

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	203	All
Examination	Date	Pages
Final	April 2013	3
Instructors:	Course Examiners	
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Special Instructions:	Only calculators approved by the Department are allowed	

MARKS

[10] 1. (a) Let $f(x) = \sqrt{4-x^2}$. Find $h = f \circ f$ and determine the domain and the range of f and the domain of h .

(b) Given the function $f = \log_3(2^x - 4)$ find the inverse function f^{-1} , and determine the domain of f and the domain of f^{-1} .

[12] 2. Evaluate the limits Do not use l'Hôpital rule:

(a) $\lim_{x \rightarrow -3} \frac{x^2 + 5x + 6}{x^2 - 9}$ (b) $\lim_{x \rightarrow 1} \frac{\sqrt{x+8} - 3}{x-1}$ (c) $\lim_{x \rightarrow \infty} \frac{x\sqrt{2+16x^2+x^4}}{1-3x^3}$

[5] 3. Calculate both one-sided limits of $f(x) = \frac{|x^2 - 5x|}{x-5}$ at the point(s) where the function f is discontinuous.

$$\begin{cases} (x^2 - 5x) & x \leq 5 \\ (x^2 - 5x) & x > 5 \end{cases}$$

[16] 4. Find the derivatives of the following functions:

(a) $f(x) = \frac{\sqrt{x^5 - x^2} + e\sqrt{x}}{x^{3/2}}$ Regular

(b) $f(x) = \ln \frac{x^3}{(x+3)^4}$ $(\ln(F(x)))' = \frac{F'(x)}{F(x)}$ $F'(x) = \frac{F'(x)}{F(x)}$

(c) $f(x) = e^{-x}(x^2 \arctan x)$

(d) $f(x) = \sin(x + \sin(x + \sin x))$

(e) $f(x) = (1+2x)^{\ln x}$ (use logarithmic differentiation)

$$d^a f(x) = f'(x) \cdot \ln a \cdot a^{f(x)}$$

$$\left[U(x) \cdot V(x) \right]' = U(x) \cdot V'(x) + [\ln(U(x)) \cdot V(x)]'$$

- [16] 5. (a) Verify that the point $(1, -2)$ belongs to the curve defined by the equation $xy - x\sqrt{5 + y^2} + 6 = x^2$, and find the equation of the tangent line to the curve at that point. ○
- (b) A particle is moving on the trajectory $(x(t), y(t))$ described by the equation $x^2 + 4y^2 = 8$ in the (x, y) plane. At an instant t when the (x, y) coordinates are $(2, 1)$ the x -coordinate is changing at the rate $\frac{dx}{dt} = 16 \frac{\text{m}}{\text{sec}}$. How fast is the y -coordinate changing at that instant?
- (c) Use l'Hôpital's rule to evaluate the limit $\lim_{x \rightarrow 0} \frac{\sin(3x) - 3x}{3x^4 - x^3}$.
- [6] 6. Let $f(x) = x^3 - 3x + 2$.
- (a) Find the slope m of the secant line joining the points $(0, f(0))$ and $(2, f(2))$.
- (b) Find all points $x = c$ (if any) on the interval $[0, 2]$ such that $f'(c) = m$.
- [9] 7. Consider the function $f(x) = \sqrt{1 + 3x}$.
- (a) Use the definition of the derivative to find the formula for $f'(x)$.
- (b) Write the linearization formula for f at $a = 5$
- (c) Use this linearization to approximate the value of $f(6) = \sqrt{18}$
- [12] 8. (a) Find the absolute extrema of $f(x) = \frac{x-1}{3+2x}$ on the interval $[-1, 2]$.
- (b) Find the radius r and the height h of the cylinder that has a given volume V , but has the smallest possible surface area including both its bottom and the top (i.e. express r and h as functions of V).

- [14] 9. Given the function $f(x) = x^4 - 6x^2$.
- (a) Find the domain of f and check for symmetry. Find asymptotes of f (if any).
 - (b) Calculate $f'(x)$ and use it to determine intervals where the function is increasing, intervals where it is decreasing, and the local extrema (if any).
 - (c) Calculate $f''(x)$ and use it to determine intervals where the function is concave upward, intervals where the function is concave downward, and the inflection points (if any).
 - (d) Sketch the graph of the function $f(x)$ using the information obtained above.

[5] **Bonus Question**

We know that a function f is differentiable on the interval $[0,2]$ and has values $f(0) = 0$, $f(1) = 1$ and $f(2) = -1$. Is this information sufficient to claim, using the Mean Value theorem, that the tangent line to the graph of $f(x)$ must be horizontal at least at one point x in the interval $(0,2)$? Explain why yes or why not.