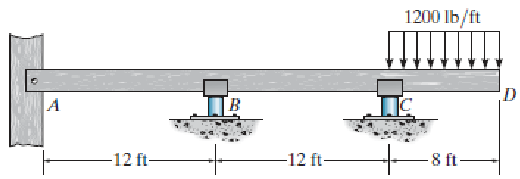
Displacement Methods

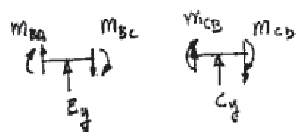
A- *Slope Deflection Method*

Problem 1

Determine the moments at B and C of the overhanging beam, then draw the bending moment diagram. EI is constant. Assume the beam is supported by a pin at A and rollers at B and C.



SOLUTION



$\psi_{AB} = \psi_{BC} = 0$

Applying Eqs. 11–8 and 11–10,

$$M_{BA} = \frac{3EI}{12}(\theta_B) + 0$$

$$M_{BC} = \frac{2EI}{12}(2\theta_B + \theta_C) + 0$$

$$M_{CB} = \frac{2EI}{12}(2\theta_C + \theta_B) + 0$$

$$M_{CD} = -(1.20)(8)(4) = -38.4\ \text{k}\cdot\text{ft}$$ **Ans.**

Moment equilibrium at B and C:

$$M_{BA} + M_{BC} = 0$$

$$M_{CB} + M_{CD} = 0$$

$$\frac{3EI}{12}\theta_B + \frac{2EI}{12}(2\theta_B + \theta_C) = 0$$

$$\frac{2EI}{12}(2\theta_C + \theta_B) - 38.4 = 0$$

$$\theta_B = \frac{-38.4}{EI}; \qquad \theta_C = \frac{134.4}{EI}$$

Thus,

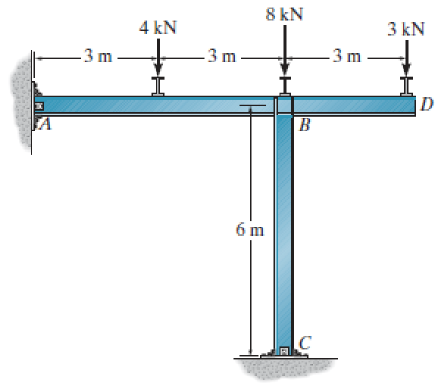
$$M_{BA} = \frac{3EI}{12}\left(\frac{-38.4}{EI}\right) = -9.60\ \text{k}\cdot\text{ft}$$ **Ans.**

$$M_{BC} = \frac{2EI}{12}\left(2\left(\frac{-38.4}{EI}\right) + \frac{134.4}{EI}\right) = 9.60\ \text{k}\cdot\text{ft}$$ **Ans.**

$$M_{CB} = \frac{2EI}{12}\left(2\left(\frac{134.4}{EI}\right) + \left(\frac{-38.4}{EI}\right)\right) = 38.4\ \text{k}\cdot\text{ft}$$ **Ans.**

Problem 2

Determine the moments at the ends of each member of the frame. The supports at A and C and joint B are fixed connected. EI is constant.



SOLUTION

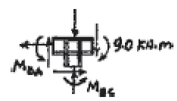
$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = \frac{2EI}{6}(0 + \theta_B) - \frac{(4)(6)}{8}$$

$$M_{BA} = \frac{2EI}{6}(2\theta_B) + \frac{4(6)}{8}$$

$$M_{BC} = \frac{2EI}{6}(2\theta_B)$$

$$M_{CB} = \frac{2EI}{6}(\theta_B)$$



Equilibrium

$$M_{BC} - 9 = 0$$

$$\frac{2EI}{6}(2\theta_B) + \frac{4(6)}{8} + \frac{2EI}{6}(2\theta_B) - 9 = 0$$

$$\theta_B = \frac{4.5}{EI}$$

Thus,

$$M_{AB} = \frac{2EI}{6}\left(0 + \frac{4.5}{EI}\right) - \frac{4(6)}{8} = -1.50\ \text{kN}\cdot\text{m}$$ **Ans.**

$$M_{CB} = \frac{2EI}{6}\left(\frac{4.5}{EI}\right) = 1.50\ \text{kN}\cdot\text{m}$$ **Ans.**

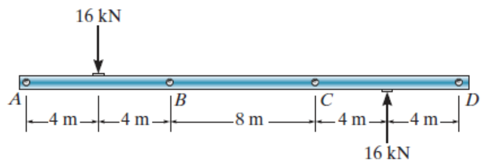
$$M_{BA} = \frac{2EI}{6}\left(2\frac{4.5}{EI}\right) - \frac{4(6)}{8} = 6.00\ \text{kN}\cdot\text{m}$$ **Ans.**

$$M_{BC} = \frac{2EI}{6}\left(2\left(\frac{4.5}{EI}\right)\right) = 3.00\ \text{kN}\cdot\text{m}$$ **Ans.**

B- Moment Distribution Method

Problem 1

The bar is pin connected at each indicated point. If the normal force in the bar can be neglected, determine the vertical reaction at each pin. EI is constant.



