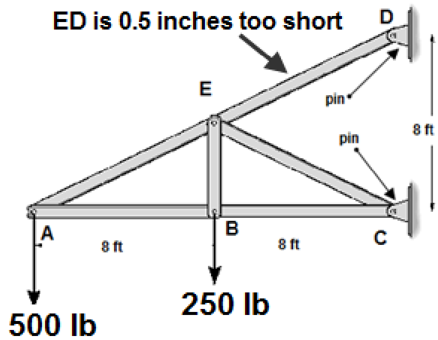
SOLUTION - A3

Problem 1

For the truss shown below determine the vertical displacement at **joint A** …



Working in [Kips, ft] …

Member	*N (kips)*	*n (kips)*	*L (ft)*	*nNL*	*nΔL*
AB	-1	-2	8	16.0	0
BC	-1	-2	8	16.0	0
AE	1.118	2.24	8.944	22.4	0
ED	1.398	2.24	8.944	28.0	2.24 * -0.5/12 = -0.0933
BE	0.250	0	4	0.0	0
CE	-0.280	0	8.944	0.0	0
			Total	82.4	-0.0933

$$\Delta_A = \left[\frac{82.4}{\left(2*\frac{1}{12^2}\right)\times\left(29\times10^3*12^2\right)}\right] + [0] + [-0.0933]$$

$$= 0.00142\,ft - 0.0933\,ft = (-0.0921\,ft) = 1.11\,in. \quad \text{upwards}$$

or ….

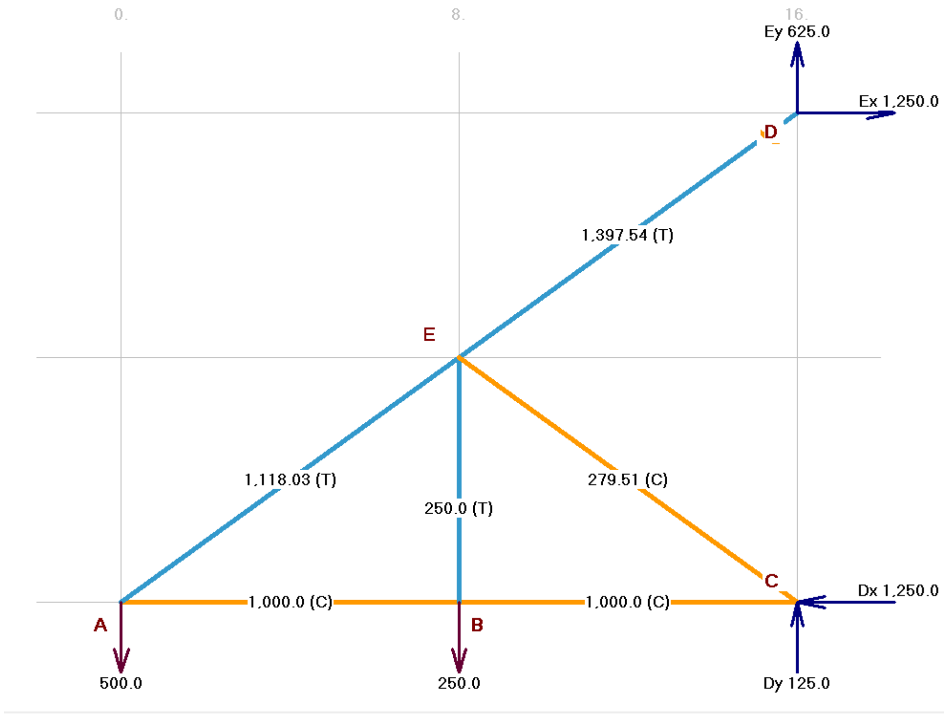
Working in [Kips, in] …

Member	*N (kips)*	*n (kips)*	*L (in)*	*nNL*	*nΔL*
AB	-1	-2	8 x12	16.0 x 12	0
BC	-1	-2	8 x12	16.0 x 12	0
AE	1.118	2.24	8.944 x12	22.4 x 12	0
ED	1.398	2.24	8.944 x12	28.0 x 12	2.24 * -0.5 = -1.12
BE	0.250	0	4 x12	0.0	0
CE	-0.280	0	8.944 x12	0.0	0
			Total	82.4 x 12	-1.12

