MCV4U-A

Unit

4

Lines and Planes

December 6, 2010

Table of Contents

Introduction

In Unit 4, you will continue your exploration of vectors. In Unit 3, you learned about the sum and difference of two vectors. What about the product? You will now learn about two types of vector multiplication: dot product and cross product. You will also learn about their applications.

In previous grades, you learned how to represent lines in two-dimensional space using the slope and y-intercept form. How do you represent a line in a three-dimensional space? What about a flat surface? In this unit, you will learn how to represent lines and planes and explore the intersection of lines and planes. Finally, you will make connections between solving a system of equations and the intersection of planes.

Overall Expectations

After completing this unit, you will be able to

- determine the dot product and cross product of two vectors
- solve problems in geometry and physics involving the cross product and dot product
- solve systems of linear equations in two-space by graphing
- describe the geometric significance of linear equations in two-space and in three-space
- identify the various ways in which planes can intersect in three-space
- describe the different ways a line or a plane can be represented
- determine the solution of a linear system of equations with three variables and interpret the solution geometrically
- determine distance using the projection of a vector onto a vector

At the end of the unit there is a Practice Test. It will help you prepare for the Final Test. You will not be marked on the Practice Test, but you should try to answer all the questions.

The Practice Test is on your course page on the ILC website along with Practice Test Suggested Answers so you can mark it yourself. Instructions will be given at the end of Lesson 20.

MCV4U-A

16

The Dot and Cross Product of Two Vectors

Introduction

So far, you have studied the sum and difference of two vectors and the multiplication of a vector by a scalar. What about the multiplication of two vectors? Can you define such multiplication?

In this lesson, you will look at two different types of vector multiplication, the dot product and the cross product. You will also develop properties of each and look at their applications.

Estimated Hours for Completing This Lesson	
The Dot Product	1
Properties of Dot Products	1
The Cross Product	1.5
Key Questions	1.5

What You Will Learn

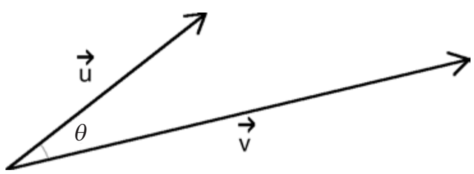
After completing this lesson, you will be able to

- determine the dot product of two vectors represented as directed line segments
- calculate the dot product of two vectors represented in Cartesian form
- describe applications of the dot product
- explain properties of the dot product
- calculate the cross product of two vectors in three-space represented in Cartesian form

The Dot Product

Given two non-zero vectors $\vec{u}$ and $\vec{v}$ arranged tail to tail, where the angle between them is θ and $0° \leq \theta \leq 180°$, the dot product is calculated with the following formula:

$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos\theta$



The dot product is the product of the magnitude of the two vectors and the cosine of the angle between the two vectors arranged tail to tail. The dot product of two vectors is a scalar.

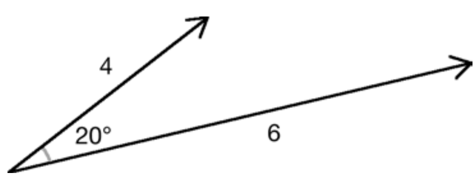
Examples

Find the dot product of each pair of vectors:

a) $|\vec{u}| = 5$, $|\vec{v}| = 6$, and $\theta = 60°$

b) $|\vec{a}| = 6$, $|\vec{b}| = 4$, and $\theta = 45°$

c)



Solutions

a) $$\begin{aligned}\vec{u} \cdot \vec{v} &= |\vec{u}||\vec{v}|\cos\theta \\ &= (5)(6)\cos(60°) \\ &= 30(0.5) \\ &= 15\end{aligned}$$

b) $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$= (6)(4)\cos(45°)$

$= 24(0.707)$

≈ 16.97

c) $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos\theta$

$= (6)(4)\cos(20°)$

$= 24(0.93969)$

$= 22.55$

Dot Product of Two Perpendicular Vectors

What can you say about the dot product of any two perpendicular vectors?