SOLUTION

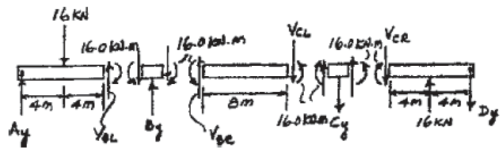
Use antisymmetric load and symmetric beam.

$$K_{BA} = \frac{3EI}{8} \qquad K_{BC} = \frac{6EI}{8}$$

$$(\text{DF})_{BA} = \frac{\frac{3EI}{8}}{\frac{3EI}{8} + \frac{6EI}{8}} = 0.3333$$

$$(\text{DF})_{BC} = \frac{\frac{6EI}{8}}{\frac{3EI}{8} + \frac{6EI}{8}} = 0.6667$$

$$\text{FEM}_{BA} = \frac{(3)(16)(8)}{16} = 24 \text{ kN}\cdot\text{m}$$



Joint	A	B		
Member	AB	BA	BC	
DF	1	0.3333	0.6667	
FEM		24		
		–8	–16	
ΣM	0	16	–16	kN·m

Segment AB:

$\circlearrowleft +\Sigma M_B = 0; \quad -A_y(8) + 16(4) - 16 = 0 \quad A_y = 6 \text{ kN}$ **Ans.**

$\uparrow +\Sigma F_y = 0; \quad V_{BL} + 6 - 16 = 0 \quad V_{BL} = 10 \text{ kN}$

Segment BC:

$\circlearrowleft +\Sigma M_C = 0; \quad -V_{BR}(8) + 16 + 16 = 0 \quad V_{BR} = 4 \text{ kN}$

$\uparrow +\Sigma F_y = 0; \quad -V_{CL} + 4 = 0 \quad V_{CL} = 4 \text{ kN}$

Segment CD:

$\circlearrowleft +\Sigma M_C = 0; \quad -D_y(8) + 16(4) - 16 = 0 \quad D_y = 6 \text{ kN}$ **Ans.**

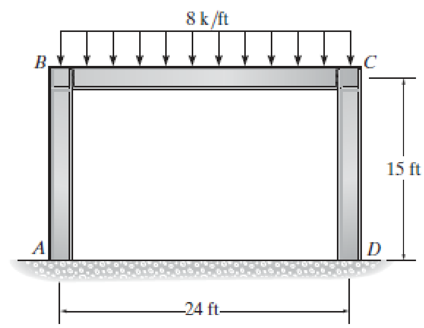
$\uparrow +\Sigma F_y = 0; \quad -V_{CR} - 6 + 16 = 0 \quad V_{CR} = 10 \text{ kN}$

$B_y = V_{BL} + V_{BR} = 10 + 4 = 14 \text{ kN}$ **Ans.**

$C_y = V_{CL} + V_{CR} = 4 + 10 = 14 \text{ kN}$ **Ans.**

Problem 2

Determine the reactions at A and D. Assume the supports at A and D are fixed and B and C are fixed connected. EI is constant.



SOLUTION

$(\text{DF})_{AB} = (\text{DF})_{DC} = 0$

$(\text{DF})_{BA} = (\text{DF})_{CD} = \dfrac{I/15}{I/15 + I/24} = 0.6154$

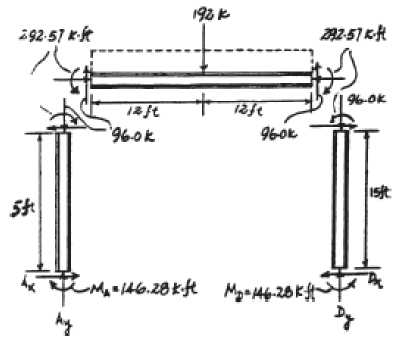
$(\text{DF})_{BC} = (\text{DF})_{CB} = 0.3846$

$(\text{FEM})_{AB} = (\text{FEM})_{BA} = 0$

$(\text{FEM})_{BC} = \dfrac{-8(24)^2}{12} = -384\ \text{k} \cdot \text{ft}$

$(\text{FEM})_{CB} = 384\ \text{k} \cdot \text{ft}$

$(\text{FEM})_{CD} = (\text{FEM})_{DC} = 0$



Joint	A	B		C		D
Mem.	AB	BA	BC	CB	CD	DC
DF	0	0.6154	0.3846	0.3846	0.6154	0
FEM			−384	384		
		236.31	147.69	−147.69	−236.31	
	118.16		−73.84	73.84		−118.16
		45.44	28.40	−28.40	−45.44	
	22.72		−14.20	14.20		−22.72
		8.74	5.46	−5.46	−8.74	
	4.37		−2.73	2.73		−4.37
		1.68	1.05	−1.05	−1.68	
	0.84		−0.53	0.53		−0.84
		0.32	0.20	−0.20	−0.33	
	0.16		−0.10	0.10		−0.17
		0.06	0.04	−0.04	−0.06	
	0.03		−0.02	0.02		−0.03
		0.01	0.01	−0.01	−0.01	
ΣM	146.28	292.57	−292.57	292.57	−292.57	−146.28

Thus from the free-body diagrams:

$A_x = 29.3\ \text{k}$ **Ans.**

$A_y = 96.0\ \text{k}$ **Ans.**

$M_A = 146\ \text{k} \cdot \text{ft}$ **Ans.**

$D_x = 29.3\ \text{k}$ **Ans.**

$D_y = 96.0\ \text{k}$ **Ans.**

$M_D = 146\ \text{k} \cdot \text{ft}$ **Ans.**