A

MAT 1320 B Fall 2016 November 16th, 11:30 Prof. Desjardins

TEST #2

Max = 15

Name: Solutions

Student Number: ______

Circle the DGD which you attend (this is where you will pick up your graded midterm):

Nazanin	Nicole	Mona	Alex	Mona
10:00 (DGD1)	11:30 (DGD2)	13:00 (DGD3)	14:30 (DGD4)	16:00 (DGD5)
MRT 211	SMD 330	FTX 137	MRT 219	SMD 226

• Time: 80 min.

• No calculators are permitted.

• There are 5 multiple choice questions worth 1 mark each and 3 problems worth 10 marks.

• For the multiple choice questions, circle the letter of your choice.

• Work all problems in the space provided. Use the backs of the pages for rough work if necessary. Do not use any other paper.

• Problems require complete and clearly presented solutions and carry part marks if there is substantial correct work toward the solution.

• Cellular phones, unauthorized electronic devices or course notes are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam, academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam. By signing below, you acknowledge that you have ensured that you are complying with the above statement.

Signature: ______

A

1. [1 point] If we use the linear approximation of $f(x) = \arctan(2x)$ at $a = 0$ to approximate $\arctan(0.05)$, it will give?

(A) 0 (B) 0.05 (C) 0.10 (D) 0.15 (E) 0.20 (F) 0.25

$f(a) = f(0) = \arctan(0) = 0$

$f'(x) = \frac{2}{1+(2x)^2} \Rightarrow f'(a) = f'(0) = 2$

$L(x) = f(0) + f'(0)(x-0) = 2x$

2. [1 point] What is $\frac{d}{dx}\left(\int_x^{x^2} f(t)\,dt\right)$?

(A) $f(x^2)$ (B) $2xf(x^2)$ (C) $f(x^2) - f(x)$

(D) $2xf(x^2) - f(x)$ (E) $f(x) - f(x^2)$ (F) $f(x) - 2xf(x^2)$

3. [1 point] If we wish to integrate $\int \frac{x^3}{\sqrt{9-x^2}}\,dx$ with a trigonometric substitution, we would let $x = $?

(A) $3\tan\theta$ (B) $3\sin\theta$ (C) $\sec\theta$ (D) $\tan\theta$ (E) $\sin\theta$ (F) $9\sin\theta$

A

4. [1 point] The length of the sides of a cube is increasing at a rate of 2 cm/s. At the moment when the volume is 64 cm^3, at what rate is the volume increasing (in cm^3/s) ?

(A) 16 (B) 32 (C) 48 (D) 64 (E) 96 (F) 128

$V = l^3$ told $\frac{dl}{dt} = 2$ cm/s

want $\frac{dV}{dt}$ when $V = 64\ \text{cm}^3 \Rightarrow l = 4$ cm

$\frac{dV}{dt} = 3l^2 \frac{dl}{dt} = 3(4)^2(2)$

5. [1 point] The derivative of $f(x) = x^{\cos x}$ has the form $f'(x) = f(x)\,p(x)$. What is $p(x)$?

(A) $x^{\cos x}$ (B) $x^{-\sin x}$ (C) $\frac{\cos x}{x} - \sin x \ln x$

(D) $\frac{\cos x}{x} + \sin x \ln x$ (E) $\frac{\sin x}{x} - \cos x \ln x$ (F) $\frac{-\sin x}{x} + \cos x \ln x$

$\ln f(x) = \ln\left(x^{\cos x}\right) = \cos x \ln x$

so $\frac{1}{f(x)} f'(x) = -\sin x \ln x + \frac{\cos x}{x}$

A

6. [2 points] Find $\frac{dy}{dx}$ if $5x^3y^4 + 3xy^3 = 7x$.

