Problem Set #1 - General Equilibrium with Exchange

November 22, 2011

Problem 1 *Consider an exchange economy, with two goods, x and y, and two individuals, A and B. Let*

$$\begin{aligned} U^A &= x_A^{0.5} y_A^{0.5} \\ U^B &= x_B^{1/3} y_B^{1/3} \end{aligned}$$

where x_A, x_B, y_A, y_B denote the consumptions of good x and good y by individuals A and B. Suppose that $\omega^A = (10, 15)$, that is, individual A has an endowment of 10 units of good x and 15 units of good y, and $\omega^B = (12, 5)$. Find the general equilibrium of this economy.

1. Solve the consumer optimisation problem to find the demand functions for each good by each individual. Note: when you solve for the demand functions, keep revenue exogenous.
2. Substitute the expressions for income (i.e., price times endowment) into each demand function. Set demand equal to supply for both goods (note: the supply side comes from the endowments).
3. Choose a numeraire commodity.
4. Solve for the price of the other commodity.
5. Calculate the equilibrium quantities demanded of each good.

Problem 2 *Consider an exchange economy, with two goods, x and y, and two individuals, A and B. There are N_A individuals of type A, and N_B individuals of type B. Let*

$$\begin{aligned} U^A &= .3lx_A + .2 \ln y_A \\ U^B &= .7 \ln x_B + .1 \ln y_B \end{aligned}$$

where x_A, x_B, y_A, y_B denote the consumptions of good x and good y by individuals of type A and B. Suppose that $\omega^A = (5, 5)$, that is, each individual A has an endowment of 5 units of good x and 5 units of good y, and $\omega^B = (20, 20)$. Find the general equilibrium of this economy. How would a change (remember - this is a code word!) in N_A affect the equilibrium outcome?

Problem 3 *Consider an exchange economy, with two goods, x and y, and two individuals, A and B. There are N_A individuals of type A, and N_B individuals of type B. Let*

$$\begin{aligned} U^A &= \alpha l x_A + \beta \ln y_A, \alpha + \beta < 1 \\ U^B &= \gamma \ln x_B \ + \delta \ln y_B, \gamma + \delta < 1 \end{aligned}$$

where x_A, x_B, y_A, y_B denote the consumptions of good x and good y by individuals of type A and B. Suppose that $\omega^A = (\overline{X}_A, \overline{Y}_A)$ and $\omega^B = (\overline{X}_B, \overline{Y}_B)$. Find the general equilibrium of this economy. How would a change (remember - this is a code word!) in N_A affect the equilibrium outcome?How would a change in any of the endowments affect the equilbrium outcome?

Problem 4 *Consider an exchange economy, with two goods, x and y, and three individuals, A,B and C. There are N_A individuals of type A, N_B individuals of type B and N_C individuals ot type C. Let*

$$\begin{aligned} U^A &= .3 l x_A + .2 \ln y_A \\ U^B &= .7 \ln x_B \ + .1 \ln y_B \\ U^C &= .5 \ln x_C \ + .5 \ln y_C \end{aligned}$$

where $x_A, x_B, x_C, y_A, y_B, y_C$ denote the consumptions of goods x and good y by individuals of type A, B and C. Suppose that $\omega^A = (5, 5)$, that is, each individual A has an endowment of 5 units of good x and 5 units of good y, $\omega^B = (20, 20)$ and $\omega^C = (5, 25)$. Find the general equilibrium of this economy.

Problem 5 *Consider an exchange economy, with two goods, x and y, and two individuals, A and B. Let*

$$\begin{aligned} U^A &= x_A^{0.5} y_A^{0.5} \\ U^B &= x_B^{1/2} y_B^{1/2} \end{aligned}$$

where x_A, x_B, y_A, y_B denote the consumptions of good x and good y by individuals A and B. Suppose that $\omega^A = (10, 20)$, that is, individual A has an endowment of 10 units of good x and 15 units of good y, and $\omega^B = (20, 10)$. Find the general equilibrium of this economy. Now calculate the equilibrium if the endowments are $\omega^A = (20, 20)$ and $\omega^B = (20, 20)$. Comment on the difference between these two outcomes: explain why there is trade in one case, and not in the other; comment on the difference (if there is a difference) in the structure of equilibrium prices.