Unit 2

Lesson 6

1. a)

Point (x, y)	Value of x	Value of $f(x) = 1.7^x$	Value of $f'(x)$	$k = \frac{f'(x)}{f(x)}$
(0, 1)	0	1	0.53	0.53
	1	1.7	0.90	0.529
	2	2.89	1.54	0.532
	3	4.913	2.60	0.529
	4	8.352	4.44	0.531
	5	14.198	7.54	0.531

An estimate for k is 0.531.

b)

Point (x, y)	Value of x	Value of $g(x) = 2.3^x$	Value of $g'(x)$	$k = \frac{g'(x)}{g(x)}$
(0, 1)	0	1	0.83	0.830
	1	2.3	1.91	0.830
	2	5.29	4.42	0.835
	3	12.17	10.1	0.830
	4	27.98	23.21	0.829

An estimate of k is 0.830.

2. The derivative of $f(x) = 3^x$ is $f'(x) = 3^x \ln 3$, so the instantaneous rate of change at the point where $x = 2$ is $3^2 \ln 3 = 9 \ln 3 \approx 9.8875$.

3. a) $y' = e^{3x} \cdot 3$

   b) $y' = 6x - e^{x^2} \cdot 2x$

   c) $f'(x) = 2^x \ln 2 - 2x$

   d) $g'(x) = 6^{x^2 - 3x + 4} \cdot \ln 6 \cdot (2x - 3)$

Suggested Answers to Support Questions

4. The slope of the tangent to $f(x)$ is the value of the derivative at $x = 1$. Use the chain rule to find the derivative of $f(x)$:

$f'(x) = (x^2 + 1)'3^{x^2+1} \ln(3) - 2x = (2x)3^{x^2+1} \ln(3) - 2x$

$f'(1) = 2(1)3^{1^2+1} \ln(3) - 2(1)$
$= (2)3^{1+1} \ln(3) - 2 = 18\ln(3) - 2 \approx 17.77$

The equation of the tangent is

$y = 17.77(x - 1) + 8$

$y = 17.77x - 17.77 + 8$

$y = 17.77x - 9.77$

Lesson 7

5. a) This is a chain-rule type of question, where the outer function is the sine function and the inner function is x^2. The derivative of $f(x) = \sin(x^2)$ is $f'(x) = \cos(x^2) \cdot 2x = 2x\cos(x^2)$.

   b) This is a chain-rule type of question where the outer function is the exponential function and the inner function is $\cos x$. The derivative of $g(x) = e^{\cos x}$ is $g'(x) = e^{\cos x} \cdot (-\sin x) = -\sin x \cdot e^{\cos x}$.

6. a) $y = \dfrac{1}{x^3 - 3x^2 + 6}$

   Rewrite the function as $y = (x^3 - 3x^2 + 6)^{-1}$. Apply the chain rule, with outer function u^{-1}, to conclude that the derivative is $y' = (-1)(x^3 - 3x^2 + 6)^{-2}(3x^2 - 6x)$.

   This can also be written
   $y' = \dfrac{-(3x^2 - 6x)}{(x^3 - 3x^2 + 6)^2}$

   b) $y = \dfrac{x - 2}{x + 4}$

   Rewrite the function as $y = (x - 2)(x + 4)^{-1}$.
   Apply the product rule to conclude that
   $y' = (x - 2)'(x + 4)^{-1} + (x - 2)((x + 4)^{-1})'$

Since the chain rule says that the derivative of u^{-1} is $(-1)u^{-2} \cdot u'$, the answer is
$y' = 1 \cdot (x + 4)^{-1} + (x - 2)(-1)(x + 4)^{-2} \cdot 1$

This can be simplified slightly to give $y' = \dfrac{1}{x+4} - \dfrac{x-2}{(x+4)^2}$

c) $y = (3x^2 - 4)^{\frac{1}{2}}$

By the chain rule, the derivative of $u^{\frac{1}{2}}$ is $\left(\frac{1}{2}\right)u^{-\frac{1}{2}} \cdot u'$. The derivative is $y' = \left(\frac{1}{2}\right)(3x^2 - 4)^{-\frac{1}{2}} \cdot 6x$

This can also be written as $y' = \dfrac{3x}{(3x^2 - 4)^{\frac{1}{2}}}$

7. a) $f(x) = \dfrac{\sin x}{5x^2}$

Rewrite the function as a product: $f(x) = (\sin x)(5x^2)^{-1}$.

