CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	203	All
Examination	**Date**	**Pages**
Final	December 2013	3
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Special Instructions:	Only calculators approved by the Department are allowed	

MARKS

[11] **1.** (a) Solve for x : $\log_2(x-4)+\log_2(x+2)-4=0$

(b) Let $f(x)=\sqrt{4-x^2}$ and $g(x)=\sqrt{1-x}$. Find $f\circ g$ and determine its domain.

(c) Given the function $f=\dfrac{3}{2^x+1}$, find the inverse function f^{-1}, the range of f and the range of f^{-1}.

[7] **2.** Find the limit if it exists :

(a) $\lim\limits_{x\to 2}\dfrac{|x-2|}{x^2+x-6}$ (b) $\lim\limits_{x\to 1}\dfrac{x-1}{3-\sqrt{x^2+8}}$

[6] **3.** Find all horizontal and vertical asymptotes of the function

$$f(x)=\frac{x\sqrt{4x^2+2x+1}+2x^2}{x^2-25}$$

[15] **4.** Find the derivatives of the following functions (you don't need to simplify your final answer, but you must show how you calculate it):

(a) $f(x)=(x^{1/2}+x^{-1/2})(x^{1/2}-x^{-1/2})\sqrt{x}$

(b) $f(x)=\dfrac{\ln(x^2)}{\sqrt{x^2+e^2}}$

(c) $f(x)=\ln[e^{-2x}+\cos(x^3+3x)]$

(d) $f(x)=\dfrac{e^x+e^{-x}}{\tan(x)+\sin(x)}$

(e) $f(x)=(1+x^2)^{x^2+1}$ (use logarithmic differentiation)

[12] 5. Consider the function $y = \sqrt{3+x^2}$.

(a) Use the **definition of derivative** to find the formula for dy/dx.

(b) Find the linearization $L(x)$ of the function $y(x)$ at $a = 1$

(c) Find the differential dy and evaluate it for the values $x = 1$ and $dx = 0.1$.

[7] 6. Let $f(x) = x^3 - 2x + 3$.

(a) Find the slope m of the secant line joining the points $(-2, f(-2))$ and $(0, f(0))$.

(b) Find all points $x = c$ (if any) on the interval [-2,0] such that $f'(c) = m$.

[17] 7. (a) Verify that the point (2,1) belongs to the curve defined by the equation $y^2 + x\sqrt{3+y} = 1 + x^2$, and find an equation of the tangent line to the curve at this point.

(b) A particle is moving on an elliptic trajectory $(x(t), (y(t))$ described by the equation $x^2 + 2x + y^2 = 19$. At the point $(3, 2)$ the y-coordinate changes at the rate $\frac{dy}{dt} = 25 \frac{m}{sec}$. How fast is the x-coordinate changing at that instant?

(c) Use l'Hôpital's rule to evaluate the $\lim_{x \to 0} \frac{e^{x^2}-1}{1-\cos(2x)}$.

[11] 8. (a) Find the point (x_0, y_0) on the line $y = 2x + 5$ that is closest to the origin.

(b) If $A = 1200\,\text{cm}^2$ of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

[14] 9. Given the function $f(x) = xe^{-x^2}$.

(a) Calculate $f'(x)$ and use it to determine intervals where the function is increasing, intervals where it is decreasing, and the local extrema (if any).

(b) Calculate $f''(x)$ and use it to determine intervals where the function is concave upward, intervals where the function is concave downward, and the inflection points (if any).

(c) Sketch the graph of the function $f(x)$ using the information obtained above.

[5] **Bonus Question.** Show that $\frac{x}{1+x^2} < \arctan(x)$ if $x > 0$ (HINT: use Rolle's Theorem.)