The angle between two perpendicular vectors is 90° and $\cos(90°) = 0$.

$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos(90°)$

$= 0$

You can conclude that the dot product of any two perpendicular vectors is zero.

Dot Product of a Vector With Itself

Since the angle between a vector and itself is zero, and $\cos(0°) = 1$, you can conclude that

$\vec{u} \cdot \vec{u} = |\vec{u}||\vec{u}|\cos(0°)$

$= |\vec{u}||\vec{u}|$

$= |\vec{u}|^2$

Therefore, the dot product of a vector with itself is its magnitude squared.

Dot Product of Vectors in Cartesian Form

You can also find the dot product from the Cartesian coordinates of a vector. As you saw with the line segments in the previous sections, the dot product of two vectors in Cartesian form is a scalar.

Given two vectors in two-space, $\vec{u} = (x_1, y_1)$ and $\vec{v} = (x_2, y_2)$, then $\vec{u} \cdot \vec{v} = x_1x_2 + y_1y_2$.

In three-space, given $\vec{u} = (x_1, y_1, z_1)$ and $\vec{v} = (x_2, y_2, z_2)$, then $\vec{u} \cdot \vec{v} = x_1x_2 + y_1y_2 + z_1z_2$.

Examples

Find the dot product of each pair of vectors:

a) $\vec{u} = (2, -3)$ and $\vec{v} = (4, -1)$

b) $\vec{u} = (2, -3, 1)$ and $\vec{v} = (-4, -1, 2)$

Solutions

a)
$$\begin{aligned}\vec{u} \cdot \vec{v} &= x_1x_2 + y_1y_2\\ &= (2)(4) + (-3)(-1)\\ &= 11\end{aligned}$$

b)
$$\begin{aligned}\vec{u} \cdot \vec{v} &= x_1x_2 + y_1y_2 + z_1z_2\\ &= (2)(-4) + (-3)(-1) + (1)(2)\\ &= -3\end{aligned}$$

Angle Between Two Vectors

Using the two definitions of the dot product that you have learned (line segment and Cartesian), it is possible to calculate the angle between two vectors given their Cartesian coordinates. You do this by finding the dot product of the two vectors using the Cartesian definition and rearranging $\vec{u} \cdot \vec{v} = |\vec{u}| \cdot |\vec{v}|\cos\theta$ to solve for θ:

$$\vec{u} \cdot \vec{v} = |\vec{u}| \cdot |\vec{v}| \cos\theta$$

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| \cdot |\vec{v}|}$$

Examples

Find the angle between each pair of vectors:

a) $\vec{u} = (1, 3)$ and $\vec{v} = (2, -1)$

b) $\vec{u} = (2, -3, 1)$ and $\vec{v} = (2, -1, 1)$

Solutions

a) $\vec{u} = (1, 3)$ and $\vec{v} = (2, -1)$

Find the dot products of the vectors in Cartesian form:

$$\begin{aligned} \vec{u} \cdot \vec{v} &= (1, 3) \cdot (2, -1) \\ &= (1)(2) + (3)(-1) \\ &= -1 \end{aligned}$$

Then find the magnitude of $\vec{u}$ and $\vec{v}$:

$$\begin{aligned} |\vec{u}| &= \sqrt{(1)^2 + (3)^2} \\ &= \sqrt{10} \end{aligned}$$

$$\begin{aligned} |\vec{v}| &= \sqrt{2^2 + (-1)^2} \\ &= \sqrt{5} \end{aligned}$$

Finally, put the values into the equation:

$$\begin{aligned} \cos\theta &= \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| \cdot |\vec{v}|} \\ &= \frac{-1}{\left(\sqrt{10}\right)\left(\sqrt{5}\right)} \\ &= \frac{-1}{5\sqrt{2}} \\ &\approx -0.14 \end{aligned}$$

Use a calculator to find the angle θ:

$\cos(\theta) \approx -0.14$

$$\theta \approx \cos^{-1}(-0.14)$$

$$\theta \approx 98°$$

The angle between $\vec{u}$ and $\vec{v}$ is 98°.