Apply the product rule to conclude that the derivative is
$f'(x) = (\sin x)'(5x^2)^{-1} + (\sin x)((5x^2)^{-1})'$

Recall that the derivative of $\sin x$ is $\cos x$, and use the chain rule to conclude that the derivative of u^{-1} is $(-1)u^{-2} \cdot u'$.
The derivative is $f'(x) = (\cos x)(5x^2)^{-1} + (\sin x)(-1)(5x^2)^{-2}(10x)$

This can also be written as $f'(x) = \dfrac{\cos x}{5x^2} - \dfrac{10x\sin x}{(5x^2)^2}$

b) $y = x\cos x^2$

By the product rule, the derivative is
$y' = (x)'(\cos x^2) + (x)(\cos x^2)'$

Apply the chain rule, where the outer function is the cosine function and the inner function is x^2. The derivative is
$y' = 1 \cdot \cos x^2 + x \cdot (-\sin x^2) \cdot 2x$.

This simplifies slightly to give $y' = \cos x^2 - 2x^2\sin x^2$.

Suggested Answers to Support Questions

c) $g(x) = \dfrac{\sqrt{x^2 - 5}}{e^x}$

Rewrite the function as a product: $g(x) = (x^2 - 5)^{\frac{1}{2}}(e^x)^{-1}$

By the product rule, the derivative is

$g'(x) = ((x^2 - 5)^{\frac{1}{2}})'(e^x)^{-1} + (x^2 - 5)^{\frac{1}{2}}((e^x)^{-1})'$.

Use the chain rule to differentiate expressions of the form $u^{\frac{1}{2}}$ or u^{-1}:

$$g'(x) = \left(\frac{1}{2}\right)(x^2 - 5)^{-\frac{1}{2}}(2x)(e^x)^{-1} + (x^2 - 5)^{\frac{1}{2}}(-1)(e^x)^{-2}(e^x)$$

8. The derivative of $y = x^3 - 4x^2 + 5x - 6$ is $y' = 3x^2 - 8x - 6$. If you're looking for points where the slope of the tangent line is 1, you need to set the derivative to 1 and solve. This gives

$3x^2 - 8x + 5 = 1$

$3x^2 - 8x + 5 - 1 = 0$

$3x^2 - 8x + 4 = 0$

$(3x - 2)(x - 2) = 0$

You must have $x = \dfrac{2}{3}$ or $x = 2$.

The question asked you for the points, so plug $x = \dfrac{2}{3}$ and $x = 2$ into the original formula for y in order to find the corresponding y-coordinates.

$x = \dfrac{2}{3}$, you have

$$y = \left(\frac{2}{3}\right)^3 - 4\left(\frac{2}{3}\right)^2 + 5\left(\frac{2}{3}\right) - 6 = \left(\frac{8}{27}\right) - 4\left(\frac{4}{9}\right) + 5\left(\frac{2}{3}\right) - 6 = -\frac{112}{27}.$$

$x = 2$, you have $y = 2^3 - 4(2)^2 + 5(2) - 6 = -4$.

The two points are $(2, -4)$ and $\left(\dfrac{2}{3}, -\dfrac{112}{27}\right)$.

Lesson 8

9. (A): derivative is (iii)

   (B): derivative is (i)

   (C): derivative is (ii)

10. a) From the graph of the derivative:

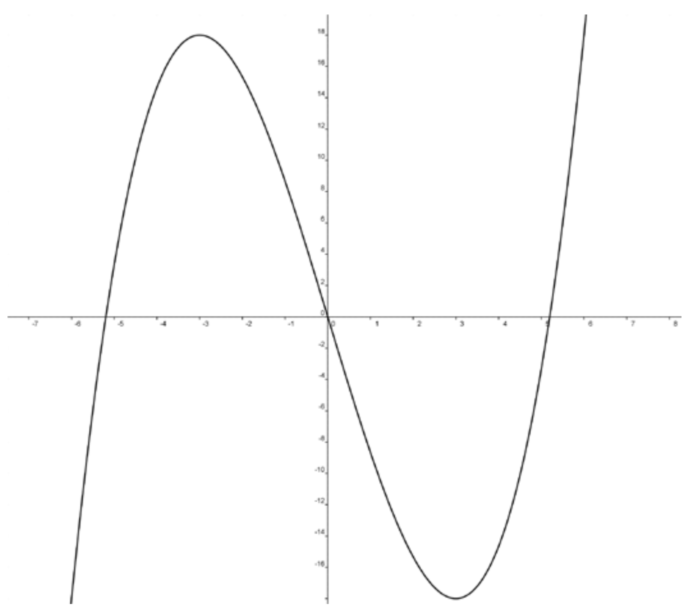
    $f'(x) < 0$ when $-3 < x < 3$ and hence $f(x)$ is decreasing

    $f'(x) > 0$ when $x < -3$ and $x > 3$ and hence $f(x)$ is increasing

    $f'(-3) = f(3) = 0$

    Therefore, $(-3, f(-3))$ is a local maximum and $(3, f(3))$ is a local minimum of f.

    b)



    c) Since the slope of the tangent to the curve has a zero slope at $x = 0$, the second derivative has a zero slope at $x = 0$. This is not enough to assert that $(0, f(0))$ is an inflection point; you still need to confirm that $f''(x)$ changes signs.