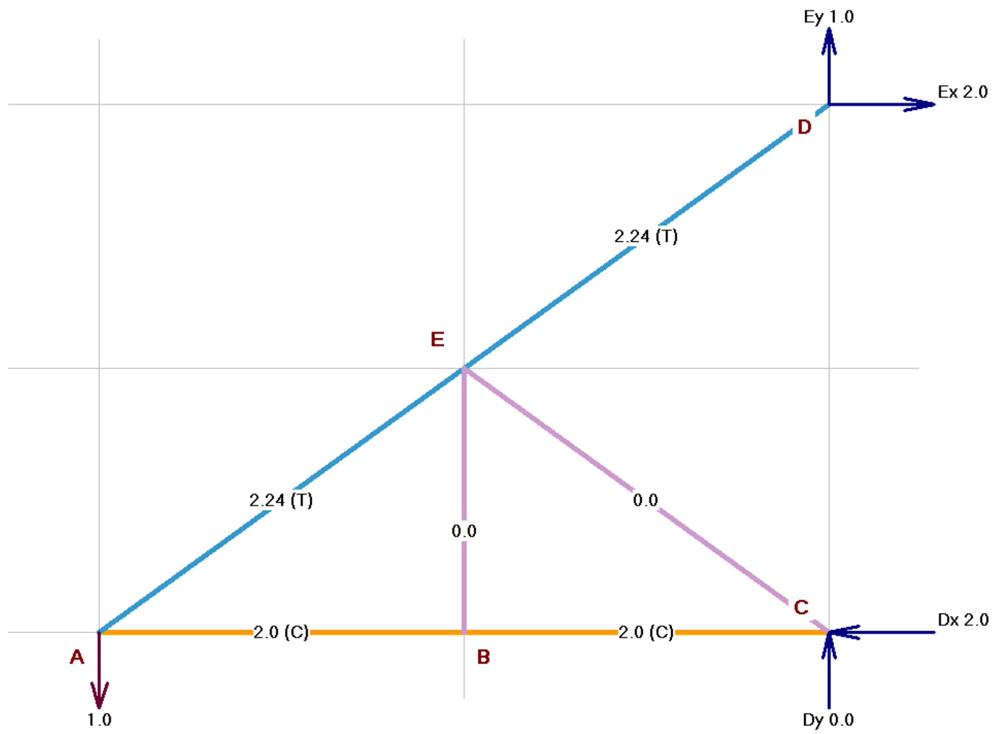
$$\Delta_A = \left[\frac{988.8}{\left(29\times10^3\right)(2)}\right] + [0] + [-1.12]$$

$$= 0.017in - 1.12in = 1.11\,in. \quad \text{upwards}$$

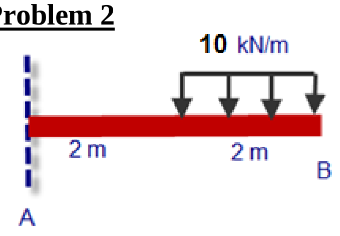
Under real loads: get "N" forces (shown in lbs but we will put in KIPs in the table)