b) $\vec{u} = (2, -3, 1)$ and $\vec{v} = (2, -1, 1)$

Find the dot product:

$$\begin{aligned}\vec{u} \cdot \vec{v} &= (2, -3, 1) \cdot (2, -1, 1) \\ &= 4 + 3 + 1 \\ &= 8\end{aligned}$$

Then find the magnitude of $\vec{u}$ and $\vec{v}$:

$$\begin{aligned}\left|\vec{u}\right| &= \sqrt{(2)^2 + (-3)^2 + (1)^2} \\ &= \sqrt{14}\end{aligned}$$

$$\begin{aligned}\left|\vec{v}\right| &= \sqrt{2^2 + (-1)^2 + (1)^2} \\ &= \sqrt{6}\end{aligned}$$

Finally, put the values into the equation:

$$\begin{aligned}\cos\theta &= \frac{\vec{u} \cdot \vec{v}}{\left|\vec{u}\right| \cdot \left|\vec{v}\right|} \\ &= \frac{8}{\left(\sqrt{14}\right)\left(\sqrt{6}\right)} \\ &\approx 0.87\end{aligned}$$

Use a calculator to find the angle θ:

$\cos(\theta) \approx 0.87$

$$\theta \approx \cos^{-1}(0.87)$$

$$\theta \approx 29.54°$$

The following table is a summary of the formulas used to find the dot product or the angle between two vectors:

Dot product of line segments: $\vec{u} \cdot \vec{v} = \lvert \vec{u} \rvert \lvert \vec{v} \rvert \cos\theta$
Dot product of Cartesian coordinates in two-space: $\vec{u} = (x_1, y_1)$ and $\vec{v} = (x_2, y_2)$, then $\vec{u} \cdot \vec{v} = x_1x_2 + y_1y_2$
Dot product of Cartesian coordinates in three-space: $\vec{u} = (x_1, y_1, z_1)$ and $\vec{v} = (x_2, y_2, z_2)$, then $\vec{u} \cdot \vec{v} = x_1x_2 + y_1y_2 + z_1z_2$
Angle between two vectors: $\cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{\lvert\vec{u}\rvert \cdot \lvert\vec{v}\rvert}$

Support Questions

(do not send in for evaluation)

1. Calculate the dot product of $\vec{u}$ and $\vec{v}$:

   a) $\lvert \vec{u} \rvert = 2$, $\lvert \vec{v} \rvert = 3$, and $\theta = 30°$

   b) $\lvert \vec{u} \rvert = 5$, $\lvert \vec{v} \rvert = 3$, and $\theta = 135°$

2. Find the dot product of each pair of vectors:

   a) $\vec{u} = (-1, 2)$ and $\vec{v} = (4, 3)$

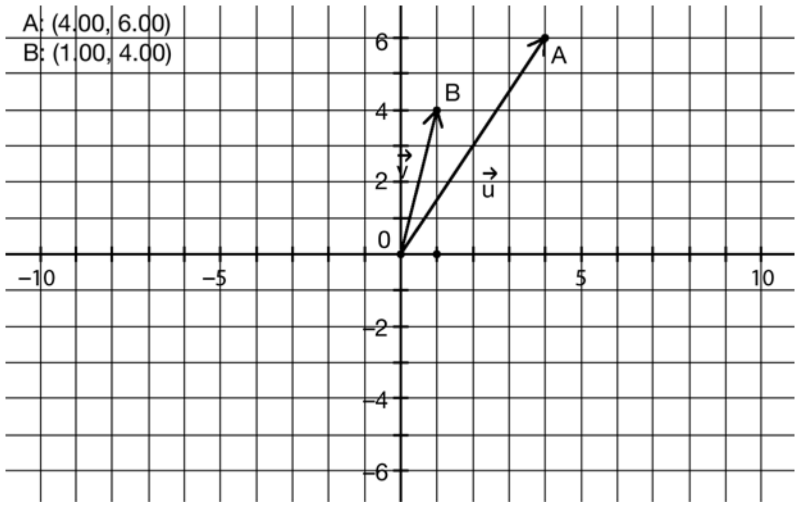
   b) $\vec{u} = (4, -2, 3)$ and $\vec{v} = (0, -1, 5)$

3. Find the angle between each pair of vectors:

   a) $\vec{u} = (-1, 2)$ and $\vec{v} = (-3, -1)$

   b) $\vec{u} = (-1, 2, 1)$ and $\vec{v} = (4, 0, 3)$

4. Determine the angle between the vectors $\vec{u}$ and $\vec{v}$:



There are Suggested Answers to Support Questions at the end of this unit.

