THE UNIVERSITY OF CALGARY

FACULTY OF ENGINEERING

Final Examination

PROBABILITY AND STATISTICS FOR ENGINEERS
ENGG 319

December, 2000 Time: 3 hrs.

THE EXAM IS OPEN TEXT
CALCULATORS ARE ALLOWED

There are 40 questions.
Answer all 40 questions by indicating the letter of
the correct answer on the scoring sheet.

Each question answered correctly is awarded 1 mark.
Each question answered incorrectly is awarded 0 marks.

Total possible for entire exam is 40 marks.

01 FixRight Inc. manufacturers mounting plates for car stereos. The process engineer responsible for overseeing the production of these plates is concerned about the large number of plates that have been produced with drill holes which fall outside the specified tolerances of ± 1 mm. A sample of 31 mounting plates has been inspected, revealing that 9 of these plates were defective (plates with drill holes outside the specified tolerances). The engineer wants to obtain an estimate of the proportion defective, accurate to within an error of 0.03 using a 99% confidence level. How many mounting plates should be inspected to achieve this estimate?

$z_{\alpha/2} = z_{0.005} = 2.576$

$n = \frac{z_{\alpha/2}^2 \hat{p}\hat{q}}{e^2}$

$\hat{p} = 9/31 = 0.29 \quad \hat{q} = 0.71$

$e = 0.03$

(a) 1238
(b) 1519
(c) 1730
(d) 1844

$n = 2.576^2 \times 0.29 \times 0.71 / 0.03^2 = 1519$

02 A water utility employs a calibrated portable water meter with a true standard deviation of 0.1 litres/second to check the calibration of residential water meters. Residential water meters, being cheaper units, have a true standard deviation of 0.3 litres/second when properly calibrated. To verify the accuracy of a residential meter, a technician connects the portable meter to the residential supply line and 7 measurements are taken using both meters. The sample variances are then compared to check the accuracy of the residential meter. What is the ratio of sample variance of a residential meter to sample variance of the portable meter that is exceeded only 5% of the time, if the residual meter is properly calibrated?

$\sigma_p = 0.1 \quad \sigma_r = 0.3 \quad n_p = 7$

$\sigma_p^2 = 0.01 \quad \sigma_r^2 = 0.09 \quad n_r = 7$

(a) 0.48
(b) 4.28
(c) 12.84
(d) 38.52

$F_{0.05} = \frac{\sigma_p^2 s_r^2}{\sigma_r^2 s_p^2} \Rightarrow \frac{s_r^2}{s_p^2} = F_{0.05} \frac{\sigma_r^2}{\sigma_p^2}$

$F_{0.05}(6,6) = 4.28 \quad \frac{s_r^2}{s_p^2} = 4.28 \times \frac{0.09}{0.01} = 38.52$

03 A high school grade teacher has obtained the following figures for the time (minutes) that ten randomly picked students in her class took to complete a task:

15 10 12 8 11 10 10 13 12 9

$\bar{x} = 11$

$s^2 = 4.22$

$n = 10$

What is the 95% confidence interval for the true variance of the time?

(a) 0.97, 6.85
(b) 1.414, 3.75
(c) 2.00, 14.07
(d) 1.414, 14.07

$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$

$\chi^2_{0.025,9} = 19.023 \quad \chi^2_{0.975,9} = 2.700$

$\frac{9 \times 4.22}{19.023} < \sigma^2 < \frac{9 \times 4.22}{2.700}$ (2.00, 14.07)

04 A software engineer is developing an automated parts delivery system in a warehouse. Data have been collected on the number of requests for parts received in a 15 minute interval:

Number of Parts	Frequency
0	5
1	2
2	2
3	1
4	0
5	1

What is the median of this sample?

$n = 11$

$\tilde{X} = X_6 = 1$

(a) 0.5
(b) 1.0
(c) 1.3
(d) 2.5

05 The Little Plane Airline of Nebraska is concerned over the possibility that flight delays at busy airports are reducing the time between failures of air conditioning systems. A study has been undertaken, and the following data have been collected. x is the cumulative number of hours in delays since the conditioning system was last overhauled, and y is number of hours between system failures.

x	3.4	2.1	4.2	1.1	3.9	1.4	2.7	1.9	3.1	2.3	3.3	1.7	2.5	4.5
y	45.6	75.3	23.6	98.9	43.2	90.6	63.8	76.8	56.9	72.0	48.7	87.4	67.3	21.1

