MAT 2371
Mid-Term test

Fall 2016 **Professor Mahmoud Zarepour**
Duration: 80 minutes

Student Number:__________

First Name: ______________ **Last Name:** ______________________

- This is a closed book exam. Formula sheets and the required tables are attached to this exam.
- Only the calclators TI 30, TI 34, Casio fx-260 and Casio fx-300 are allowed.
- This exam has two parts: A (detailed answers) and B (mutiple choice questions). For part (B), write the true answers for each question in the table provided in the next page. For part (A), write your detailed responses in the space provided.
- At the end of exam you need to submit the complete exam booklet.

Cellular phones, unauthorized electronic devices or course notes (unless an open-book exam) are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam and academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam.
By signing below, you acknowledge that you have ensured that you are complying with the above statement.

X____________________________________

For use of Professors only:

Part A		
Part B	1	
	2	
	3	
	4	
TOTAL		

Part A: detailed questions. 15 points for each question.

1. We perform Bernoulli experiments with the success probability of 0.2, until we obtain a success. We define X as the number of fails until we obtain a success.

(i) Compute $P(X = i)$, with $i = 0, 1, 2, ...$
(ii) Compute $P(X > 3|X > 2)$.
(iii) Find $E[X]$.

solution (i). X has a geometric distribution as follows:

$$P(X = i) = (0.2)(0.8)^i, i = 0, 1, 2, \ldots.$$

(ii)

$$P(X > 3|X > 2) = P(X \geq 4|X \geq 3) = \frac{\frac{(0.2)(0.8^4)}{1-0.8}}{\frac{(0.2)(0.8^4)}{1-0.8}} = 0.8.$$

(iii). We have

$$E(X) = \sum_{i=0}^{\infty} i(0.2)(0.8)^i = (0.2)(0.8) \sum_{i=1}^{\infty} i(0.8)^{i-1}.$$

Using the last formula on the formula sheet concludes

$$E(X) = (0.2)(0.8)(1 - 0.8)^{-2} = 4.$$

2. Suppose A and B are two events such that $P(A) = 1/5$ and $P(B) = 3/10$.

(i) If A and B are independent find $P(A \cup B)$.
(ii) If $P(A \cap B) > 0$, calculate $P(A \cup B|A \cap B)$.
(iii) If A and B are independent and $P(A \cap B) > 0$ find $P(A \cap B|A \cup B)$.

solution (i):

$$P(A \cup B) = P(A) + P(B) - P(A)P(B) = 0.2 + 0.3 - (0.2)(0.3) = 0.44.$$

(ii)

$$P(A \cup B|A \cap B) = \frac{P(A \cap B)}{P(A \cup B)} = 1.$$

(iii)

$$P(A \cap B|A \cap B) = \frac{P(A \cap B)}{P(A \cup B)} = \frac{(0.2)(0.3)}{0.44} = \frac{3}{22}.$$

3. An urn contatins 7 red balls and 10 white balls. Draw at random a sample of 5 balls from this urn. Find the probabaility that 2 balls are white under two different sampling schemes:

(i) sampling is with replacement.
(ii) Sampling is without replacement.

solution (i):

$$P(\text{2 White balls}) = \binom{5}{2}(10/17)^2(7/17)^3 = 0.2415736.$$

(ii)

$$P(\text{2 White balls}) = \frac{\binom{10}{2}\binom{7}{3}}{\binom{17}{5}} = 0.2545249.$$

As you can results are not very different. This is due to the fact that box contains 17 balls and both sampling with replacement and sampling with replacement should be close.

4. The probability distribution of X, the number of typographical errors per page in online manuscripts are given by

x	0	1	2	3
$f(x)$	$3c/7$	$2c/7$	$c/7$	c

(i) Find c.
(ii) Find $E(X)$ and $Var(X)$
(iii) Find the moment generating function for X.

solution (i): We need to have

$$\sum_{x=0}^{3} f(x) = 1.$$

This gives $c = 7/13$.

(ii).

$$E(X) = \sum_{x=0}^{3} x f(x) = 25c/7 = 25/13 \approx= 1.923.$$

$$E(X^2) = \sum_{x=0}^{3} x^2 f(x) = 69c/7 = 69/13 \approx 5.308.$$

Therefore

$$\sigma_X^2 = E(X^2) - (E(X))^2 = 69/13 - (25/13)^2 = 1.609467.$$

$$M(t) = E(e^{tX}) = 3/13 + 2e^t/13 + e^{2t}/13 + 7e^{3t}/13 = \frac{3 + 2e^t + e^{2t} + 7e^{3t}}{13}.$$

Part B: mutiple choice questions. 10 points for each question.

