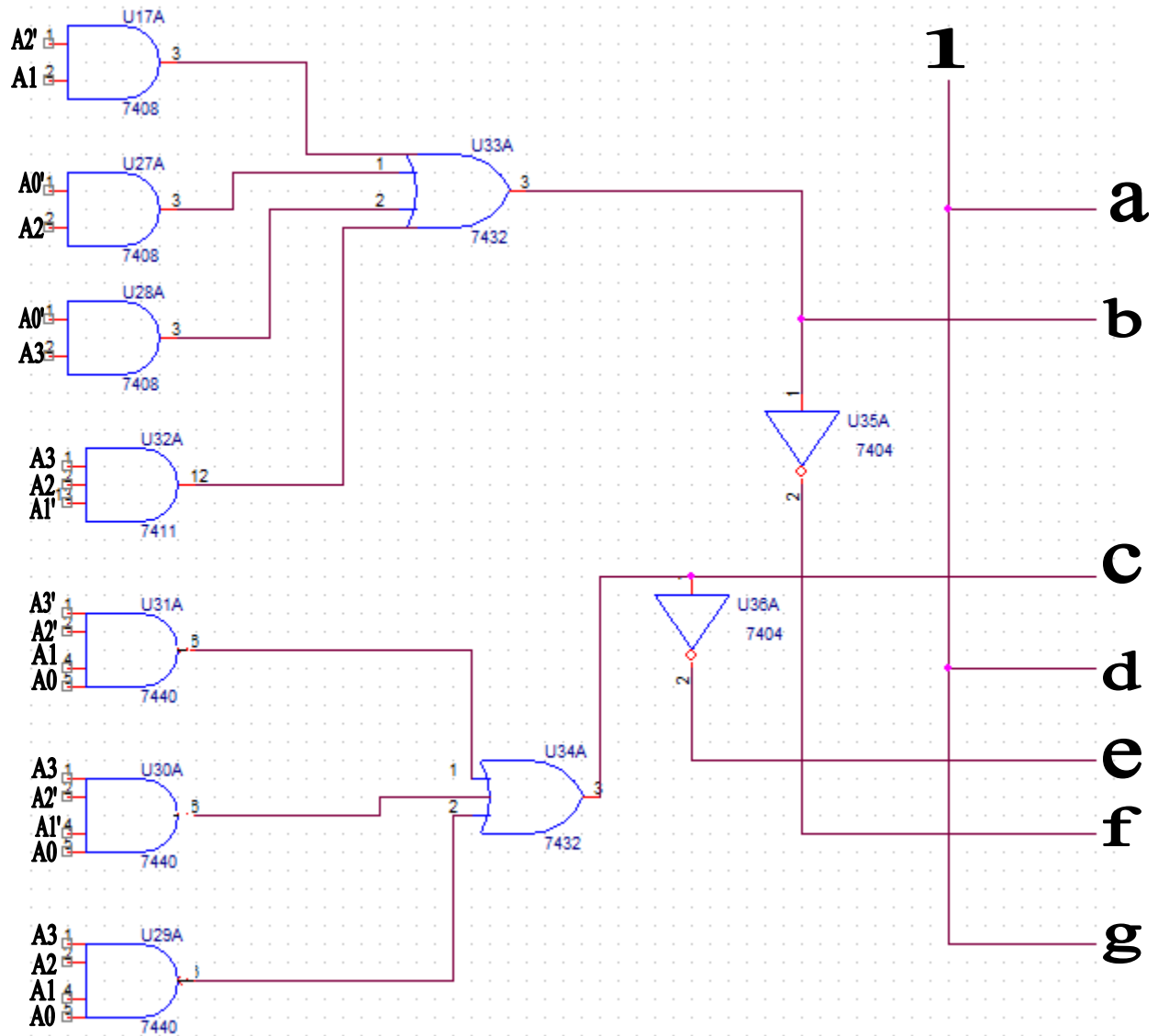
$$a=d=g=\sum_m(0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15)$$

$$b=\sum_m(2,3,4,6,8,9,10,12,14,15)$$

$$c=\sum_m(3,9,15)$$

$$e=\sum_m(0,1,2,4,5,6,7,8,10,11,12,13,14)$$

$$f=\sum_m(0,1,5,7,11,13)$$



Q3

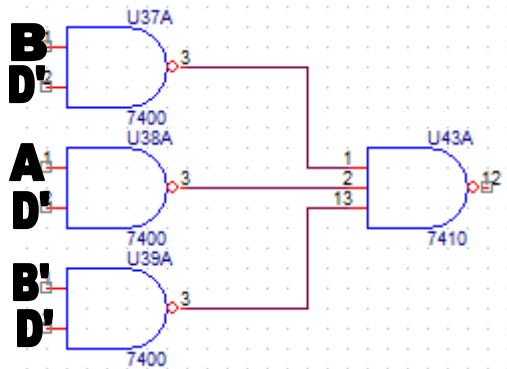
a)