Suggested Answers to Support Questions

Observe that the slope of the tangent to the curve changes from negative when x is just less than 0 to positive when x is just more than 0. Therefore, $f(x)$ is concave down when $x < 0$ and concave up when $x > 0$. Hence $(0, f(0))$ is an inflection point.

11. From the graph of $f(x)$, you can observe the following:

The function is decreasing, and hence $f'(x) < 0$ on the following intervals: $x < -2$ and $-1 < x < 1$

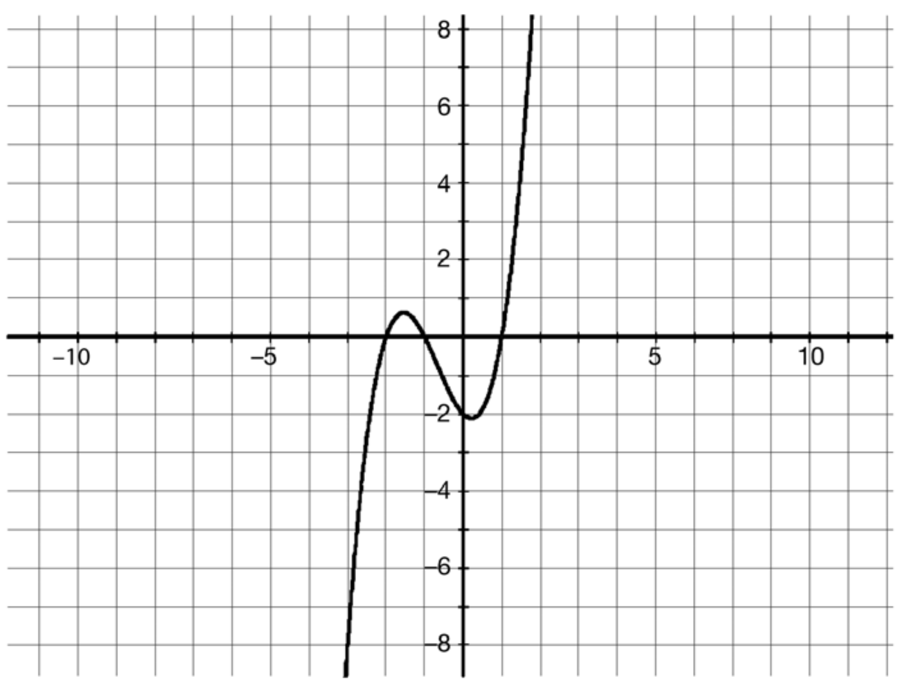
The function is increasing and hence $f'(x) > 0$ on the following intervals: $-2 < x < -1$ and $x > 1$

The function is concave up and hence $f''(x) > 0$ when $x < -1.5$ and $x > 0$.

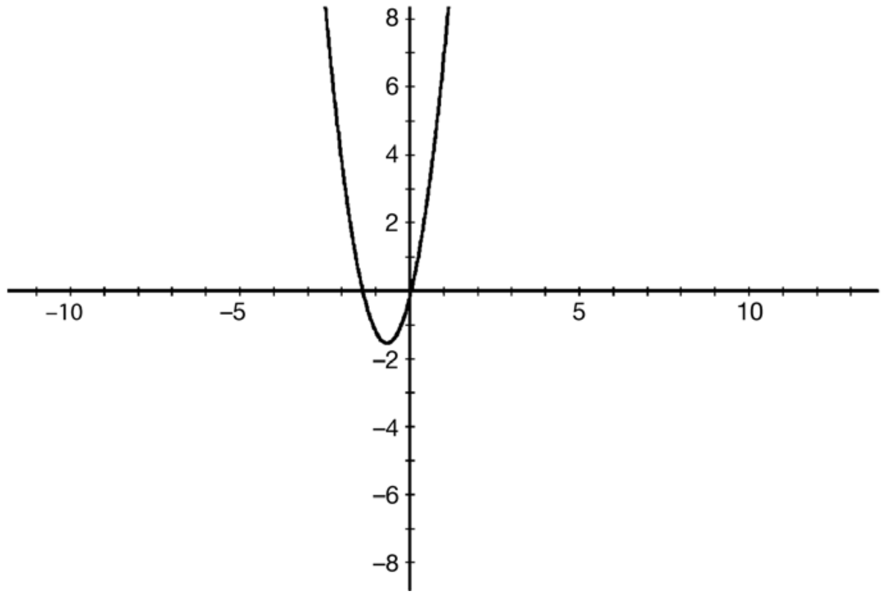
The function is concave down and hence $f''(x) < 0$ when $-1.5 < x < 0$.