Properties of Dot Products

At this point in the lesson, you should be familiar with the two definitions of the dot product and how to use them to calculate the angle between two vectors. In this section, you will explore properties of dot products.

The Commutative Law

As you have already learned, the commutative law says that the order in which you multiply two numbers does not matter. The product of real numbers satisfies the commutative law of multiplication; that is, $xy = yx$. For example, $(2)(3) = (3)(2) = 6$.

Does the dot product satisfy the commutative law? Yes, it does, and this will be demonstrated using the two definitions of the dot product.

Line Segment Definition

$$\vec{u} \cdot \vec{v} = |\vec{u}| \cdot |\vec{v}| \cos\theta$$
$$= |\vec{v}||\vec{u}| \cos\theta$$
$$= \vec{v} \cdot \vec{u}$$

The magnitude of each vector is a real number, so the commutative law of multiplication of real numbers applies.

Cartesian Definition

Here is another way to define the dot product of two vectors given their Cartesian components:

If $\vec{u} = (x_1, y_1, z_1)$ and $\vec{v} = (x_2, y_2, z_2)$, then

$$\vec{u} \cdot \vec{v} = x_1x_2 + y_1y_2 + z_1z_2$$
$$= x_2x_1 + y_2y_1 + z_2z_1$$
$$= \vec{v} \cdot \vec{u}$$

The Cartesian coordinates are real numbers, so the commutative law of multiplication of real numbers applies.

The Distributive Law

Recall, with the distributive law:

$$\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$$

You get the same answer when you multiply a group of numbers by a number as when you do each multiplication separately and then add them together. Here's an example to show the distributive property in three-space. You will do the two-space case as an exercise later in the lesson.

Let $\vec{u} = (x_1, y_1, z_1)$, $\vec{v} = (x_2, y_2, z_2)$, and $\vec{w} = (x_3, y_3, z_3)$

$$\begin{aligned}\vec{u}\cdot(\vec{v}+\vec{w}) &= (x_1, y_1, z_1)\cdot\big((x_2, y_2, z_2)+(x_3, y_3, z_3)\big)\\ &= (x_1, y_1, z_1)\cdot(x_2+x_3, y_2+y_3, z_2+z_3)\\ &= x_1(x_2+x_3)+y_1(y_2+y_3)+z_1(z_2+z_3)\\ &= x_1x_2+x_1x_3+y_1y_2+y_1y_3+z_1z_2+z_1z_3\\ &= x_1x_2+y_1y_2+z_1z_2+x_1x_3+y_1y_3+z_1z_3\\ &= \vec{u}\cdot\vec{v}+\vec{u}\cdot\vec{w}\end{aligned}$$

The Associative Law

The associative law of multiplication says that the product of any three real numbers is the same regardless of how they are grouped together. It is meaningless to talk about the associative property of the dot product, $\vec{u}\cdot(\vec{v}\cdot\vec{w})$, because the dot product is calculated only between vectors and the dot product of $\vec{v}\cdot\vec{w}$ is a scalar. It is possible, however, to show that $(k\vec{u})\cdot\vec{v} = k(\vec{u}\cdot\vec{v})$, where k is a real number.

