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1 a) $\operatorname{curl} \vec{F} = \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y^2 & 2xy + e^{3z} & 3ye^{3z} \end{pmatrix}$

$= \langle 3e^{3z} - 3e^{3z}, 0-0, 2y-2y \rangle = \langle 0, 0, 0 \rangle$

b) $\varphi_x = y^2 \Rightarrow \varphi = y^2 x + g(y,z)$

$\Rightarrow \varphi_y = 2xy + g_y(y,z) = 2xy + e^{3z} = g_y(y,z) = e^{3}$

$\Rightarrow g(y,z) = ye^{3z} + h(z)$

$\Rightarrow \varphi = y^2 x + ye^{3z} + h(z)$

$\Rightarrow \varphi_z = 3ye^{3z} + h'(z) = 3ye^{3z}$

$\Rightarrow h'(z) = 0 \Rightarrow h(z)$ constant (say 0)

$\Rightarrow \boxed{\varphi = y^2 x + ye^{3z}}$

c) $\varphi(36, e^{15}, 7) - \varphi(0, 1, 7)$

$= (e^{15})^2 \cdot 36 + e^{15} e^{21} - 0 - 1 \cdot e^{21}$

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(2) $z = f(x,y) = 7 - x^2 - 2y^2$

$\nabla f = \langle -2x, -4y \rangle = \langle -2, -4 \rangle$ at $(1,1)$

a) DD in direction $\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

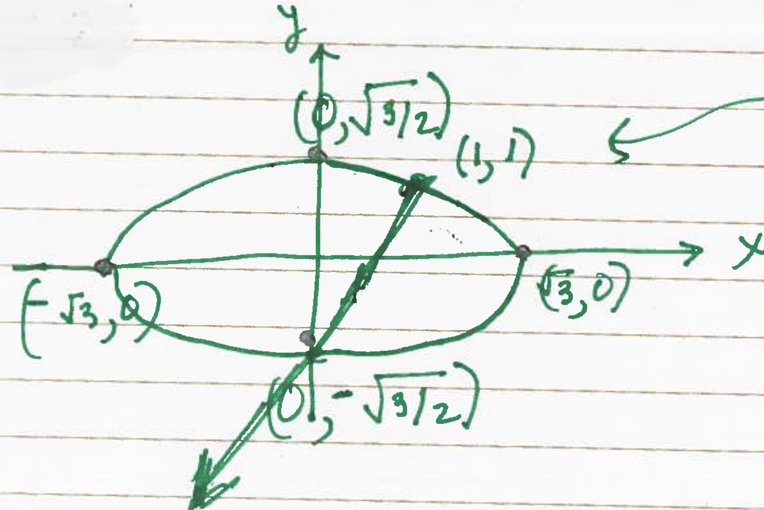
is $\langle -2, -4 \rangle \cdot \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle = -\frac{6}{\sqrt{2}} = -3\sqrt{2}$

b) $\frac{-\nabla f}{\|\nabla f\|} = \frac{\langle 2, 4 \rangle}{\sqrt{20}}$ is direction

DD is

$\nabla f \cdot \frac{\langle 2,4 \rangle}{\sqrt{20}} = \langle -2, -4 \rangle \cdot \frac{\langle 2, 4 \rangle}{\sqrt{20}} = -\sqrt{20}$

c) $f(x,y) = 4$ becomes $x^2 + 2y^2 = 3$



grad. is $\perp$ to curve

not to scale

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(3) $\theta = 45^\circ \therefore \cos\theta = \sin\theta = \sqrt{2}/2$

$$x(t) = \left(\sqrt{18}\cos\theta\right)t = 3t$$

$$y(t) = -\frac{1}{2}\cdot 32t^2 + \left(\sqrt{18}\sin\theta\right)t + 100$$

$$= -16t^2 + 3t + 100$$

a) $y(t) = 0 \implies t \approx 2.6$ sec (by quadratic formula)

b) $x(2.6) \cong 7.8$ ft

(4) $\vec{r} = \langle \cos 2t, \sin 2t, 6t \rangle$

$\vec{r}\,' = \langle -2\sin 2t, 2\cos 2t, 6 \rangle$

$\vec{r}\,'' = \langle -4\cos 2t, -4\sin 2t, 0 \rangle$

$\|\vec{r}\,'\| = \sqrt{40}$

a) $L = \int_0^1 \sqrt{40}\, dt = \sqrt{40}$

b) $K = \dfrac{\|\vec{r}\,'(0) \times r''(0)\|}{\|r'(0)\|^3}$

at $t=0$

$$\begin{pmatrix} i & j & k \\ 0 & 2 & 6 \\ -4 & 0 & 0 \end{pmatrix}$$

$$= \frac{\|\langle 0, -24, 8 \rangle\|}{40^{3/2}}$$

$$= \frac{\sqrt{640}}{40^{3/2}} = .1$$

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(5) $f(x,y,z) = 2x^3 - e^{xy} - z = 0$

$$\nabla f = \langle 6x^2, -xe^{xy}, -1 \rangle$$

$$= \langle 6, -1, -1 \rangle \quad \text{at} \quad (1,0,1)$$

TP : $\begin{pmatrix} 6 \\ -1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} x-1 \\ y-0 \\ z-1 \end{pmatrix} = 0$

$$6x - y - z = 5$$

NL $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} 6 \\ -1 \\ -1 \end{pmatrix}$

$x = 1 + 6t$

$y = -t$

$z = 1 - t$

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⑥

a) $$\text{Area} = \frac{1}{2}\left\|\langle 2,3,4\rangle \times \langle 7,0,3\rangle\right\|$$

$$\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 3 & 4 \\ 7 & 0 & 3 \end{vmatrix}$$

$$\langle 9, 22, -21\rangle$$

$$\therefore \quad \text{Area} = \frac{1}{2}\sqrt{81 + 22^2 + 21^2}$$

$$= \frac{1}{2}\sqrt{1006} \approx 15.86$$

b) Think of volume of parallelopiped:

$$|\vec{a} \cdot (\vec{b} \times \vec{c})|$$

$$\left| \langle 1,4,7\rangle \cdot \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & -1 & 4 \\ 0 & -9 & 18 \end{vmatrix} \right|$$

$$= |\langle 1,4,7\rangle \cdot \langle 18, -36, -18\rangle|$$

$$= 0 \quad \therefore$$ 3 vectors coplanar (0-volume)