$$\frac{d}{dx}(5x^3y^4 + 3xy^3) = \frac{d}{dx}(7x)$$

$$15x^2y^4 + 20x^3y^3y' + 3y^3 + 9xy^2y' = 7$$

so $$\boxed{y' = \frac{7 - 15x^2y^4 - 3y^3}{20x^3y^3 + 9xy^2}}$$

7. [2 points] Set up the approximation of the definite integral $\int_0^1 e^x\,dx$ with $n = 4$ rectangles using the right-hand endpoints (*ie* R_4). You do not need to evaluate the sum.

$$n = 4 \Rightarrow \Delta x = \frac{1-0}{4} = 0.25$$

so $x_0 = 0$, $x_1 = 0.25$, $x_2 = 0.5$, $x_3 = 0.75$, $x_4 = 1$

$$R_4 = \sum_{j=1}^{4} f(x_j)\,\Delta x$$

$$= \frac{1}{4}\left[e^{x_1} + e^{x_2} + e^{x_3} + e^{x_4}\right]$$

$$= \boxed{\frac{1}{4}\left(e^{0.25} + e^{0.5} + e^{0.75} + e^{1}\right)}$$

A

8. [6 points] Evaluate the following integrals:

(a) $\int \sin^7 x \cos^3 x \, dx$

$= \int \sin^7 x \cos^2 x \cos x \, dx$

$= \int \sin^7 x (1 - \sin^2 x) \cos x \, dx$

$= \int (\sin^7 x - \sin^9 x) \cos x \, dx$

$= \boxed{\frac{1}{8} \sin^8 x - \frac{1}{10} \sin^{10} x + C}$

(b) $\int_1^e \left(\frac{4(\ln x)^3}{x} - \frac{2(\ln x)^{1/3}}{x} \right) dx$

$\left(\begin{array}{ll} \text{let } u = \ln x & x = 1 \Rightarrow u = 0 \\ du = \frac{1}{x} dx & x = e \Rightarrow u = 1 \end{array} \right)$

$= \int_0^1 (4u^3 - 2u^{1/3}) \, du$

$= u^4 - \frac{3}{2} u^{4/3} \Big|_0^1$

$= \left(1 - \frac{3}{2}\right) - (0) = \boxed{-1/2}$

(c) $\int x^2 e^{-x} \, dx$

$\left(\begin{array}{ll} u = x^2 & dv = e^{-x} dx \\ du = 2x \, dx & v = -e^{-x} \end{array} \right)$

$= -x^2 e^{-x} - \int 2x (-e^{-x}) dx$

$= -x^2 e^{-x} + 2 \int x e^{-x} \, dx$

$\left(\begin{array}{ll} u = x & dv = e^{-x} dx \\ du = dx & v = -e^{-x} \end{array} \right)$

$= -x^2 e^{-x} + 2 \left(-x e^{-x} - \int -e^{-x} dx \right)$

$= \boxed{-x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + C}$

(B)

MAT 1320 B   Fall 2016.   November 16th, 11:30   Prof. Desjardins

TEST #2

Max = 15

Name: Solutions

Student Number: ______________________

Circle the DGD which you attend (this is where you will pick up your graded midterm):

Nazanin	Nicole	Mona	Alex	Mona
10:00 (DGD1)	11:30 (DGD2)	13:00 (DGD3)	14:30 (DGD4)	16:00 (DGD5)
MRT 211	SMD 330	FTX 137	MRT 219	SMD 226

• Time: 80 min.

• No calculators are permitted.

• There are 5 multiple choice questions worth 1 mark each and 3 problems worth 10 marks.

• For the multiple choice questions, circle the letter of your choice.

• Work all problems in the space provided. Use the backs of the pages for rough work if necessary. Do not use any other paper.

• Problems require complete and clearly presented solutions and carry part marks if there is substantial correct work toward the solution.

• Cellular phones, unauthorized electronic devices or course notes are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam, academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam. By signing below, you acknowledge that you have ensured that you are complying with the above statement.

Signature: ______________________

B

1. [1 point] If we use the linear approximation of $f(x) = \arctan(3x)$ at $a = 0$ to approximate $\arctan(0.05)$, it will give?