This is true for vectors in two-space or three-space. Three-space is shown:

Given $\vec{u} = (x_1, y_1, z_1)$ and $\vec{v} = (x_2, y_2, z_2)$

$$\begin{aligned}(k\vec{u})\cdot\vec{v} &= \big(k(x_1, y_1, z_1)\big)\cdot(x_2, y_2, z_2)\\ &= (kx_1, ky_1, kz_1)\cdot(x_2, y_2, z_2)\\ &= kx_1x_2+ky_1y_2+kz_1z_2\\ &= k(x_1x_2+y_1y_2+z_1z_2)\\ &= k(\vec{u}\cdot\vec{v})\end{aligned}$$

Now practise some of the properties of the dot product.

Examples

If $\vec{u} = (0, 1, 3)$, $\vec{v} = (2, 1, 0)$, and $\vec{w} = (4, 1, -1)$, verify the following:

a) $\vec{u}\cdot(\vec{v}+\vec{w}) = \vec{u}\cdot\vec{v}+\vec{u}\cdot\vec{w}$

b) $\vec{v}\cdot\vec{v} = |\vec{v}|^2$

c) $(\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v}) = |\vec{u}|^2+|\vec{v}|^2+2\vec{u}\cdot\vec{v}$

Solutions

a) $$\begin{aligned}\vec{u}\cdot(\vec{v}+\vec{w}) &= (0,1,3)\cdot\left[(2,1,0)+(4,1,-1)\right]\\ &= (0,1,3)\cdot(6,2,-1)\\ &= (0)(6)+(1)(2)+(3)(-1)\\ &= -1\end{aligned}$$

$$\begin{aligned}\vec{u}\cdot\vec{v}+\vec{u}\cdot\vec{w} &= (0,1,3)\cdot(2,1,0)+(0,1,3)\cdot(4,1,-1)\\ &= (0)(2)+(1)(1)+(3)(0)+(0)(4)+(1)(1)+(3)(-1)\\ &= -1\end{aligned}$$

Observe that $\vec{u}\cdot(\vec{v}+\vec{w})=\vec{u}\cdot\vec{v}+\vec{u}\cdot\vec{w}$.

b) $$\begin{aligned}\vec{v}\cdot\vec{v} &= (2,1,0)\cdot(2,1,0)\\ &= 5\end{aligned}$$

$$\begin{aligned}|\vec{v}|^2 &= (2)^2+(1)^2+(0)^2\\ &= 5\end{aligned}$$

Observe that $\vec{v}\cdot\vec{v}=|\vec{v}|^2$.

c) $$\begin{aligned}(\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v}) &= \left[(0,1,3)+(2,1,0)\right]\cdot\left[(0,1,3)+(2,1,0)\right]\\ &= (2,2,3)\cdot(2,2,3)\\ &= (2)(2)+(2)(2)+(3)(3)\\ &= 17\end{aligned}$$

$$\begin{aligned}|\vec{u}|^2+|\vec{v}|^2+2\vec{u}\cdot\vec{v} &= 10+5+2(0,1,3)\cdot(2,1,0)\\ &= 15+2(1)\\ &= 17\end{aligned}$$

Observe that $(\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v})=|\vec{u}|^2+|\vec{v}|^2+2\vec{u}\cdot\vec{v}$.

Support Questions
(do not send in for evaluation)

5. If $\vec{u} = (1, -1, 2)$, $\vec{v} = (1, -1, 3)$, and $\vec{w} = (-1, 0, 2)$, verify the following:

   a) $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$

   b) $\vec{w} \cdot \vec{w} = |w|^2$

   c) $(\vec{u} + \vec{v}) \cdot (\vec{u} + \vec{v}) = |\vec{u}|^2 + |\vec{v}|^2 + 2(\vec{u} \cdot \vec{v})$

6. Verify the distributive property for the dot product for vectors in two-space using the technique demonstrated in the lesson.

The Cross Product

The dot product of two vectors is a scalar. In this section, you will learn about the cross product. The cross product of two vectors in three-space, $\vec{u}$ and $\vec{v}$, is a third vector. The new vector is perpendicular to both $\vec{u}$ and $\vec{v}$.

Let $\vec{u} = (u_1, u_2, u_3)$ and $\vec{v} = (v_1, v_2, v_3)$. The cross product of $\vec{u}$ and $\vec{v}$, denoted $\vec{u} \times \vec{v}$, is defined by the following:

$$\vec{u} \times \vec{v} = \left(u_2v_3 - v_2u_3, u_3v_1 - v_3u_1, u_1v_2 - v_1u_2\right)$$

Note the cross product is calculated only for vectors in three-space. The following example demonstrates how to calculate the cross product:

Example

Find the cross product of vectors $\vec{u} = (1, -1, 2)$ and $\vec{v} = (2, 1, 3)$.

Solution

$$\begin{aligned}\vec{u} \times \vec{v} &= \left(u_2v_3 - v_2u_3, u_3v_1 - v_3u_1, u_1v_2 - v_1u_2\right)\\ &= \left((-1)(3) - (1)(2), (2)(2) - (3)(1), (1)(1) - (2)(-1)\right)\\ &= \left(-5, 1, 3\right)\end{aligned}$$

The formula of the cross product is a bit complicated to remember. An easier way to find the cross product is to use the following steps, which have been applied to the vectors from the previous example:

$\vec{u} = (1, -1, 2)$ and $\vec{v} = (2, 1, 3)$

Step 1: Write each vector in a row, starting with the second component and ending with it:

–1	2	1	–1
1	3	2	1

Step 2: Subtract the diagonal products to get the components of the new vector:

–1	2	1	–1
1	3	2	1

$(-1)(3) - (1)(2) = -5$

$(2)(2) - (3)(1) = 1$

$(1)(1) - (2)(-1) = 3$

The cross product $\vec{u} \times \vec{v} = (-5, 1, 3)$

How can you check that your answer is perpendicular to both $\vec{u}$ and $\vec{v}$? Recall the dot product of perpendicular vectors is zero. You can calculate the dot product of $\vec{u} \times \vec{v}$ with $\vec{u}$ and $\vec{v}$ to check if your answer is correct.

$$\begin{aligned}(\vec{u}\times\vec{v})\cdot\vec{u} &= (-5,1,3)\cdot(1,-1,2)\\ &= (-5)(1)+(1)(-1)+(3)(2)\\ &= -5-1+6\\ &= 0\end{aligned}$$

$$\begin{aligned}(\vec{u}\times\vec{v})\cdot\vec{v} &= (-5,1,3)\cdot(2,1,3)\\ &= (-5)(2)+(1)(1)+(3)(3)\\ &= -10+1+9\\ &= 0\end{aligned}$$

Since both dot products are zero, the vector (–5, 1, 3) is perpendicular to both $\vec{u}$ and $\vec{v}$.

Example

Given the points $A(-1, 2, 3)$, $B(2, -2, 1)$, and $C(3, 1, 2)$, determine a vector that is perpendicular to both $\vec{AB}$ and $\vec{BC}$.

Solution

To find a vector perpendicular to both $\vec{AB}$ and $\vec{BC}$, you need to find their cross product.

First calculate the components of $\vec{AB}$ and $\vec{BC}$:

$$\begin{aligned}\vec{AB} &= (2, -2, 1) - (-1, 2, 3)\\ &= (3, -4, -2)\end{aligned}$$

$$\begin{aligned}\vec{BC} &= (3, 1, 2) - (2, -2, 1)\\ &= (1, 3, 1)\end{aligned}$$

Next, find the cross product by following the procedure in the previous example:

$$\begin{array}{cccc} -4 & -2 & 3 & -4 \\ 3 & 1 & 1 & 3 \end{array}$$

$$(-4)(1) - (3)(-2) = 2$$

$$(-2)(1) - (1)(3) = -5$$

$$(3)(3) - (1)(-4) = 13$$

The vector perpendicular to $\vec{AB}$ and $\vec{BC}$ is $\vec{AB} \times \vec{BC}$ = (2, –5, 13).