There are two inflection points: $x = -1.5$ and $x = 0$

The graph of the derivative:



The graph of the second derivative:



12. a) $y' = 24x^5 - 8x - 5$

$y'' = 120x^4 - 8$

b) Rewrite $f(x) = (x^2 - 3)^{\frac{1}{2}}$

$$f'(x) = (x^2 - 3)'\frac{1}{2}(x^2 - 3)^{-\frac{1}{2}}$$

$$f'(x) = x\left(x^2 - 3\right)^{-\frac{1}{2}}$$

$$f''(x) = (x^2 - 3)^{-\frac{1}{2}} + x\left(-\frac{1}{2}\right)(2x)(x^2 - 3)^{-\frac{3}{2}}$$

$$f''(x) = (x^2 - 3)^{-\frac{1}{2}} - x^2(x^2 - 3)^{-\frac{3}{2}}$$

It is acceptable to leave the answer like this, or to rewrite $f''(x)$:

$$f''(x) = \frac{1}{\sqrt{x^2 - 3}} - \frac{x^2}{(x^2 - 3)\sqrt{x^2 - 3}}$$

Suggested Answers to Support Questions

c) Rewrite $g(x) = (x^2 - 1)x^{-1}$:

$$g'(x) = (2x)x^{-1} - (x^2 - 1)x^{-2}$$
$$= 2 - (x^2 - 1)x^{-2}$$

$$g''(x) = -2(x)x^{-2} - (x^2 - 1)(-2)x^{-3}$$
$$= -2x^{-1} + 2(x^2 - 1)x^{-3}$$

Lesson 9

13. a) $f'(x) = 3x^2 + 6x - 24$

$f''(x) = 6x + 6$

Use the quadratic formula to solve $f'(x) = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(3)(-24)}}{2(3)}$$
$$x = \frac{-6 \pm \sqrt{36 + 288}}{6}$$
$$x = \frac{-6 \pm 18}{6}$$

$x = -4$ or $x = 2$ are the two roots of $3x^2 + 6x - 24 = 0$

To check when the derivative is positive and negative, check the value of the derivative for values of x in each of the following intervals: $x < -4$, $-4 < x < 2$, and $x > 2$.

$x < -4$

$f'(-5) = 3(-5)^2 + 6(-5) - 24 = 21 > 0$. The function is increasing when $x < -4$.

$-4 < x < 2$

$f'(0) = -24 < 0$. The function is decreasing when $-4 < x < 2$.

$x > 2$

$f'(3) = 3(3)^2 + 6(3) - 24 = 21 > 0$. The function is increasing when $x > 2$.

b) $(-4, 81)$ is a local maximum and $(2, -27)$ is a local minimum.

c) $f''(x) = 6x + 6$

$f''(x) = 0$ when $x = -1$

$f''(x) < 0$ and $f(x)$ is concave down when $x < -1$, and $f''(x) > 0$ and $f(x)$ is concave up when $x > -1$.

d) $(-1, 27)$ is a point of inflection.

14. a) $f'(x) = 4x^3 - 6x^2 + 4x - 2$

Find when the derivative is zero in order to find the critical points. $f'(x)$ is a polynomial of degree 3. There is no easy way to find the solutions of a polynomial equation of degree 3, but you may be able to find one root by trial and error.

x	$f'(x) = 4x^3 - 6x^2 + 4x - 2$
-4	$f'(x) = 4(-4)^3 - 6(-4)^2 + 4(-4) - 2$ $= -370$
-3	$f'(-3) = -176$
-2	$f'(-2) = -66$
-1	$f'(-1) = -16$
0	$f'(0) = -2$
1	$f'(1) = 0$
2	$f'(2) = 14$
3	$f'(3) = 64$
4	$f'(4) = 174$

$f'(1) = 0$, which shows that $(x - 1)$ is a factor of $f'(x)$. Use long division to find the other factor of $f'(x)$. (You should be familiar with long division of polynomials. If not, check the online resource for the course.)

Suggested Answers to Support Questions

$$
\begin{array}{r|l}
\multicolumn{2}{r}{4x^2 - 2x + 2} \\
\cline{2-2}
x-1 & 4x^3 - 6x^2 + 4x - 2 \\
 & \underline{4x^3 - 4x^2 } \\
 & -2x^2 + 4x - 2 \\
 & \underline{ -2x^2 + 2x } \\
 & 2x - 2 \\
 & \underline{ 2x - 2} \\
 & 0
\end{array}
$$

$f'(x) = (x - 1)(4x^2 - 2x + 2)$

Use the quadratic formula to find the roots of $4x^2 - 2x + 2 = 0$:

The discriminant ($b^2 - 4ac$) of $4x^2 - 2x + 2$ is $(-2)^2 - 4(4)(2) < 0$, which means $4x^2 - 2x + 2 = 0$ has no real roots.