Write the correct answers (the corresponding letter) in the space provided in the table below:

Question	1	2	3	4
Response				

1. From an announcement by Air Canada it is known that 10% of clients reserve first class seats. Among the next 20 reservations, what is the probability that at least one of them reserves a first class seat if reservations are made independently ?

   (a) 0.2701
   (b) 0.1216
   (c) 0.8784
   (d) 0.9865
   (e) 0.9999.

   Answer is c

2. Determine how many different words (meaningful or meaningless) can be created by reordering the letters used in the word

   STATISTICS.

   (a) 3628800
   (b) 100
   (c) 50400
   (d) 66000
   (e) 125000

   Answer is c

3. If $E(X^2) = E((X-2)^2) = 5$ for some random variable X, find its variance σ_X^2.

(a) 2.75
(b) 3
(c) 3.25
(d) 3.5
(e) 4

Answer is (e)

4. In a certain city, 25 percent of all cars emit excessive amount of pollutants. If the probability is 0.99 that a car emitting excessive amount of pollutants will fail in an emission test, and the probability is 0.17 that a car not emitting excessive pollutants will fail the test, what is the probability that a car that fails the test actually emits excessive amount of pollutants

(a) 0.55
(b) 0.45
(c) 0.66
(d) 0.001
(e) 0.39

(USE BAYE's RULE). Answer is c.

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Final Exam Formula Sheet

- Addition Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Conditional probability of A given B:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- Total probability rule:
  $P(A) = P(A \cap B) + P(A \cap B') = P(A|B)P(B) + P(A|B')P(B')$
- Bayes' rule

$$P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|B')P(B')}$$

- Events A and B are independent if $P(A \cap B) = P(A)P(B)$
- Expected value of a discrete random variable X:

$$\mu = E(X) = \sum_x x f(x), \quad \text{where} \quad f(x) = P(X = x)$$

- Variance of a discrete random variable X:

$$\sigma^2 = \mathrm{Var}(X) = \sum_x (x-\mu)^2 f(x) = \sum_x x^2 f(x) - \mu^2, \quad \text{where} \quad f(x) = P(X = x)$$

- Cumulative distribution function of a random variable X: $F(x) = P(X \leq x)$
- If X is a binomial random variable with n trials and probability p of success, then

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad k = 0, 1, \ldots, n$$

  and its mean and variance are respectively $n\,p$ and $n\,p\,(1-p)$.
- If X is a geometric random variable with probability p of success, then

$$P(X = k) = (1-p)^{k-1}\,p, \quad k = 1, 2, 3, \ldots$$

  and its mean and variance are respectively $1/p$ and $(1-p)/p^2$.

- If X is a Poisson random variable with mean λ, then

$$P(X=k)=e^{-\lambda}\frac{\lambda^x}{x!}, \quad k=0,1,2,\ldots$$

and its mean and variance are both λ.

- If X has Poisson distribution (with $E[X]=\lambda>5$) or a binomial distribution (with $n\,p>5$ and $n\,(1-p)>5$), then X has an approximate normal distribution. A continuity correction should be applied in the approximation.

- If X has an exponential distribution with mean $E[X]=1/\lambda$. Its p.d.f. is

$$f(x)=\lambda\,e^{-\lambda\,x}, \quad \text{for } x>0,$$

and its variance is $1/\lambda^2$.

- Standardization: If X is a normal random variable with mean μ and variance σ^2, then

$$Z=\frac{X-\mu}{\sigma} \quad \text{has a standard normal distribution}$$

- Sample mean of the observations $x_1,\ldots,x_n$: $\bar{x}=\frac{1}{n}\sum_{i=1}^{n}x_i$

- Central Limit Theorem: for a random sample $X_1,\ldots,X_n$ from a population with a mean μ and a variance σ^2, then

$$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \text{ has approximately a standard normal distribution, when } n \text{ is large}$$

- The number of combinations, subsets of size r that can be selected from a set of n elements, is

$$C_r^n=\binom{n}{r}=\frac{n!}{(n-r)!\,r!}.$$

- The number of permutations, arrangements of size r that can be selected from a set of n elements, is

$$P_r^n=\frac{n!}{(n-r)!}.$$

- For $|x|<1$, we have

$$\sum_{i=1}^{\infty} ix^{i-1}=(1-x)^{-2}.$$