<u>A</u> <u>B</u>				
<u>C</u> <u>D</u>	00	01	11	10
00	1		1	X
01		1	X	
11		1	X	
10	1		1	X

$$F = BD + AD' + B'D'$$

$$b) F = (BD + AD' + B'D')' = ((BD)'(AD')'(B'D'))'$$

Assuming double rail



c)

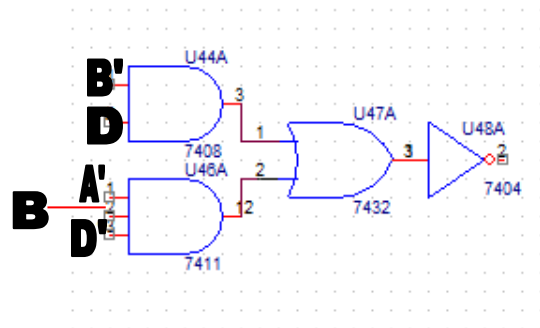
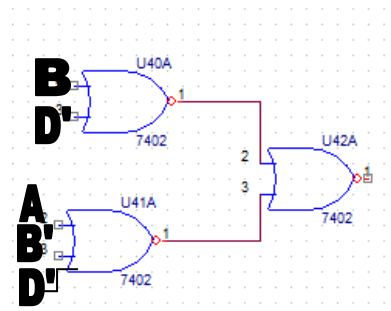
<u>A</u> <u>B</u>				
<u>C</u> <u>D</u>	00	01	11	10
00		0		X
01	0		X	0
11	0		X	0
10		0		X

$$F = (B + D')(A + B' + D)$$

d)

<u>A</u> <u>B</u>				
<u>C</u> <u>D</u>	00	01	11	10
00		1		X
01	1		X	1
11	1		X	1
10		1		X

$$F' = B'D + A'BD'$$



Q4

a) $L \Rightarrow$ output

$N \Rightarrow$ someone is home

$T \Rightarrow$ Time is between 8 PM & 5 AM

$H \Rightarrow$ is a holiday

$$L = N' + H'$$

b) Many ways of doing this problem. This is the simplest:

$$a'c + b'c' + ab = a'b' + bc + ac' \text{ expand both sides}$$

$$a'c(b+b') + b'c'(a+a') + ab(c+c') = a'b'(c+c') + bc(a+a') + ac'(b+b')$$

$$a'bc + a'b'c + ab'c' + a'b'c' + abc + abc' = a'bc + a'b'c + ab'c' + a'b'c' + abc + abc'$$

$$\sum_m(0,1,3,5,6,7) = \sum_m(0,1,3,5,6,7)$$

c) $ac' + ab'd + a'b'c + a'cd' + b'c'd'$

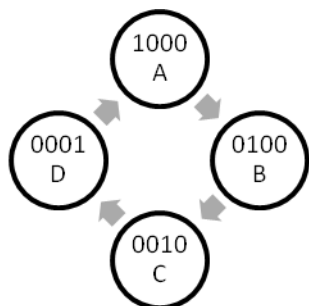
$$\begin{aligned}
 & \overbrace{ab'd + a'b'c}^{ab'd + a'b'c + b'cd} \leftarrow (\text{eg } ab + a'c + bc = ab + a'c) \\
 & ac' + ab'd + a'b'c + \overbrace{b'cd + a'cd' + b'c'd'}^{ac' + ab'd + b'cd = ac' + b'cd} \\
 & ac' + b'cd + a'b'c + a'cd' + b'c'd' \\
 & \overbrace{b'cd + a'b'c + a'cd'}^{b'cd + a'b'c + a'cd' = b'cd + a'cd'} \\
 & f = ac' + b'cd + a'cd' + b'c'd'
 \end{aligned}$$

d)