$f'(x) = 0$ when $x = 1$

$f(1) = 3$

(1, 3) is a critical point.

Find the value of $f'(x)$ in each of the following intervals, $x < 1$ and $x > 1$.

$f'(0) = -2 < 0$, so the function is decreasing when $x < 1$.

$f'(2) = 14 > 0$, so the function is increasing when $x > 1$.

b) (1, 3) is a local minimum. Confirm this by looking at the concavity of $f(x)$.

c) $f''(x) = 12x^2 - 12x + 4$

The discriminant of the second derivative $12x^2 - 12x + 4$ is less than zero so $f''(x)$ has no real roots. $f''(x) > 0$ for all x. The function is always concave up.

d) The function has no inflection points. (1, 3) is a local minimum. It is actually a global minimum of the function; that is, the smallest value the function can be.

15. Since the second derivative is not zero, you can assume that the function is a polynomial of degree 2. Start with $g(x) = ax^2 + bx + c$ where a, b, and c are real numbers.

    Use the usual differentiation rules to get the following. Note that a, b, and c are treated like constant numbers:

    $g'(x) = 2ax + b$

    $g''(x) = 2a$

    Now substitute the given values for $g''(2) = 1$, $g'(2) = 3$, and $g(2) = 0$:

    $g'(x) = 2ax + b$

    $g'(2) = 2a(2) + b$

    $3 = 4a + b$ **[1]**

    $g''(x) = 2a$

    $g''(2) = 2a$

    $1 = 2a$

    $a = \frac{1}{2}$

    Substitute the value of $a = \frac{1}{2}$ into **[1]**:

    $3 = 4(\frac{1}{2}) + b$

    $b = 1$

    $ax^2 + bx + c = 0$

    $a(2)^2 + b(2) + c = 0$ **[2]**

    Substitute $a = \frac{1}{2}$ and $b = 1$ into **[2]** and solve for c.

    $2 + 2 + c = 0$

    $c = -4$

    A possible value of $g(x)$ is $(\frac{1}{2})x^2 + x - 4$.

Suggested Answers to Support Questions

16. From the graph, observe that $h'(x)$ is a linear function that implies that the degree of $h'(x)$ is one. Therefore, $h(x)$ is a quadratic function, since $h(x)$ has one degree more than its derivative:

    $h(x) = ax^2 + bx + c$

    $h'(x) = 2ax + b$

    From the graph, observe that $h'(-5) = 0$ and $h'(-4) = 2$. This gives you two equations with two unknowns:

    $h'(-5) = 2a(-5) + b$

    $0 = -10a + b$

    $b = 10a$ **[1]**

    $h'(-4) = 2a(-4) + b$

    $2 = -8a + b$ **[2]**

    Substitute $b = 10a$ into **[2]**:

    $2 = -8a + 10a$

    $2 = 2a$

    $a = 1$

    From **[1]** you get $b = 10$.

    Substitute a and b into $h(x)$, $h(x) = x^2 + 10x + c$

    How can you find the value of c? You know that $h(1) = 3$.

    $(1)^2 + 10(1) + c = 3$

    $11 + c = 3$

    $c = -8$

    Therefore, $h(x) = x^2 + 10x - 8$.

Lesson 10

17. a) The polynomial contains only odd powers of x, so it is an odd function. Alternatively, you can see that $f(-x) = (-x)^3 - 5(-x) = -x^3 + 5x = -f(x)$, so $f(x)$ is an odd function.

    b) The polynomial contains only even powers of x (including the constant term 3, which counts as an even power since it is the same as $3x^0$), so it is an even function. Alternatively, you can see that $g(-x) = (-x)^4 - 3 = x^4 - 3 = g(x)$, so $g(x)$ is an even function.

    c) The graph looks the same if reflected through the y-axis, so it represents an even function.