$\sum_i x_i = 38.1$ $\sum_i x_i^2 = 118.07$ $\bar{x} = 2.72$

$\sum_i y_i = 871$ $\sum_i y_i^2 = 61538$ $\bar{y} = 62.2$

$\sum_i x_i y_i = 2049.1$

$S_{xx} = 14.38$ $S_{yy} = 7323.9$ $S_{xy} = -321.85$

$b = \frac{14 \times 2049.1 - 38.1 \times 871}{14 \times 61538 - (871)^2}$

$b = -0.0437$

$a = 2.72 - (-0.0437) \times 62.2$

$a = 5.44$

What is the regression line for this relationship $x = g(y)$ (using the method of least squares) and the associated coefficient of correlation?

(a) $x = 5.44 - 0.0437\,y$, with a correlation coefficient of - 0.992
(b) $x = 5.44 - 0.0437\,y$, with a correlation coefficient of - 0.002
(c) $x = 122.96 - 22.34\,y$, with a correlation coefficient of - 0.992
(d) $x = 122.96 - 22.34\,y$, with a correlation coefficient of - 0.002

$r = b\sqrt{\frac{S_{yy}}{S_{xx}}}$

$= -0.0437 \times \left(\frac{7323.9}{14.38}\right)^{1/2}$

$r = -0.99$

06 A hypothesis test has been developed to monitor the proportion of defective power supply units produced by the *Flash in the Pan* Corporation. The test has been structured with a H_o: $p \leq 0.04$ and a H_1: $p > 0.04$, and an $\alpha = 5\%$. The last sample of 100 units revealed 9 defective units. The production superintendent has determined that this last sample corresponds to a Z statistic of 1.84. What is the most appropriate conclusion regarding the true proportion of defectives being produced?

(a) $z_{obs} > 1.645$, Reject H_o and therefore more than 4% defectives are being produced.
(b) $z_{obs} > -1.645$, Accept H_o and therefore 4% defectives are being produced.
(c) $z_{obs} < 1.960$, Reject H_o and therefore more than 4% defectives are being produced.
(d) $z_{obs} > -1.960$, Accept H_o and therefore 4% defectives are being produced.

$z_{obs} = 1.84 > z_{0.05} = 1.645$ $\therefore$ Reject H_o

07 What is the significance level associated with a one-tailed test in which the mean cannot be more than 1.64 standard errors from the true mean?

(a) 0.5 %
(b) 1 %
(c) 5 %
(d) 10 %

$z_\alpha = 1.64 \Rightarrow \alpha = 0.05$

08 A city health department wishes to determine if the mean bacteria counts per unit volume of water at a lake beach is below the safety level of 200. A researcher collected 10 water samples of unit volume and found the bacteria counts to be:

175 190 215 198 184 207 210 193 196 180

$\bar{x} = 194.8$
$s = 13.14$
$n = 10$

What is the value of the appropriate statistic, and do the data indicate that there is no cause for concern at the 0.01 significance level? *(Assume that the normally requirement is met.)*

(a) -1.25; accept H_o
(b) -1.25; reject H_o
(c) 1.25; reject H_o
(d) -2.821; accept H_o

$H_o: \mu = 200$
$H_1: \mu < 200$

σ^2 is unknown

$$t_{obs} = \frac{194.8 - 200}{13.14/\sqrt{10}} = -1.25$$

$\alpha = 0.01$ $-t_{0.01,9} = -2.821$

Since $t_{obs} > -t_{0.01,9}$ Accept H_o

09 *BrainView Inc.* operates a number of Magnetic Resonance Imaging (MRI) scanners in Alberta. One of the critical settings for the proper operation of an MRI scanner is the signal frequency of 85 MHz. Five independent measurements are therefore taken every 2 weeks to make sure that no drift has occurred in the signal frequency. A hypothesis test has been defined, where H_o: $\mu = 85$ MHz and H_1: $\mu \neq 85$ MHz. The true standard deviation of the frequency has been independently established as 1.6 MHz. The mean of the latest sample is 85.9 MHz. If the true mean frequency is 86.5 MHz and α= 5% has been selected, what is the probability of making a Type II Error? *(Assume that the signal frequency follows the normal distribution.)*

$H_0: \mu = 85$ $\sigma = 1.6$
$H_1: \mu \neq 85$ $n = 5$

(a) 0.2005
(b) 0.4443
(c) 0.8660
(d) 0.8686

$L = 85 - z_{0.025}\, 1.6/\sqrt{5} = 83.6$ $z_{0.025} = 1.960$
$U = 85 + z_{0.025}\, 1.6/\sqrt{5} = 86.4$

