

Department of Mathematics & Statistics

Course	Number	Section(s)
Mathematics	203	All
Examination	Date	Pages
Final	December 2012	3
Instructors	Course Examiners	
J. Brody, S. Hachem, M. Padamadan, N. Rossokhata, J. Schulz, P. Zorin	A. Atoyan, H. Proppe	

Special Instructions

- ▷ Only Sharp EL 531 or Casio FX 300 MS calculators are allowed.

MARKS

- [10] 1. (a) Suppose $f(x) = \sqrt{1-x}$ and $g(x) = \cos^2 x$. Find $F = f \circ g$ and simplify it. Use F to find $g \circ f \circ g$.

- (b) Find the inverse of the function $f(x) = e^{x^5} - 1$. Determine the domain and range of f and f^{-1} .

- [8] 2. Evaluate the limits (Do not use l'Hôpital's rule.):

(a) $\lim_{x \rightarrow 2} \frac{\sqrt{7x+2} - 4}{2x^3 - 8x}$ (b) $\lim_{x \rightarrow \infty} \frac{4x^2(\sqrt{x} - 1)^3}{(1 - 2x)^3(\sqrt{x} + 1)}$

- [10] 3. (a) Consider the function $f(x) = \frac{|x+3|}{x^2 + x - 6}$.

Calculate both one-sided limits at the point(s) where the function is undefined.

- (b) Find numbers a and b such that the function

$$f(x) = \begin{cases} 2x^2, & \text{if } x \leq -1 \\ ax + b, & \text{if } -1 < x \leq 0 \\ x^2 - 2, & \text{if } x > 0 \end{cases}$$

will be continuous at every point.

Sketch the graph of this function.

- [16] 4. Find derivatives of the functions (you do not have to simplify the answers):

(a) $f(x) = \frac{x^{2/5} - \sqrt[5]{x} + x^{-2/5}}{x^{1/5}};$

(b) $f(x) = \left(x + \frac{1}{x}\right)^3 \sin(2x);$

(c) $f(x) = \frac{(\arctan x)^2}{1+x^2};$

(d) $f(x) = \tan(x^2 + \ln(3x));$

(e) $f(x) = (\arcsin x)^{\sqrt{x}}$ (use logarithmic differentiation).

- [9] 5. Given the function $f(x) = \frac{x+1}{x-1},$

(a) Use the definition of derivative to find $f'(x).$

(b) Use the appropriate differentiation rule(s) to verify your answer in part (a).

(c) Find the differential dy and evaluate it when $x = 2$ and $dx = 0.05.$

(d) Find the linear approximation $L(x)$ to $f(x)$ at $a = 2$ and find $L(2.05).$
Explain the connection between this and your answer in part (c).

- [16] 6. (a) The equation of a curve defined implicitly is $4(x^2 + y^2)^2 = 25xy^2.$

Verify that the point $(1, 2)$ belongs to the curve. Find an equation of the tangent line to the curve at this point.

(b) The sides of a square decrease in length at a rate of 1 m/sec. At what rate is the area of the square changing when the sides are 5 m long?

(c) Use l'Hôpital's rule to evaluate $\lim_{x \rightarrow 0} \frac{e^{x^2} - 1}{1 - \cos 2x}.$

- [12] 7. (a) Find the absolute maximum and minimum values of the function $g(x) = x^{2/3}(2-x)$ on the interval $[-1, 2].$

(b) An airline policy states that all baggage must be box-shaped with the sum of the length, width and height not to exceed 192 cm. What are the dimensions of a box with a square base that has the largest volume acceptable by the airline, and what is the largest volume?

- [6] 8. Let $f(x) = \frac{x}{x+2}$.
- (a) Find the slope m of the secant line joining the points $(1, f(1))$ and $(4, f(4))$.
- (b) Find the point(s) $x = c$ on the interval $[1, 4]$ such that $f'(c) = m$.

- [13] 9. You are given the following information about the function f :

$$f(x) = \frac{10x^3}{x^2 - 1}; \quad f'(x) = \frac{10x^2(x^2 - 3)}{(x^2 - 1)^2}; \quad f''(x) = \frac{20x(x^2 + 3)}{(x^2 - 1)^3}$$

- (a) Find the domain of f and check for symmetry. Find asymptotes (if any).
- (b) Determine interval(s) where f is increasing, interval(s) where f is decreasing, and also find all local extrema (if any).
- (c) Determine interval(s) where f is concave upward, interval(s) where f is concave downward and inflection point(s) (if any).
- (d) Sketch the graph of f .

[5] **Bonus Question**

Use the Mean Value Theorem to prove that if $x > 1$ then

(a) $\ln x < x - 1$, and

(b) $\frac{x-1}{x} < \ln x$.

[Hint: apply the Mean Value Theorem to the function \ln on the interval $[1, x]$ for part (a), and again to the function \ln on the interval $[1/x, 1]$ for part (b)].