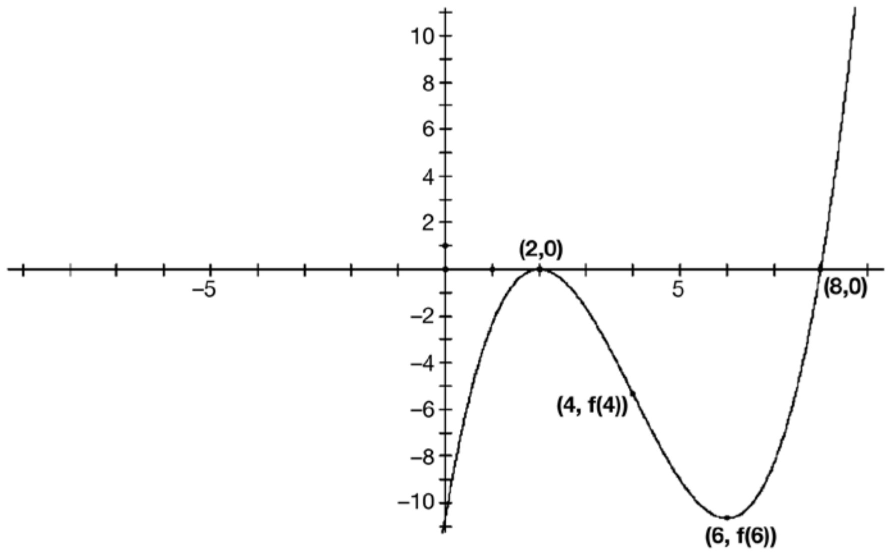
    d) The graph does not look the same if reflected through the y-axis, and does not look the same if rotated 180 degrees around the origin. Therefore, the function is neither even nor odd.

18. From the first graph, $f'(x)$ is positive when $x < 2$, $f'(x)$ is negative when $2 < x < 6$, and $f'(x)$ is positive when $x > 6$. Therefore, $f(x)$ must be increasing when $x < 2$, decreasing when $2 < x < 6$, and increasing when $x > 6$. This means $f(x)$ must have a maximum at $x = 2$ and a minimum at $x = 6$.

    From the second graph, $f''(x)$ is negative when $x < 4$ and is positive when $x > 4$. Therefore, $f(x)$ must be concave down when $x < 4$ and concave up when $x > 4$. This means $x = 4$ is an inflection point.

    To construct a reasonable graph of $f(x)$, plot the given points (2, 0) and (8, 0) and then make $f(x)$ increasing or decreasing, and concave up or concave down, on the appropriate intervals. There isn't enough information to know exact values of $f(x)$ at points other than $x = 2$ and $x = 8$, but you can illustrate the correct overall behaviour of $f(x)$.

Suggested Answers to Support Questions



19. $f(x) = x^3 - 3x^2 - x + 3$.

Symmetry: Since $f(x)$ is a polynomial containing both even and odd powers of x, $f(x)$ is neither an even function nor an odd function and there is no obvious symmetry.

Intercepts: Find the y-intercept by substituting $x = 0$. $f(0) = 3$, so the y-intercept is the point $(0, 3)$. Find x-intercepts by solving $f(x) = 0$. Ordinarily, it is difficult to factor a cubic, but if you happen to find one root, then you have one factor. If you experiment with simple numbers, you find that $x = 1$ is a root, which means $x - 1$ is a factor. Dividing $x^3 - 3x^2 - x + 3$ by $x - 1$, $f(x)$ factors as

$(x - 1)(x^2 - 2x - 3)$

$(x - 1)(x + 1)(x - 3)$

The roots of $f(x)$ are $x = 1$, $x = -1$, and $x = 3$, and so the x-intercepts are the points $(-1, 0)$, $(1, 0)$, and $(3, 0)$.

First derivative: The first derivative is $f'(x) = 3x^2 - 6x - 1$. You can find where $f'(x)$ is positive or negative by first finding where $f'(x) = 0$. It looks as if you cannot solve $3x^2 - 6x - 1 = 0$ by factoring, so use the quadratic formula. The solutions to $3x^2 - 6x - 1 = 0$ are

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(-1)}}{2(3)} = \frac{6 \pm \sqrt{36 + 12}}{6} = \frac{6 \pm \sqrt{48}}{6} \approx \frac{6 \pm 6.93}{6}$$

The solutions to $f'(x) = 0$ are approximately $x = 12.93/6 = 2.15$ and $x = -0.93/6 = -0.15$. You also know that the graph of $3x^2 - 6x - 1$ is a parabola opening upward, so the end behaviour of the graph is that $3x^2 - 6x - 1$ is positive when x moves far to the right or far to the left. Therefore, $f'(x)$ is positive when $x < -0.15$, $f'(x)$ is negative when $-0.15 < x < 2.15$, and $f'(x)$ is positive when $x > 2.15$.

Intervals of increase or decrease: Now that you've found where $f'(x)$ is positive and negative, you can say that $f(x)$ is increasing when $x < -0.15$, $f(x)$ is decreasing when $-0.15 < x < 2.15$, and $f(x)$ is increasing when $x > 2.15$.

Extreme points: Now that you know where $f(x)$ is increasing and decreasing, you can say that $f(x)$ has a maximum at $x = -0.15$ and a minimum at $x = 2.15$.

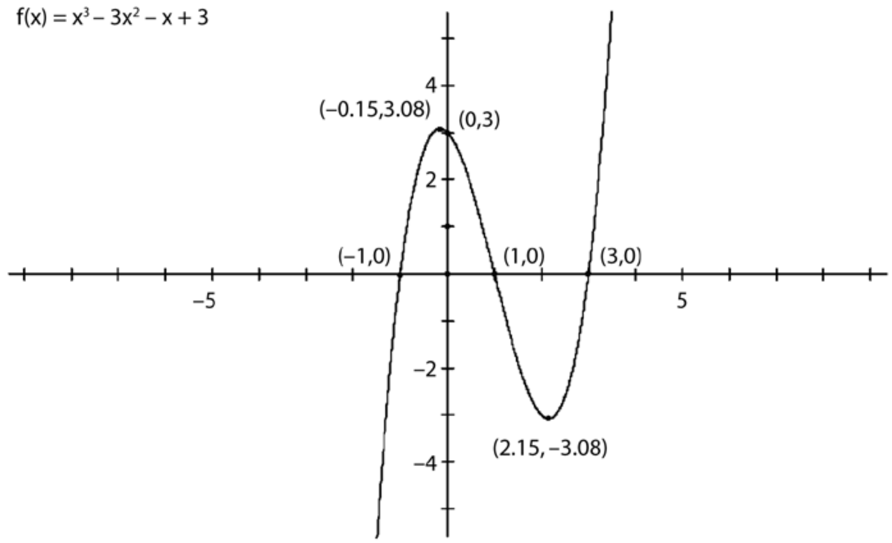
Second derivative: The second derivative is $f(x) = 6x - 6$. The only zero of the second derivative is $x = 1$. $f''(x)$ is negative when $x < 1$ and $f''(x)$ is positive when $x > 1$.

Concavity: $f(x)$ is concave down when $x < 1$, and $f(x)$ is concave up when $x > 1$.

Inflection points: The only inflection point is at $x = 1$, since that is the only point where the graph has a change in concavity.

Combine all this information into a graph of $f(x)$:

Suggested Answers to Support Questions



20. a) Symmetry: Since y is a polynomial containing both even and odd powers of x, y is neither an even function nor an odd function and we have no obvious symmetry.

b) Intercepts: Find the y-intercept by substituting $x = 0$. This gives $y = –1$, so the y-intercept is the point (0, –1). Try to find x-intercepts by solving $–3x^2 + 4x – 1 = 0$. Solve by factoring:
$–3x^2 + 4x – 1 = –(x – 1)(3x – 1)$
The roots are $x = 1$ and $x = \frac{1}{3} \approx 0.33$, and so the x-intercepts are the points (1, 0), and (0.33, 0).

c) First derivative: The first derivative is $y' = –6x + 4$. $y' = 0$ when $x = \frac{4}{6} = \frac{2}{3} \approx 0.67$. y' is positive when $x < \frac{2}{3}$, and y' is negative when $x > \frac{2}{3}$.
Intervals of increase or decrease: Based on the previous paragraph, y is increasing when $x < \frac{2}{3}$ and y is decreasing when $x > \frac{2}{3}$.

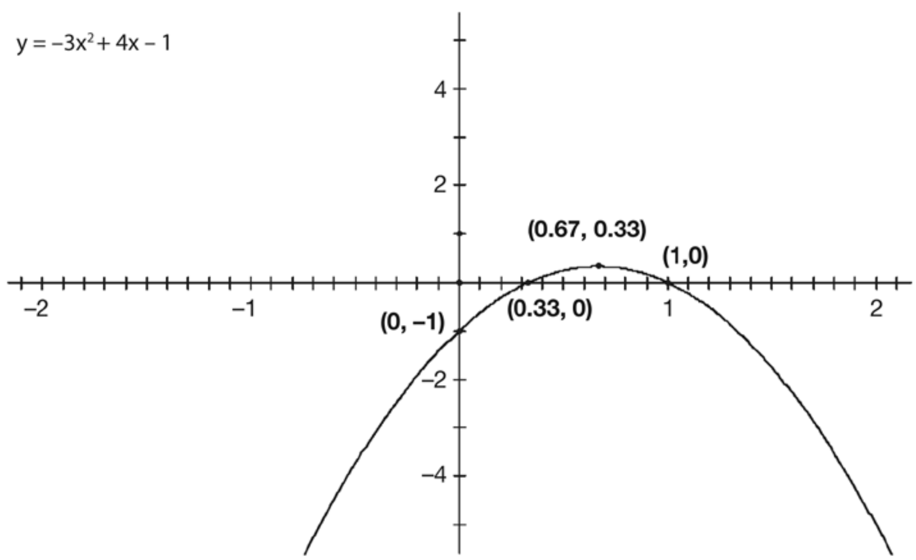
Extreme points: Based on the previous paragraph, y has a maximum at $x = \frac{2}{3}$.

Second derivative: The second derivative is $y'' = -6$. This means the second derivative is negative for all values of x.

Concavity: Based on the previous paragraph, y is concave down for all values of x.

Inflection points: Based on the previous paragraph, there are no inflection points because there are no changes of concavity.

d) Combine all this information into a graph of y:



21. a) Domain: $x + 1 = 0$ when $x = -1$, so the domain is all real numbers except $x = -1$.

b) Vertical asymptote: $\lim_{x \to -1^-} \frac{2x-6}{x+1} = \frac{2(-1^-)-6}{(-1^-)+1}$

The numerator is negative, as is the denominator. Conclude that $\lim_{x \to -1^-} \frac{2x-6}{x+1} = +\infty$.

Suggested Answers to Support Questions

Similarly, $\lim\limits_{x \to -1^+} \dfrac{2x-6}{x+1} = -\infty$
$x = -1$ is a vertical asymptote.

Horizontal asymptote: $\lim\limits_{x \to \infty} \dfrac{2x-6}{x+1} = \dfrac{x\left(2-\dfrac{6}{x}\right)}{x\left(1+\dfrac{1}{x}\right)} = 2$

$y = 2$ is a horizontal asymptote.

c) Intercepts: $f(0) = \dfrac{2(0)-6}{(0)+1} = -6$

$(0, -6)$ is the y-intercept.
$f(x) = 0$, $\dfrac{2x-6}{x+1} = 0$ when $2x - 6 = 0$, $x = 3$.
$(3, 0)$ is the x-intercept.

d) Calculate the first and second derivative, the intervals on which the function is increasing and decreasing, inflection points and concavity.

We write $f(x)$ as a product and differentiate:

$$f(x) = \frac{2x-6}{x+1} = (2x-6)(x+1)^{-1}$$

$$f'(x) = -(2x-6)(x+1)^{-2} + 2(x+1)^{-1}$$

$$= \frac{-(2x-6)}{(x+1)^2} + \frac{2}{(x+1)} = \frac{-2x+6+2(x+1)}{(x+1)^2} = \frac{8}{(x+1)^2}$$

$f'(x)$ is always positive, since $(x + 1)^2 > 0$ for all x.
Conclude that the function is always increasing with no local maximums or minimums.

To find the second derivative, write $f'(x)$ as a product and differentiate:

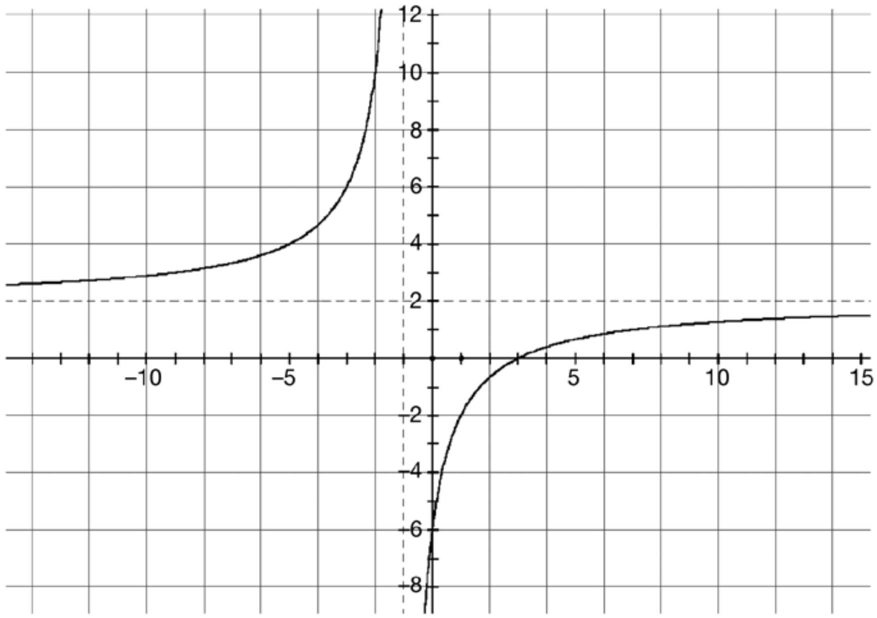
$$f'(x) = \frac{8}{(x+1)^2} = 8(x+1)^{-2}$$

$$f''(x) = -16(x+1)^{-3}$$

$f''(x) > 0$ when $x < -1$ and $f''(x) < 0$ when $x > -1$. Hence, the function is concave up when $x < -1$ and concave down when $x > -1$.

There are no inflection points, since $f''(x)$ is never zero.

e)



Suggested Answers to Support Questions