CONCORDIA UNIVERSITY
Department of Mathematics and Statistics

Course	**Number**		**All sections**
MATH	251/2		
Examination	**Date**	**Time**	**Pages**
Sample final 1	December 2016	14:00 - 17:00	3

Course examiner: Prof. J. Harnad

Instructions: Answer all parts of all numbered questions 1-6. The value for each part is indicated in square brackets in the margin (out of a possible total of 60). Lined examination booklets will be provided. Write all relevant calculations, proofs and results on the right hand pages. Left hand pages are for rough work only, and will not be read or included in the grading. Only calculators of the type authorized by the Department of Mathematics and Statistics may be used. Any books, notes, or other recorded materials may not be used.

In the following, $P_n(\mathbf{R})$ denotes the space of polynomials in one variable of degree less than or equal to n, with real coefficients.

1. Let $T : P_3(\mathbf{R}) \to P_4(\mathbf{R})$ be the map defined by

$$T(p(x)) = xp(1) - p(0) - \int_1^x p(y)dy.$$

[2] **a.** Prove that it is linear.

[4] **b.** Find a basis for the null space $N(T)$ and find its dimension.

[4] **c.** Find a basis for the range $R(T)$ and find its dimension.

[10] **2.** In $\mathbb{R}^2$ let L be the line $y = mx$ where $m \neq 0$. Find an expression for $T(x, y)$ where

1. T is the reflection of $\mathbb{R}^2$ about L

2. T is the projection on L about the line perpendicular to L

[5] **3a.** Let A be an $m \times n$ matrix. Prove that if λ is any nonzero scalar, then $\text{rank}(\lambda A) = \text{rank}(A)$.

[5] **3b.** Let

$$A := \begin{pmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & -1 & 2 \\ 2 & 0 & 1 & 0 \\ 0 & -1 & 1 & -3 \end{pmatrix}.$$

Compute $\text{rank}(A)$, and find a basis for the space generated by its column vectors.

[5] **4a.** Let

$$A := \begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{pmatrix}.$$

where a, b and c are distinct real numbers. Show that A is invertible and compute its inverse.

[5] **4b.** Using Cramer's rule, solve the linear system

$$\begin{aligned} x + y + z &= 1 \\ ax + by + cz &= m \\ a^2x + b^2y + c^2z &= m^2 \end{aligned}$$

where a, b and c are distinct real numbers and m is an arbitrary constant.

5. Let

$$A = \begin{pmatrix} -4 & 3 & 3 \\ 0 & 2 & 0 \\ -6 & 3 & 5 \end{pmatrix}.$$

[5] **a.** Find all the eigenvalues of A.

[5] **b.** Find a basis for $\mathbf{R}^3$ consisting of eigenvectors of A.

[10] **6.** By diagonalizing the matrix of coefficients, find the general solution $(x(t), y(t), z(t))$ of the system of differential equations

$$\frac{dx}{dt} = x + 2y$$
$$\frac{dy}{dt} = 2x + y + 2z$$
$$\frac{dz}{dt} = 2y + z.$$