<u>ab</u>				
c	00	01	11	10
0			1	
1	1		1	

$$F = \prod M(0,2,3,4,5)$$

Q5



	present	next	R	Y	B	G
00	A	B	1	0	0	0
01	B	C	0	1	0	0
10	C	D	0	0	1	0
11	D	A	0	0	0	1

$$\begin{array}{c}
 \underline{Y1} \\
 Y0 \quad 0 \quad 1 \\
 0 \quad \begin{array}{|c|c|} \hline & 1 \\ \hline \end{array} \\
 1 \quad \begin{array}{|c|c|} \hline 1 & \\ \hline \end{array} \\
 Y_1^+ = Y_1 \oplus Y_0
 \end{array}$$

$$\begin{array}{c}
 \underline{Y1} \\
 Y0 \quad 0 \quad 1 \\
 0 \quad \begin{array}{|c|c|} \hline 1 & 1 \\ \hline \end{array} \\
 1 \quad \begin{array}{|c|c|} \hline & \\ \hline \end{array} \\
 Y_0^+ = Y_0'
 \end{array}$$

present		next		output			
Y1	Y0	Y1	Y0	R	Y	B	G
0	0	0	1	1	0	0	0
0	1	1	0	0	1	0	0
1	0	1	1	0	0	1	0
1	1	0	0	0	0	0	1

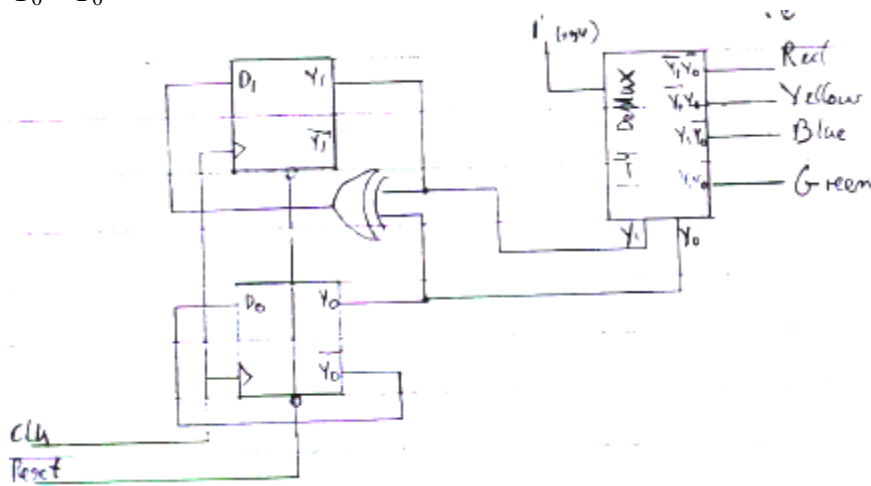
A=00 B=01 C=10 D=11

Using D-FF

$$Y^+ = D$$

$$Y_1^+ = Y_1 \oplus Y_0$$

$$Y_0^+ = Y_0'$$



Frequency of operation: $1\text{sec} * 4 = 4\text{sec} = .25\text{ Hz}$

Q6

$$\left. \begin{aligned} Y^+ &= D \\ Y_2^+ &= X \\ Y_1^+ &= Y_2 \\ Y_0^+ &= Y_1 \\ Z &= Y_0 \end{aligned} \right\} \text{Excitation Equations}$$

Transition Table

				X=0			X=1			output
	Y2	Y1	Y0	Y2+	Y1+	Y0+	Y2+	Y1+	Y0+	Z
A	0	0	0	0	0	0	1	0	0	0
B	0	0	1	0	0	0	1	0	0	1
C	0	1	0	0	0	1	1	0	1	0
D	0	1	1	0	0	1	1	0	1	1
E	1	0	0	0	1	0	1	1	0	0
F	1	0	1	0	1	0	1	1	0	1
G	1	1	0	0	1	1	1	1	1	0
H	1	1	1	0	1	1	1	1	1	1

State Table

present	Next		output
	X=0	X=1	z
A	A	E	0
B	A	E	1
C	B	F	0
D	B	F	1
E	C	G	0
F	C	G	1
G	D	H	0
H	D	H	1

The function of this circuit is a 3-bit right shift register.

State Diagram:

