



Université d'Ottawa - University of Ottawa

Faculté des sciences Faculty of Science
Mathématiques et de statistique Mathematics and Statistics

Calculus III for Engineers

MAT 2322B - Fall 2016

Midterm I

Professor: C.Rada

Time limit: 80 minutes. Closed books.

Name: _____ ID Number _____

Cellular phones, unauthorized electronic devices or course notes are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam and academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam. By signing below, you acknowledge that you have ensured that you are complying with the above statement.

(Signature): _____

Instructions

- The only calculators which are allowed are Texas Instruments TI-30, TI-34, Casio fx-260 and fx-300, scientific and non programmable.
- The exam has 8 pages. Read each question carefully before answering.
- Questions 1 to 3 are multiple choice. These questions are worth 2 points each and no partial marks are possible. **Please write your answers in the corresponding boxes in the grid below entitled "Answers to multiple choice Qs".**
- Questions 4 to 6 are long answer questions. Questions 4 and 6 are worth 6 marks each, and question 5 is worth 7 marks, so organize your time accordingly. **A correct answer requires a full, clearly-written and detailed solution.** Answer each question in the space provided, using backs of pages or the extra pages at the end if necessary.
- Do not unstaple the test. Good luck!

Answers to multiple choice Qs

1	2	3
F	C	F

Grid below is used for grading
(do not write in this grid)

MCQ	4	5	6	Total
	2/6	4/6	3/7	6/6
				15/25

1. For all possible unit vectors \vec{u} , what is the largest possible value of $D_{\vec{u}}f(3,2)$, where $f(x,y) = x^2 - 3y^2$?

- A. $8\sqrt{7}$
- B. $7\sqrt{6}$
- C. $6\sqrt{5}$
- D. $5\sqrt{4}$
- E. $4\sqrt{3}$
- F. $3\sqrt{2}$

$$f_x = 2x \quad f_x(3,2) = 6 \quad \vec{u} = \frac{1}{\sqrt{2}}(\vec{i} + \vec{j}) = \frac{\sqrt{2}}{2}(\vec{i} + \vec{j})$$

$$f_y = -6y \quad f_y(3,2) = -12 \quad \frac{\sqrt{3}}{2}$$

$$(6\vec{i} - 12\vec{j}) \left(\frac{\sqrt{2}}{2}\vec{i} + \frac{\sqrt{2}}{2}\vec{j} \right)$$

$$3\sqrt{2} + 6\sqrt{2} = -3\sqrt{2}$$

$$2\sqrt{3} + 4\sqrt{3}$$

2. Which of the following statements is true concerning the critical points of the function $f(x,y) = x^4 - 4x^3 + 4x^2 + y^2$?

- A. $(0,0)$ is a local minimum, $(1,0)$ is a saddle, $(2,0)$ is a local minimum
- ~~B. f has no critical points~~
- C. $(0,0)$ is a local maximum, $(1,0)$ is a saddle, $(2,0)$ is a local maximum
- ~~D. $(0,0)$ is a local minimum, $(1,0)$ is a local maximum, $(2,0)$ is a local minimum~~
- ~~E. $(0,0)$ is a local maximum, $(1,0)$ is a local minimum, $(2,0)$ is a local maximum~~
- ~~F. None of the above~~

$$f_x = 4x^3 - 12x^2 + 8x \quad \cancel{f_x(x^2)} \Rightarrow x=1 \quad x=2$$

$$f_y = 2y \quad y=0 \quad x=0$$

$$f_{xx} = 12x^2 - 24x + 8 \quad (0,0) \quad (1,0) \quad (2,0)$$

$$f_{yy} = 2 \quad f_{xy} = 0 \quad f_{yx} = 0$$

$$(0,0) \quad 8 + 0 + 0 = 16 \quad f_{xx} =$$

$$(1,0) \quad -4 + 0 + 0 = -8 < 0 \rightarrow \text{saddle}$$

$$(2,0) \quad 8 + 0 + 0 = 16$$

3. For the composite function $z(u, v) = f(x(u, v), y(u, v))$, which of the following correspond to the chain rule for $\frac{\partial z}{\partial v}$?

A. $\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial u}$

B. $\frac{\partial z}{\partial v} = \frac{\partial z}{\partial u} \frac{\partial u}{\partial v} + \frac{\partial z}{\partial v}$

C. $\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial v}$

D. $\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial v}$

E. $\frac{\partial z}{\partial v} = \frac{\partial z}{\partial y} \frac{\partial y}{\partial v}$

F. $\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial v}$

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial v}$$

4. Compute the tangent plane to the graph of the function $z = f(x, y) = x \cos y + y \sin x$ at the point $(x, y, z) = (\pi/2, \pi/2, \pi/2)$.

$$f_x = \cos y - y \cos x \quad f_x\left(\frac{\pi}{2}, \frac{\pi}{2}\right) = 0 - 0 = 0$$

$$f_y = x \sin y + \sin x \quad f_y\left(\frac{\pi}{2}, \frac{\pi}{2}\right) = \frac{\pi}{2} + 1 = \frac{2\pi}{2}$$

$$z = \frac{\pi}{2} + 0(x-1) + \frac{2\pi}{2}(y-1)$$

$$z = \frac{\pi}{2} + \frac{2\pi}{2}(y-1) \quad \times$$

$$f_x = \cos y - y \sin x$$

$$f_y = x \sin y + \sin x$$

5. Find the global extrema of the function $f(x, y) = x^2 + y^2$ over the elliptical domain $D = \{(x, y) \in \mathbb{R}^2 \mid 3x^2 + 2xy + 3y^2 \leq 8\}$ sketched below. For part of your solution, you must use the method of Lagrange multipliers.