(A) 0 (B) 0.05 (C) 0.10 (D) 0.15 (E) 0.20 (F) 0.25

$f(0) = 0$ $f'(x) = \frac{3}{1+(3x)^2} \Rightarrow f'(0) = 3$

$L(x) = 0 + 3(x-0) = 3x$

2. [1 point] What is $\frac{d}{dx}\left(\int_{x^2}^{x} f(t)\,dt\right)$?

(A) $f(x^2)$ (B) $2xf(x^2)$ (C) $f(x^2) - f(x)$

(D) $2xf(x^2) - f(x)$ (E) $f(x) - f(x^2)$ (F) $f(x) - 2xf(x^2)$

3. [1 point] If we wish to integrate $\int \frac{x^3}{\sqrt{9+x^2}}\,dx$ with a trigonometric substitution, we would let $x = $?

(A) $3\tan\theta$ (B) $3\sin\theta$ (C) $\sec\theta$ (D) $\tan\theta$ (E) $\sin\theta$ (F) $9\sin\theta$

B

4. [1 point] The length of the sides of a cube is increasing at a rate of 2/3 cm/s. At the moment when the volume is 64 cm^3, at what rate is the volume increasing (in cm^3/s) ?

(A) 16 (B) 32 (C) 48 (D) 64 (E) 96 (F) 128

$$\frac{dV}{dt} = 3l^2 \frac{dl}{dt} = 3(4)^2(2/3)$$

5. [1 point] The derivative of $f(x) = x^{\sin x}$ has the form $f'(x) = f(x)\,p(x)$. What is $p(x)$?

(A) $x^{\cos x}$ (B) $x^{-\sin x}$ (C) $\frac{\cos x}{x} - \sin x \ln x$

(D) $\frac{\cos x}{x} + \sin x \ln x$ (E) $\frac{\sin x}{x} + \cos x \ln x$ (F) $\frac{-\sin x}{x} + \cos x \ln x$

$$\ln(f(x)) = \sin x \ln x$$

so $\frac{1}{f(x)} f'(x) = \cos x \ln x + \frac{\sin x}{x}$

(B)

6. [2 points] Find $\frac{dy}{dx}$ if $4x^2y^5 + 2xy^4 = 5x$.

$$\frac{d}{dx}(4x^2y^5 + 2xy^4) = \frac{d}{dx}(5x)$$

$$8xy^5 + 20x^2y^4y' + 2y^4 + 8xy^3y' = 5$$

so $\boxed{y' = \frac{5 - 8xy^5 - 2y^4}{20x^2y^4 + 8xy^3}}$

7. [2 points] Set up the approximation of the definite integral $\int_0^1 e^x\,dx$ with $n = 4$ rectangles using the left-hand endpoints (*ie* L_4). You do not need to evaluate the sum.

$n = 4 \Rightarrow \Delta x = 0.25$

$x_0 = 0,\ x_1 = 0.25,\ x_2 = 0.5,\ x_3 = 0.75,\ x_4 = 1$

$$L_4 = \sum_{j=1}^{4} f(x_{j-1})\Delta x$$

$$= \frac{1}{4}(e^{x_0} + e^{x_1} + e^{x_2} + e^{x_3})$$

$$= \boxed{\frac{1}{4}(e^0 + e^{0.25} + e^{0.5} + e^{0.75})}$$

(B)

8. [6 points] Evaluate the following integrals:

(a) $\displaystyle\int \cos^5 x \sin^3 x\, dx$

$= \int \cos^5 x \sin^2 x \sin x\, dx$

$= \int \cos^5 x (1-\cos^2 x) \sin x\, dx$

$= \int (\cos^5 x - \cos^7 x) \sin x\, dx$

$= \boxed{\frac{1}{8}\cos^8 x - \frac{1}{6}\cos^6 x + C}$

(b) $\displaystyle\int_1^e \left(\frac{5(\ln x)^4}{x} - \frac{3(\ln x)^{2/3}}{x}\right) dx$ $\quad (u = \ln x)$

$= \int_0^1 (5u^4 - 3u^{2/3})\, du$

$= u^5 - \frac{9}{5}u^{5/3} \Big|_0^1$

$= (1 - \frac{9}{5}) - (0) \quad = \boxed{-4/5}$

(c) $\displaystyle\int x^2 e^{3x}\, dx$ $\quad \left(\begin{array}{ll} u = x^2 & dv = e^{3x}dx \\ du = 2x\,dx & v = \frac{1}{3}e^{3x} \end{array}\right)$

$= \frac{1}{3}x^2e^{3x} - \int \frac{1}{3}(2x)e^{3x}\, dx$

$= \frac{1}{3}x^2e^{3x} - \frac{2}{3}\int xe^{3x}\, dx$ $\quad \left(\begin{array}{ll} u = x & dv = e^{3x}dx \\ du = dx & v = \frac{1}{3}e^{3x} \end{array}\right)$

$= \frac{1}{3}x^2e^{3x} - \frac{2}{3}\left(\frac{1}{3}xe^{3x} - \int \frac{1}{3}e^{3x}dx\right)$

$= \boxed{\frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x} + C}$

(C)

MAT 1320 B Fall 2016 November 16th, 11:30. Prof. Desjardins

TEST #2

Max = 15

Name: Solutions

Student Number: ______

Circle the DGD which you attend (this is where you will pick up your graded midterm):

Nazanin	Nicole	Mona	Alex	Mona
10:00 (DGD1)	11:30 (DGD2)	13:00 (DGD3)	14:30 (DGD4)	16:00 (DGD5)
MRT 211	SMD 330	FTX 137	MRT 219	SMD 226

• Time: 80 min.

• No calculators are permitted.

• There are 5 multiple choice questions worth 1 mark each and 3 problems worth 10 marks.

• For the multiple choice questions, circle the letter of your choice.

• Work all problems in the space provided. Use the backs of the pages for rough work if necessary. Do not use any other paper.

• Problems require complete and clearly presented solutions and carry part marks if there is substantial correct work toward the solution.

• Cellular phones, unauthorized electronic devices or course notes are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam, academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam. By signing below, you acknowledge that you have ensured that you are complying with the above statement.

Signature: ______

C

1. [1 point] If we use the linear approximation of $f(x) = \arctan(4x)$ at $a = 0$ to approximate $\arctan(0.05)$, it will give?

(A) 0 (B) 0.05 (C) 0.10 (D) 0.15 (E) 0.20 (F) 0.25

$f(0) = 0 \qquad f'(x) = \frac{4}{1+(4x)^2} \Rightarrow f'(0) = 4$

$L(x) = 4x$

2. [1 point] What is $\frac{d}{dx}\left(\int_x^{x^3} f(t)\,dt\right)$?

(A) $f(x^3)$ (B) $3x^2 f(x^3)$ (C) $f(x) - 3x^2 f(x^3)$

(D) $f(x^3) - f(x)$ (E) $f(x) - f(x^3)$ (F) $3x^2 f(x^3) - f(x)$

3. [1 point] If we wish to integrate $\int \frac{x^4}{16+x^2}\,dx$ with a trigonometric substitution, we would let $x = $?

(A) $2\tan\theta$ (B) $2\sin\theta$ (C) $2\sec\theta$ (D) $4\tan\theta$ (E) $4\sin\theta$ (F) $16\sin\theta$

C

4. [1 point] The length of the sides of a cube is increasing at a rate of 4/3 cm/s. At the moment when the volume is 64 cm^3, at what rate is the volume increasing (in cm^3/s) ?

(A) 16 (B) 32 (C) 48 (D) 64 (E) 96 (F) 128

$$\frac{dV}{dt} = 3l^2 \frac{dl}{dt} = 3(4)^2 (4/3)$$

5. [1 point] The derivative of $f(x) = (\sin x)^x$ has the form $f'(x) = f(x)\,p(x)$. What is $p(x)$?