$\beta = P(83.6 < \bar{x} < 86.4 \mid \mu = 86.5) = P(z < -0.14) - P(z < -4.05)$
$= 0.4443$

10 Given these six pairs of (x,y) values, what are the estimates for the least squares parameters α and β.

x	1	2	3	3	4	5
y	9	5	6	3	3	1

$n = 6$
$\sum x = 18$ $\sum x^2 = 64$
$\sum y = 27$ $\sum y^2 = 161$
$\sum xy = 63$ $\bar{x} = 3.0$
$\bar{y} = 4.5$

(a) a = -0.9, b = -1.8
(b) a = 4.3, b = 0.07
(c) a = 8.4, b = -0.9
(d) a = 9.9, b = -1.8

$b = \dfrac{6 \times 63 - 18 \times 27}{6 \times 64 - (18)^2} = -1.8$

$a = 4.5 - (-1.8) \times 3.0 = 9.9$

11 The following data have been collected as part of a study into the efficiency of a new solid fuel. What is the standard deviation of the variable y, based on the available data?

y	320.6	359.2	398.4	372.0	321.9	348.3	303.3

$n = 7$

(a) 30.8
(b) 33.3
(c) 44.2
(d) 47.6

$s = \sum(y_i - \bar{y})^2/(n-1)$

$s = 33.3$

12 An engineer wants to perform the following hypothesis test using a sample of 15 units, and $\alpha = 0.01$ and $\beta = 0.05$:

H_o: $\sigma^2 = 2300$
H_1: $\sigma^2 \neq 2300$

$\alpha = 0.01$ $\alpha/2 = 0.005$
$\nu = 15-1 = 14$

What is the acceptance region for this test?

(a) $\chi^2_{0.95,14} < \chi^2_{obs.} < \chi^2_{0.05,14}$
(b) $\chi^2_{0.975,14} < \chi^2_{obs.} < \chi^2_{0.025,14}$
(c) $\chi^2_{0.99,14} < \chi^2_{obs.} < \chi^2_{0.01,14}$
(d) $\chi^2_{0.995,14} < \chi^2_{obs.} < \chi^2_{0.005,14}$

$\chi^2_{0.995,14} < \chi^2_{obs} < \chi^2_{0.005,14}$

13 A medical researcher wishes to determine if a pill has the undesirable side effect of reducing the blood pressure of the user. The study involves recording the initial blood pressures of 15 college-age women. After they use the pill regularly for six months, their blood pressures are again recorded. The researcher wishes to draw inferences on the effect of the pill by developing confidence intervals.

Blood Pressure Measurements Before and After Use of Pill

Before	70	80	72	76	76	76	72	78	82	64	74	92	74	68	84
After	68	72	62	70	58	66	68	52	64	72	74	60	74	72	74
D	2	8	10	6	18	10	4	26	18	-8	0	32	0	-4	10

What is the 95% confidence interval to assist in drawing this inference? $\mu_{before} - \mu_{after}$

(a) -2.72, -14.88
(b) 2.72, 14.88
(c) 3.74, 13.86
(d) 5.27, 12.33

$n = 15$
$\bar{D} = 8.8$ $s_D = 10.98$ $t_{0.025,14} = 2.145$
$L = 8.8 - 2.145 \times 10.98/\sqrt{15} = 2.72$
$U = 8.8 - 2.145 \times 10.98/\sqrt{15} = 14.88$

14 *LiquidGrow* is a fertilizer with a mean phosphorus concentration of 0.2 mg/L. Extensive monitoring of 4-litre bottles at the factory has revealed that the actual concentration in a bottle follows the normal distribution with a true mean of 0.2 mg/L and a true standard deviation of 0.05 mg/L. What is the probability that a bottle selected at random from a sample of 4 bottles has a concentration that is greater than 0.2588 mg/L?

(a) 0.01
(b) 0.05
(c) 0.12
(d) 0.16

$P(x > 0.2588) = P(z > (0.2588-0.2)/0.05)$
$= P(z > 1.18)$
$= 1 - P(z < 1.18)$
$= 1 - 0.88$
$= 0.12$

15 The engineer responsible for the packaging of magnesia pelts wants to monitor this process. After 2 years of data collection, it has been established that the mean weight of the packages is 15 kg, with a standard deviation of 1.4 kg. The engineer has decided that the packaging process will be monitored using the average weight of 6 packages. What is the probability that a sample mean will be greater than 15.84?

(a) 0.07
(b) 0.27
(c) 0.73
(d) 0.93

$\mu = 15 \quad \sigma = 1.4 \quad n = 6$

$P(\bar{x} > 15.84) = P(z > (15.84-15)/(1.4/\sqrt{6}))$
$= P(z > 1.47)$
$= 1 - P(z < 1.47)$
$= 1 - 0.93 = 0.07$

16 A fitted regression line relating the dosage in milligrams (x) to the number of days of relief (y) experienced by 10 allergy patients is: $\underline{y} = -1.07 + 2.74\,x$. Furthermore, $S_{xx} = 40.9$, $S_{yy} = 370.9$, $S_{xy} = 112.1$ and $\bar{x} = 5.9$. What is the 95% prediction interval for the mean duration of relief with a dosage of 6 mg?

(a) 8.55, 22.19
(b) 12.86, 17.88
(c) 13.31, 17.43
(d) 13.35, 17.39

$s^2 = (370.9 - 2.74 \times 112.1)/8 = 7.97$
$s = 2.82$
$\hat{y} = 15.37 \qquad t_{0.025,8} = 2.306$
$15.37 \pm 2.306 \times 2.82 \times \left(1/10 + (6-5.9)^2/40.9\right)^{1/2}$
$15.37 \pm 2.06 \qquad (13.31, 17.43)$

17 Given the information in Question 16, what proportion of the variability is explained by the fitted straight line?

(a) 1 %
(b) 69 %
(c) 83 %
(d) 91 %

$r^2 = \dfrac{S_{xy}^2}{S_{xx}\,S_{yy}} = \dfrac{112.1^2}{40.9 \times 370.9} = 0.83$

18 Tests were carried out to compare strengths of two types of yarn. When 15 random tests were carried out with Yarn A, it was found that the mean strength was 140 lbs/in^2. Also, 10 random tests were carried out with Yarn B yielded a mean of 145 lbs/in^2. Assume the strength of yarn has a normal distribution for both yarns and that the population variance for Yarn A is 25 lbs/in^2 while the population variance for Yarn B is 30 lbs/in^2. What is the 80 percent confidence interval for the difference between the true means of Yarns A and B ($\mu_A - \mu_B$)?

(a) -3.19, -6.81
(b) -2.23, -7.77
(c) 0.033, -10.03
(d) 4.69, -14.69

$\bar{x}_A = 140 \quad n_A = 15 \quad \sigma_A^2 = 25$
$\bar{x}_B = 145 \quad n_B = 10 \quad \sigma_B^2 = 30$
$(140-145) \pm 1.282 \times (25/15 + 30/10)^{1/2}$
-5 ± 2.77
$(-2.23, -7.77)$

19 A sample of 4 chairs has been subjected to fatigue testing. The 4 chairs failed after 2304, 2198, 2256 and 2001 cycles, respectively. A search of the available research literature has revealed that the true standard deviation for this type of chair is 150 cycles. What is the 95% confidence interval for the mean number of cycles to failure? *(Assume that the number of cycles to failure is normally distributed.)*

$\bar{x} = 2190 \quad \sigma = 150$

(a) 2190 ± 123
(b) 2190 ± 147
(c) 2190 ± 212
(d) 2190 ± 239

$\bar{x} \pm z_{\alpha/2}\, \sigma/\sqrt{n}$

$2190 \pm 1.96 \times 150/\sqrt{4}$

2190 ± 147

20 Suppose according to a survey conducted in 1960 the probability distribution of the age of an adult over twenty years of age is as given in the first two rows of the following table. In 1983, when a sample of 1000 adults over 20 years of age was interviewed, the frequency distribution given in row 2 was obtained. Test the null hypothesis that the probability distribution in 1983 is the same as that in 1960.

	Age Group	20-29	30-39	40-49	50-59	60-69	over 70
	Probability Distribution	0.25	0.23	0.20	0.15	0.08	0.09
O_i	Observed sample frequencies of 1983	270	202	180	160	88	100
	E_i	250	230	200	150	80	90

Using a 0.05 significance level, what is the value of the appropriate statistic and associated conclusion for the test?

$\chi^2_{0.05,5} = 11.07$

(a) 9.488; reject H_o
(b) 9.586; ~~reject H_o~~ accept H_o
(c) 9.940; accept H_o
(d) 11.07; reject H_o

ignore

$$\chi^2_{obs} = \frac{(270-250)^2}{250} + \frac{(202-230)^2}{230} + \frac{(180-200)^2}{200} + \frac{(160-150)^2}{150} + \frac{(88-80)^2}{80} + \frac{(100-90)^2}{90} = 9.586$$

21 If the 99% tolerance limits that will contain 95% of the exam scores for a class of 400 is 97 and 26, what are the sample mean and standard deviation?

$1-\gamma = 0.99$
$1-\alpha = 0.95$
$n = 400$

$k = 2.138$

(a) 61.5; 4.07
(b) 61.5; 12.6
(c) 61.5; 16.6
(d) 65; 15

$\bar{x} - 2.138\,s = 26$
$\bar{x} + 2.138\,s = 97$
$\Rightarrow \bar{x} = 61.5$, $s = 16.6$

22 A mechanical engineer has been hired by a freight company to investigate the variability in fuel consumption currently observed in their truck fleet. After analyzing the available data, the engineer has developed two regression models to explain this variability in fuel consumption. The first model (Model 1) relates fuel consumption (miles per gallon), y, and average speed of a vehicle (miles per hour), x_1. The second model (Model 2) defines the relationship between fuel consumption (miles per gallon), y, and the proportion of time spent on highway travel (%), x_2.

$$y = 5.9 + 0.047x_1 \quad r_1^2 = 0.72$$
$$y = 3.6 + 0.056x_2 \quad r_2^2 = 0.83$$

Which model should be used, and why?

(a) Model 1 should be used because Model 1 explains 85% of the variability of y, compared to 91% for Model 2.
(b) Model 1 should be used because Model 1 explains 72% of the variability of y, compared to 83% for Model 2.
(c) Model 2 should be used because Model 2 explains 91% of the variability of y, compared to 85% for Model 1.
(d) Model 2 should be used because Model 2 explains 83% of the variability of y, compared to 72% for Model 1.

23 Plastic sheets produced by a machine are periodically monitored for possible fluctuations in thickness. Uncontrollable heterogeneity in the viscosity of the liquid mold makes some variation in thickness measurements unavoidable. However, if the true standard deviation of thickness exceeds 1.5 millimetres, there is a cause to be concerned about the product quality. Thickness measurements (in millimetres) of 10 specimens produced on a particular shift resulted in the following data:

226 228 226 228 232 228 227 229 225 230

The engineer undertaking the test wants to decide if the data substantiate the suspicion that the process variability exceeded the stated level on this particular shift at the 0.05 significance level. What is the value of the appropriate statistic and associated conclusion for the test?

$H_0: \sigma^2 = 1.5^2 = 2.25$

$H_1: \sigma^2 > 2.25$

$\alpha = 0.05$

$n = 10$

(a) 12.48; accept H_o
(b) 17.306; reject H_o
(c) -17.306; accept H_o
(d) 25.959; reject H_o

$s^2 = 4.32 \quad s = 2.08$

$$\chi^2_{obs} = \frac{9 \times 4.32}{2.25} = 17.28 > \chi^2_{0.05,9} = 16.919$$

So Reject H_o

24 A new alloy has been devised for use in a space vehicle. Tensile strength measurements are made on 5 pieces of the alloy, and the mean and the standard deviation of these measurements are found to be 39.9 and 2.6, respectively. What is the 90% confidence interval for the mean tensile strength of the alloy? *(Assume that the normality requirement is met.)*

$\bar{x} = 39.9 \quad s = 2.6 \quad n = 5$

(a) 39.9 ± 1.91
(b) 39.9 ± 2.34
(c) 39.9 ± 2.48
(d) 39.9 ± 5.35

$t_{0.05,4} = 2.132$

$39.9 \pm 2.132 \times 2.6/\sqrt{5}$

39.9 ± 2.48

25 The following summary statistics were obtained from a study that used regression analysis to investigate the relationship between pavement deflection and surface temperature of the pavement at various locations on a state highway. Here x = temperature (°F) and y = deflection adjustment factor ($y \geq 0$).

$\sum_i x_i = 1425 \quad \sum_i x_i^2 = 139037.25 \qquad n = 18$

$\sum_i y_i = 10.68 \quad \sum_i y_i^2 = 7.8518$

$\sum_i x_i y_i = 987.645$

$$b = \frac{18 \times 987.645 - 1425 \times 10.68}{18 \times 139037.25 - (1425)^2}$$

$b = 0.0054$

What is the regression line for this relationship $y = g(x)$ (using the method of least squares)?

$a = (10.68 - 0.0054 \times 1425)/18$

$a = 0.1658$

(a) $y = 0.1658 + 0.0054\,x$
(b) $y = 0.5874 - 0.0001\,x$
(c) $y = 23.49 + 93.83\,x$
(d) $y = 43.56 - 0.5427\,x$

26 A manufacturer of nickel-hydrogen batteries randomly selects 100 nickel plates for test cells, cycles them 2400 times, and determines that 14 of the plates have blistered. The manufacturer wants to devise a hypothesis test to establish if more than 10% of the plates blister under these conditions. What hypothesis test should be undertaken? *(Select the most appropriate answer.)*

(a) H_o: p = 0.10, H_1: p > 0.10
(b) H_o: p = 0.14, H_1: p < 0.14
(c) H_o: p = 0.10, H_1: p ≠ 0.10
(d) H_o: p = 0.14, H_1: p ≠ 0.14

27 Hexavalent chromium has been identified as an inhalation carcinogen and an air toxin of concern in a number of different communities. A study into this problem in the Town of Furdor yielded the following data on the indoor and outdoor concentrations (in nanograms per metre squared) for a sample of 9 houses.

House	1	2	3	4	5	6	7	8	9
Indoor	0.07	0.08	0.09	0.12	0.12	0.12	0.13	0.14	0.15
Outdoor	0.29	0.68	0.47	0.54	0.97	0.35	0.49	0.84	0.86

What is the most appropriate test to conduct if we are interested in determining if there is a significant difference between the indoor and outdoor concentrations of Hexavalent chromium? *(Assume that both the indoor and outdoor concentrations follow the normal distribution.)*

σ_D unknown

and paired observations

(a) $Z = \dfrac{\overline{D} - \mu_D}{\sigma_D / \sqrt{n}}$, $D = X_1 - X_2$

(b) $T = \dfrac{\overline{D} - \mu_D}{s_D / \sqrt{n}}$, $D = X_1 - X_2$

(c) $Z = \dfrac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$

(d) $T = \dfrac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{S_p \sqrt{1/n_1 + 1/n_2}}$, $\upsilon = n_1 + n_2 - 2$

28 A mobile maintenance unit is used to patrol a 1120 km length of pipeline. This unit must carry all the equipment necessary to effect repairs on the pipeline, including the installation of temporary sleeves to arrest leaks. New supplies are delivered to the unit only once a week. Previous experience suggests that the unit covers 140 km of pipeline per week, and that on average 0.0143 leaks are found per km of pipeline patrolled. Assuming that the number of leaks follows the Poisson distribution, what is the number of sleeves that should be stocked at the beginning of the week if the maintenance crew wants to be at least 99% confident that they will not run out of repair sleeves?

$\lambda t = 0.0143 \times 140 = 2.0 = \mu$

(a) 2
(b) 6
(c) 7
(d) 26

$P(x \le r) \geq 0.99 \Rightarrow r = 6$

29 Corrosion of steel reinforcing bars is an important problem for reinforced concrete structures. Studies into this problem has resulted in the development of a regression model that relates y = strength of the steel (MPa) to x = carbonation depth (mm). Based on a sample size of 18, the following information has been obtained:

$\sum_i x_i = 659.0$ $\sum_i x_i^2 = 28967.50$ $\bar{x} = 36.61$

$\sum_i y_i = 293.2$ $\sum_i y_i^2 = 5335.76$ $\bar{y} = 16.29$

$\hat{y} = 27.18 - 0.2976 \times 48$

$\hat{y} = 12.90$

$\sum_i x_i y_i = 9293.95$

$S_{xx} = 4840.78$ $r^2 = 0.766$ $s^2 = 8.202$

$s = 2.8639$

Regression Model: $y = 27.18 - 0.2976\,x$

What is the 95% confidence interval for the strength of a reinforcement rod when the carbonation depth is 48 mm? *(Assume that the strength is normally distributed.)*

(a) $12.90 \pm 2.120 \times 0.8219$
(b) $12.90 \pm 2.120 \times 2.3538$
(c) $12.90 \pm 2.120 \times 2.9796$
(d) $12.90 \pm 2.120 \times 8.5331$

$2.8639 \times \left(1 + 1/8 + \frac{(48-36.61)^2}{4840.78}\right)^{1/2}$

$= 2.9795$

$t_{0.025,16} = 2.120$

30 In a study of copper deficiency in cattle, the copper values (μg Cu/100 mL blood) were determined for cattle grazing in two different regions. Region 1 is an area with high metal concentrations, and Region 2 is an area with normal metal concentrations. The following statistics were collected:

$F_{obs} = 1.22$

$s_1 = 21.50$ $n_1 = 16$

$f_{0.01}(15,8) = 5.52$

$s_2 = 19.45$ $n_2 = 9$

$f_{0.99}(15,8) = 1/f_{0.01}(8,15) = 1/4.00 = 0.25$

Using a two-tailed test with $\alpha = 2\%$, does the above data support the hypothesis that the variances of copper in the cattle grazing in the two regions are equal?

(a) Yes, $s_1^2/s_2^2 = 1.22 > 0.18$, and therefore we accept H_o
(b) Yes, $s_1^2/s_2^2 = 1.22 > 0.25$, and therefore we accept H_o
(c) No, $s_1^2/s_2^2 = 1.22 < 4.00$, and therefore we reject H_o
(d) No, $s_1^2/s_2^2 = 1.22 < 5.52$, and therefore we reject H_o

31 A company that manufactures video cameras produces a basic model and a deluxe model. Over the past year, 40% of the cameras sold have been of the basic model. Of those buying the basic model, 30% purchase an extended warranty, whereas 55% of all deluxe purchasers do so. If you learn that a randomly selected purchaser (who purchased a camera this past year) has an extended warranty, how likely is it that this purchaser has a basic model?

$P(B) = 0.4 \quad P(D) = 0.6$

$P(E|B) = 0.3 \quad P(E|D) = 0.55$

(a) 0.120
(b) 0.218
(c) 0.267
(d) 0.450

$$P(B|E) = \frac{0.4 \times 0.3}{0.4 \times 0.3 + 0.6 \times 0.55} = 0.267$$

32 A feeding test is conducted on a herd of 10 milking cows to compare two diets, one of dewatered alfalfa and the other of field-wilted alfalfa. A sample of 6 cows randomly selected from the herd are fed dewatered alfalfa. The remaining 4 cows are fed field-wilted alfalfa. From observations of average daily milk production, the following information was obtained:

Field-wilted alfalfa: $\bar{x}_1 = 45.15$ $s_1^2 = 63.97$
Dewatered alfalfa: $\bar{x}_2 = 42.25$ $s_2^2 = 76.39$

$$S_p^2 = \frac{3 \times 63.97 + 5 \times 76.39}{8}$$

$S_p = 8.47$

What is the 95% confidence interval for the difference in mean daily milk yield (1 - 2) per cow between the two diets. *(Assume that the population variances are equal.)*

(a) -16.63, 22.43
(b) -11.17, 16.97
(c) -9.71, 15.51
(d) -9.42, 15.22

$(45.15 - 42.25) \pm t_{0.025,8}\, s_p \sqrt{1/n_1 + 1/n_2}$

$(45.15 - 42.25) \pm 2.306 \times 8.47 (1/4 + 1/6)^{1/2}$

$(-9.71, 15.51)$

33 Based on a University study, only 20% of all Calgary drivers come to a complete stop at an intersection controlled with four way stops when no other cars are visible. What is the probability that, of 16 randomly chosen drivers coming to an intersection under these conditions, at most 7 cars will not come to a complete stop?

$p = 0.80$

$n = 16$

(a) 0.0002
(b) 0.0015
(c) 0.9930
(d) 0.9998

$P(x \le 7) = 0.0015$

34 Reclaimed phosphate land in Polk County, Florida has been found to emit a higher mean radiation level than other non-mining land in the country. Suppose that the radiation level for the reclaimed land has a distribution with a mean of 5.0 and a standard deviation of 0.5. Suppose that 20 houses built on reclaimed land are randomly selected and the radiation level is measured in each. What is the probability that the sample mean for the 20 houses exceeds 5.2?

(a) ≈ 0
(b) 0.037
(c) 0.345
(d) 0.963

$$P(\bar{x} > 5.2) = P(z > (5.2-5.0)/(0.5/\sqrt{20}))$$
$$= P(z > 1.79)$$
$$= 1 - P(z < 1.79)$$
$$= 1 - 0.9633$$
$$= 0.0367$$

35 An experiment was conducted to compare the mean number of tapeworms in the stomachs of sheep that had been treated for worms against the mean number of those that were untreated. Thirteen worm-infected sheep were randomly divided into two groups; seven were injected with the drug and the remaining six were left untreated. After a 6 month period, worm counts were recorded in all thirteen sheep and shown below.

Treated	5	13	18	6	4	2	15
Untreated	40	54	26	63	21	37	

What is the 95% confidence interval for the difference in the mean worm counts ($\mu_1 - \mu_2$) between the treated (1) and untreated (2) sheep? *(Use 6 degrees of freedom and assume variances are not equal.)*

$\bar{x}_1 = 9.0$ $s_1 = 6.218$ $n_1 = 7$

$\bar{x}_2 = 40.2$ $S_2 = 16.07$ $n_2 = 6$

(a) -17.5, -44.83
(b) -14.12, -48.22
(c) 14.12, 48.22
(d) 17.5, 44.83

$$(\bar{x}_1 - \bar{x}_2) \pm t_{0.025,6}\sqrt{s_1^2/n_1 + s_2^2/n_2}$$

$$t_{0.025,6} = 2.447$$

$(-14.15, -48.25)$

36 A random sample of 1000 working-class people in Great Britain were interviewed to determine each person's political party affiliation. If 680 identified with the major left-of-centre party, what is the 95% confidence interval that estimates the true proportion of Great Britain's working class that identified with the left-of-centre party?

$\hat{p} = 0.68$

(a) 0.651, 0.709
(b) 0.655, 0.705
(c) 0.680, 0.730
(d) 705, 655

$$0.68 \pm z_{0.025} \times \left(\frac{0.68 \times 0.32}{1000}\right)^{1/2}$$

$$z_{0.025} = 1.960$$

$(0.651, 0.709)$

37 A manufacturer of a specific pesticide useful in the control of household bugs claims that his product retains most of its potency after six months and the drop in potency will exhibit at most a standard deviation of 2%. To test the manufacturer's claim, a consumer group obtained a random sample of 20 containers of pesticide from the manufacturer. Each can was tested for potency after being stored for six months and the sample potency was measured and the sample variance of the drops in potencies was computed to be 6.2. What is the value of the test statistic, and does it suggest that there is sufficient evidence to indicate that the population of potency drops has more variability than that claimed by the manufacturer? Use an $\alpha = 0.05$.

$H_0: \sigma^2 = 2^2 = 4$ $\qquad s^2 = 6.2$

$H_1: \sigma^2 > 4$

(a) 29.45; accept H_o
(b) 30.14; accept H_o
(c) 30.14; reject H_o
(d) 58.9; reject H_o

$$\chi^2_{obs} = \frac{19 \times 6.2}{4} = 29.45 < \chi^2_{0.05,19} = 30.144$$

So Accept H_o

38 An article in Applied Spectroscopy discussed the differences in the percentage of iron in laterites using both an atomic absorption method and a classical volumetric technique. The data appears below:

Sample	1	2	3	4	5	6	7	8
Difference (%)	0.94	-0.28	1.50	-0.54	0.13	0.07	0.03	0.32

What is the appropriate statistic and does it suggest a difference on average in the measurements? Use a 10% significance level. $\bar{x} = 0.271$ $\quad s = 0.659$ $\quad n = 8$

$H_o: \mu = 0$

$H_1: \mu \neq 0$

(a) -1.76; reject H_o
(b) 0.3; reject H_o
(c) 1.16; accept H_o
(d) 1.76; accept H_o

$$t_{obs} = \frac{0.271 - 0}{0.659/\sqrt{8}} = 1.16$$

$t_{0.05,7} = 1.895$

$t_{obs} < t_{0.05,7}$

So Accept H_o

39 What is the fifth percentile of the χ^2 Distribution with v = 10?

$\chi^2_{0.95,10} = 3.940$

(a) 3.247
(b) 3.940
(c) 18.307
(d) 20.483

40 A survey is to be conducted to estimate the proportion of citizens who favour imposing trade restraints on Japan. How large should the sample be so that, with 98 percent confidence, the sample proportion will not differ from the true proportion by more than 0.02?

$$n = \frac{z_{\alpha/2}^2}{4e^2} = \frac{z_{0.01}^2}{4e^2} = \frac{2.326^2}{4 \times 0.02^2} \simeq 3382$$

(a) 2,401
(b) 2,639
(c) 3,393
(d) 13,572