CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	203	All
Examination	**Date**	**Pages**
Alternate	December 2014	3
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Special Instructions:	Only approved calculators are allowed **Show all your work for full marks.**	

MARKS

[11] **1.** (a) Solve for x : $2\log_2(x) - \log_2(x+3) = 5^{\log_5(2)}$

(b) Let $f(x) = \dfrac{1}{x^2-1}$ and $g(x) = \sqrt{1+x}$. Find $f \circ g$ and determine its domain.

(c) Given the function $f(x) = (8+2^x)^{1/3}$, find the inverse function f^{-1}, the range of f and the range of f^{-1}.

[7] **2.** Find the limit if it exists (**do not** use l'Hôpital's rule.):

(a) $\lim\limits_{x\to 1} \dfrac{x^2+2x-3}{|x-1|}$ (b) $\lim\limits_{x\to -2} \dfrac{3-\sqrt{x^2-3x-1}}{x^2-4}$

[6] **3.** Find all horizontal and vertical asymptotes of the function

$$f(x) = \frac{\sqrt{9x^4+2x^2+1}}{x^2+4x}$$

[15] **4.** Find the derivatives of the following functions (you don't need to simplify your final answer, but you must show how you calculate it):

(a) $f(x) = (x^{3/2}+1)(x^{3/2}-1)\tan x$

(b) $f(x) = (x+e^{-x})\ln^2(x)$

(c) $f(x) = \sqrt{x\sin(x^3)+\sin(x^3-x)}$

(d) $f(x) = \dfrac{3^x}{3^x+3^{-x}} + 3^2$

(e) $f(x) = (\cos x + x^2)^{5x}$ (use logarithmic differentiation)

[12] **5.** Consider the function $y = 3x + x^{-1}$.

(a) Use the **definition of derivative** to find the formula for dy/dx.

(b) Find the linearization $L(x)$ of the function $y(x)$ at $a = 2$.

(c) Find the differential dy and evaluate it for the values $x = 2$ and $dx = 0.1$.

[7] **6.** Let $f(x) = \dfrac{x+1}{x+3}$.

(a) Find the slope m of the secant line joining the points $(-1, f(-1))$ and $(2, f(2))$.

(b) Find all points $x = c$ (if any) on the interval [-1,2] such that $f'(c) = m$.

[17] **7.** (a) Verify that the point (1,2) belongs to the curve defined by the equation $y^3 - x\,y - 2\sqrt{3+x^2} = 2$, and find an equation of the tangent line to the curve at this point.

(b) At 1 PM, ship A is 5 kilometers strictly to the west of ship B. Ship A is sailing west at speed 20 km/hour and ship B is sailing north at 30 km/hour. How fast (in km/hour) is the distance between ships changing at 3 PM?

(c) Use l'Hôpital's rule to evaluate the $\lim\limits_{x\to 0} \dfrac{\sin(x) - x}{e^{x^3} - 1}$.

[11] **8.** (a) Find the point (x_0, y_0) on the curve $y = 2\sqrt{x}$ that is closest to the point (3,0).

(b) A box with a square base is to be constructed with a volume of 50 m^3. The material for the bottom and the sides of the box costs \$2/m^2, and the material for the top costs \$5/m^2. Find the dimensions that minimize the cost of the box.

[14] **9.** The function $f(x)$ and its derivatives are given:

$$f(x) = \frac{(1+x)}{x^2}, \qquad f'(x) = \frac{-(x+2)}{x^3}, \qquad f''(x) = \frac{2(x+3)}{x^4}.$$

(a) Find the domain of $f(x)$, check for symmetry, and also find asymptotes (if any).

(b) Find intervals where the function $f(x)$ is increasing, intervals where it is decreasing, and the local maxima and local minima (if any) of $f(x)$.

(c) Determine intervals where the function $f(x)$ is concave upward, intervals where the function is concave downward, and the inflection points of $f(x)$.

(d) Sketch the graph of the function $f(x)$ using the information obtained above.

[5] **Bonus Question.** Let $p(x) = x^4 + a^2x^2 - 2a^2x$, where a is any real number. Prove that the graph $y = p(x)$ has at least one point of local minimum on the interval $(-1, 1)$.