To check that your answer is correct, find the dot product of $\vec{AB} \times \vec{BC}$ with $\vec{AB}$ and $\vec{BC}$:

$$\begin{aligned}\left(\vec{AB} \times \vec{BC}\right) \cdot \vec{AB} &= (2, -5, 13) \cdot (3, -4, -2) \\ &= 6 + 20 - 26 \\ &= 0\end{aligned}$$

$$\begin{aligned}\left(\vec{AB} \times \vec{BC}\right) \cdot \vec{BC} &= (2, -5, 13) \cdot (1, 3, 1) \\ &= 2 - 15 + 13 \\ &= 0\end{aligned}$$

Since both dot products are zero, the vector (2, –5, 13) is perpendicular to both $\vec{AB}$ and $\vec{BC}$.

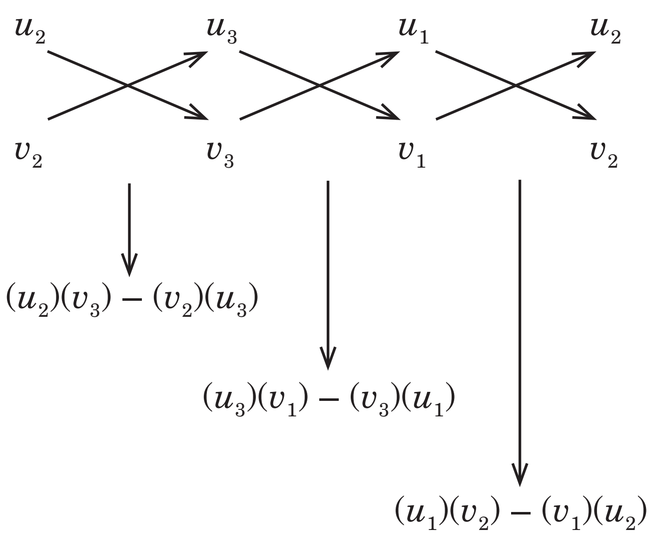
The following table summarizes cross product calculations:

Cross product of vectors in three-space:

Let $\vec{u} = (u_1, u_2, u_3)$ and $\vec{v} = (v_1, v_2, v_3)$, then

$\vec{u} \times \vec{v} = \left(u_2v_3 - v_2u_3, u_3v_1 - v_3u_1, u_1v_2 - v_1u_2\right)$

Simplified method of solving cross product:



Verify the cross product using dot product:

$(\vec{u} \times \vec{v}) \cdot \vec{u} = 0$

$(\vec{u} \times \vec{v}) \cdot \vec{v} = 0$

Support Questions

(do not send in for evaluation)

7. Find the cross product of vectors $\vec{u} = (-2, 3, 0)$ and $\vec{v} = (1, 0, 1)$ and use the dot product to confirm that your answer is correct.

8. Find a vector that is perpendicular to both (1, 0, 0) and (0, 1, 0). Confirm your answer using the dot product.

Conclusion

In this lesson, you explored two different types of vector multiplication, the dot product and its properties and the cross product. In the next lesson, you will develop properties of the cross product. You will also explore applications of both products.

Key Questions

Save your answers to the Key Questions. When you have completed the unit, submit them to ILC for marking.

(16 marks)

46. Find the angle between the vectors $\vec{u} = (-1, 0, 1)$ and $\vec{v} = (-2, 2, -1)$. **(3 marks)**

47. Find the cross product of $(-1, 3, 4)$ and $(-5, 6, 0)$ and use the dot product to confirm that your answer is correct. **(3 marks)**

48. Show that $(k\vec{u}) \cdot \vec{v} = k(\vec{u} \cdot \vec{v})$ for vectors in two-space. **(2 marks)**

49. a) Given

    $\vec{u} = (1, 3, -2)$

    $\vec{v} = (-2, 2, 2)$

    $\vec{w} = (5, 1, 4)$

    demonstrate that $\vec{u}$, $\vec{v}$, and $\vec{w}$ are all perpendicular to each other. **(5 marks)**

    b) Given two vectors $\vec{u}$ and $\vec{v}$ such that $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|$, explain what you know about the two vectors. **(3 marks)**

Now go on to Lesson 17. Do not submit your coursework to ILC until you have completed Unit 4 (Lessons 16 to 20).