Under virtual unit load: get "n" forces

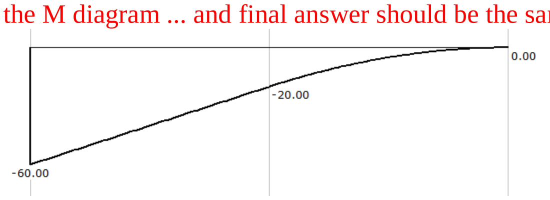


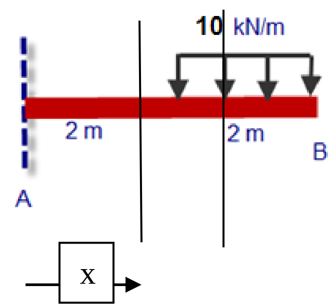
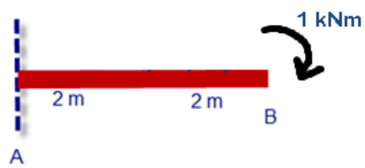
Problem 2



(a) Slope at B using direct integration

(some students might start their x from a different location so their M(x) functions will be different, you can easily check if the M function is correct by plugging x in the functions ... should give the right value on the M diagram ... and final answer should be the same)



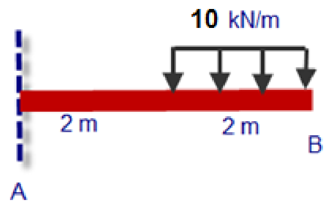
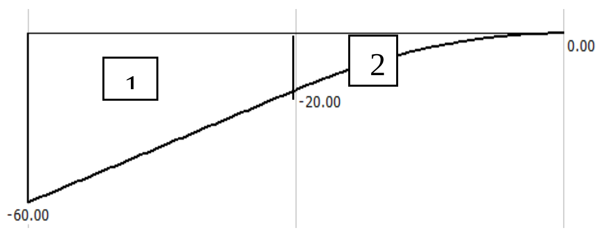
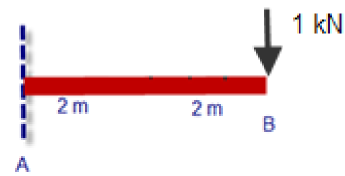
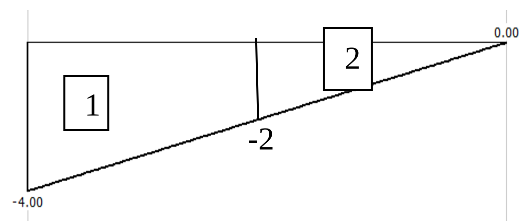
REAL BEAM	VIRTUAL BEAM
	
Cut 1: taking sum of moments gives: M1 = -60 +20x 0<x<2	Cut 1: taking sum of moments gives: m1 = -1 0<x<2
Cut 2: taking sum of moments gives: M2 = -60 +20x -10(x-2)^2/2 2<x<4	Cut 2: taking sum of moments gives: m2 = -1 2<x<4

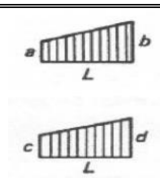
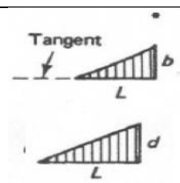
Solve virtual work equation: $\Delta = \frac{1}{EI}\int_0^2 (-1)(-60+20x)dx + \frac{1}{EI}\int_2^4 (-1)(-60+20x-5(x-2)^2)dx$

$$60x - 10x^2 \Big|_0^2 = 280 \qquad\qquad 60x - 10x^2 + \frac{5}{3}(x-2)^3 \Big|_2^4 = \frac{40}{3}$$

Therefore: $\Theta = \frac{93.33}{EI} = \frac{93.33}{(200\times10^6)(50\times10^{-6})} = =$ 0.00933 rad. CLOCKWISE

a) Displacement at B using Mohr's table

REAL BEAM	VIRTUAL BEAM
 This gives M diagram: 	 This gives m diagram: 

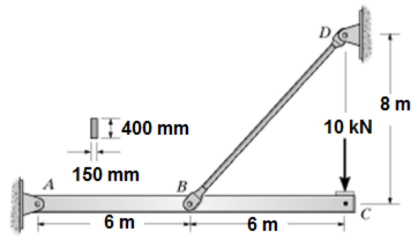
Area	$\int_0^L mM dx$ ... using Mohr's table ...
1	 $\frac{L}{6}(2ac + ad + 2bd + bc)$ $\frac{2}{6}([2*-60*-4]+[-60*-2]+[2*-20*-2]+[-20*-4]) = 253.33$
2	 $\frac{1}{4}Lbd$ $\frac{1}{4}(2*-20*-2) = 20$
$\sum$	**273.33**