(A) $x^{\cos x}$ (B) $x^{-\sin x}$ (C) $x^{\tan x}$

(D) $\ln(\cos x) - x\tan x$ (E) $x\cot x - \ln(\sin x)$ (F) $\ln(\sin x) + x\cot x$

$$\ln(f(x)) = x \ln(\sin x)$$

$$\frac{1}{f(x)} f'(x) = \ln(\sin x) + x \frac{\cos x}{\sin x}$$

C

6. [2 points] Find $\frac{dy}{dx}$ if $4x^3y^3 + 7xy^2 = 6x$.

$$\frac{d}{dx}(4x^3y^3 + 7xy^2) = \frac{d}{dx}(6x)$$

$$12x^2y^3 + 12x^3y^2y' + 7y^2 + 14xyy' = 6$$

$$\text{so } \boxed{y' = \frac{6 - 12x^2y^3 - 7y^2}{12x^3y^2 + 14xy}}$$

7. [2 points] Set up the approximation of the definite integral $\int_0^1 e^x \, dx$ with $n = 4$ rectangles using the midpoints (*ie* M_4). You do not need to evaluate the sum.

$n = 4 \Rightarrow \Delta x = 0.25$

$x_0 = 0,\ x_1 = 0.25,\ x_2 = 0.5,\ x_3 = 0.75,\ x_4 = 1$

$\bar{x}_1 = 0.125,\ \bar{x}_2 = 0.375,\ \bar{x}_3 = 0.625,\ \bar{x}_4 = 0.875$

$$M_4 = \sum_{j=1}^{4} f(\bar{x}_j)\Delta x$$

$$= \frac{1}{4}\left(e^{\bar{x}_1} + e^{\bar{x}_2} + e^{\bar{x}_3} + e^{\bar{x}_4}\right)$$

$$= \boxed{\frac{1}{4}\left(e^{0.125} + e^{0.375} + e^{0.625} + e^{0.875}\right)}$$

C

8. [6 points] Evaluate the following integrals:

(a) $\displaystyle\int \sin^5 x \cos^3 x \, dx$

$= \int \sin^5 x \cos^2 x \cos x \, dx$

$= \int \sin^5 x \, (1-\sin^2 x) \cos x \, dx$

$= \int (\sin^5 x - \sin^7 x) \cos x \, dx$

$= \boxed{\frac{1}{6}\sin^6 x - \frac{1}{8}\sin^8 x + C}$

(b) $\displaystyle\int_1^e \left(\frac{7(\ln x)^3}{x} + \frac{4(\ln x)^{2/3}}{x} \right) dx$ $\quad (u = \ln x)$

$= \int_0^1 (7u^3 + 4u^{2/3}) \, du$

$= \left(\frac{7}{4}u^4 + \frac{12}{5}u^{5/3} \right) \Big|_0^1$

$= \left(\frac{7}{4} + \frac{12}{5} \right) - 0 = \boxed{83/20}$

(c) $\displaystyle\int x^2 e^{4x} \, dx$ $\quad \begin{pmatrix} u = x^2 & dv = e^{4x}dx \\ du = 2x\,dx & v = \frac{1}{4}e^{4x} \end{pmatrix}$

$= \frac{1}{4}x^2 e^{4x} - \int \frac{1}{4}(2x)e^{4x} \, dx$

$= \frac{1}{4}x^2 e^{4x} - \frac{1}{2}\int x e^{4x} \, dx$ $\quad \begin{pmatrix} u = x & dv = e^{4x}dx \\ du = dx & v = \frac{1}{4}e^{4x} \end{pmatrix}$

$= \frac{1}{4}x^2 e^{4x} - \frac{1}{2}\left(\frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x} dx \right)$

$= \boxed{\frac{1}{4}x^2 e^{4x} - \frac{1}{8}xe^{4x} + \frac{1}{32}e^{4x} + C}$