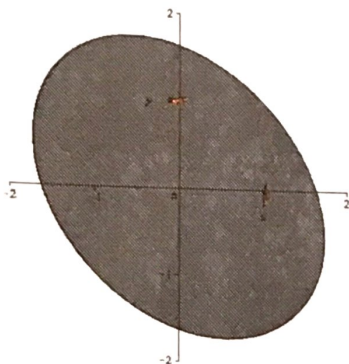


$$f_x = 2x \quad x=0$$

$$f_y = 2y \quad y=0$$

$$CP = (0, 0)$$

$$\underbrace{3x^2 + 2xy + 3y^2}_{g(x, y)} = 8$$



$$f_{xx} = 2$$

$$f_{xy} = 0$$

$$f_{yy} = 2$$

$$f_{yx} = 0$$

$$\nabla f = \lambda \nabla g \begin{cases} 2x = \lambda(6x + 2y) \\ 2y = \lambda(2x + 6y) \end{cases}$$

~~$$x=0 \implies 0 = \lambda(0 + 2y) \implies$$~~

~~$$2x = \lambda 6x + \lambda 2y \quad (6x + 2y) = 0$$~~

$$2x + 6y = 0$$

~~$$\left(-\frac{3}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \lambda\right) \implies \left(-\frac{3}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{3}{10}\right)$$~~

~~$$6x = -2y$$~~

$$2x = -6y$$

~~$$f_x\left(-\frac{3}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) = 2 \cdot \frac{3}{\sqrt{3}} = \frac{-6}{\sqrt{3}}$$~~

~~$$6(-3y) = -2y$$~~

$$x = -\frac{6y}{2} = -3y$$

~~$$f_y\left(-\frac{3}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) = 2 \cdot \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$~~

~~$$-18y = -2y$$~~

~~$$2x + 6\left(\frac{1}{\sqrt{3}}\right) = 0$$~~

~~$$x = -\frac{3}{\sqrt{3}}$$~~

$$3(-3y)^2 + 2x(-3y) + 3y^2 = 8$$

$$3(9y^2) + 6xy^2 + 3y^2 = 8$$

$$27y^2 - 6y^2 + 3y^2 = 8$$

$$2\left(-\frac{3}{\sqrt{3}}\right) = \lambda \left(6\left(-\frac{3}{\sqrt{3}}\right) - 2\left(\frac{1}{\sqrt{3}}\right)\right)$$

$$\begin{cases} 24y^2 = 8 \\ y^2 = \frac{8}{24} = \frac{1}{3} \end{cases}$$

$$\lambda = \frac{-6}{\sqrt{3}} = -\frac{6\sqrt{3}}{3} = -2\sqrt{3}$$

$$-\frac{18}{\sqrt{3}} - \frac{2}{\sqrt{3}}$$

$$= \frac{3}{10}$$

~~$$y = \frac{1}{\sqrt{3}}$$~~

(x, y)	$\nabla f = \lambda \nabla g$	
$(0, 0)$	$\frac{1}{3} \quad 0.33$	\rightarrow global max
$\left(\frac{\sqrt{2}}{2}, \frac{1}{2}\right)$	$\frac{\sqrt{2}}{3\sqrt{2}+1} \quad 0.279$	
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$	$\frac{\sqrt{2}}{3\sqrt{2}+\sqrt{2}} \quad 0.25$	
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}\right)$	$\frac{1}{3+\sqrt{2}} \quad 0.23$	\rightarrow global min

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6. Compute the double integral of the function $f(x, y) = 12x^2y^3$ over the rectangle

$$R = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1, 0 \leq y \leq 2\}$$

$$\iint_R f \, dA$$

$$\int_0^2 \int_0^1 12x^2y^3 \, dx \, dy = \int_0^2 \left. \frac{12x^3}{3} y^3 \right|_0^1 dy = \int_0^2 4 [y^3] dy$$

$$= 4 \left. \frac{y^4}{4} \right|_0^2 = 2^4 = 16$$

$$\int_0^1 \int_0^2 12x^2y^3 \, dy \, dx = \int_0^1 \left. 12x^2 \frac{y^4}{4} \right|_0^2 dx = \int_0^1 3x^2 \cdot 16 \, dx$$

$$= \int_0^1 48x^2 \, dx = 48 \left. \frac{x^3}{3} \right|_0^1 = \frac{48}{3} = 16$$