Therefore: $\Delta = \frac{273.33}{EI} = \frac{273.33}{(200\times10^6)(50\times10^{-6})} =$

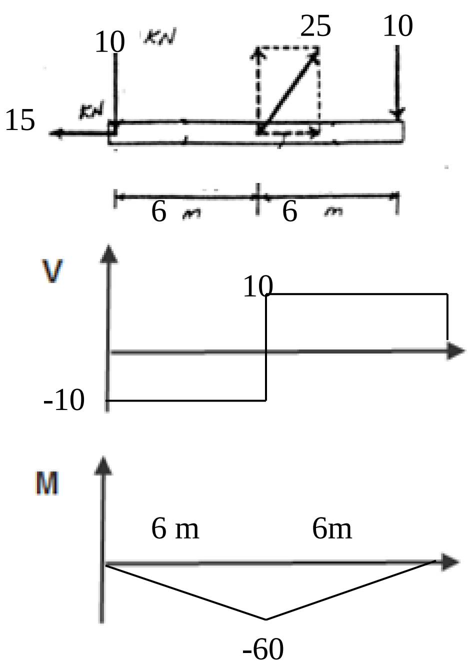
= **0.02733 m = 27.33 mm Downwards**

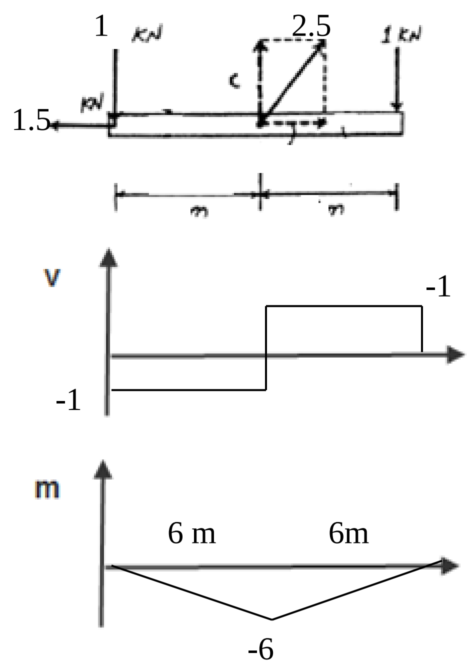
Question 3

Beam **ABC** has a rectangular cross-section of 400 mm by 150 mm.
The rod (truss member) **DB** has a diameter of 50 mm.
Find the displacement at point C due to the loading using the method of virtual work.



E = 200 GPa





For the **beam**, using Mohr's tables → $\int mM\,dx$ = 1/3 * (6*-60*-6) + 1/3 * (6*-60*-6) = **1440**

For the **rod** → $\sum nNL$ = 2.5 x 25 x 10 = **625**

Therefore (working in [kN, m]):

$$\Delta c = \frac{1440}{(EI)_{AC}} + \frac{625}{(AE)_{BD}}$$

$$= \frac{1440}{\left(E*\left[\frac{1}{12}*0.15*0.4^3\right]\right)} + \frac{625}{\left(\left[\frac{\pi}{4}*0.05^2\right]E\right)} = \frac{1.8\ x\ 10^6}{(E)} + \frac{0.319x\ 10^6}{(E)} = \frac{2.119\ x\ 10^6}{(E)}$$

E = 200 GPa, then:

$$\Delta c = \frac{2.119\ x\ 10^6}{(E)} = \frac{2.119x\ 10^6}{(200x\ 10^6)} =$$ 0.0106 m = **10.6 mm downwards**