Université d'Ottawa
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École de science informatique
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Faculty of Engineering

School of Electrical Engineering
and Computer Science

ELG 3106 / 3506 Electromagnetic Engineering / Électromagnétisme Appliqué
Fall/Automne 2013

FINAL EXAMINATION (3 hours) / EXAMEN FINAL (3 heures)

Professors H. Schriemer, K. Hinzer **Nom/Name:** ______________________________

Date Dec. 12th, 12 déc. 2013 **St./Étudiant#:**______________________________

- This booklet contains 13 pages, including the cover page.
- DO NOT SEPARATE THE PAGES OF THIS BOOKLET!
- Answer all the questions.
- This is a closed-book exam. Equations are provided on the last page of this booklet.
- Calculators are permitted, but must not be pre-programmable.
- Remember to provide units.
- Write down any assumptions that you make.

- Ce livret contient 22 pages, incluant la page de couverture.
- NE PAS SÉPARER LES PAGES DE CE LIVRET !
- Répondre à toutes les questions.
- Cet examen est à livres fermés. Les équations sont fournies à la dernière page de ce livret.
- Les calculatrices sont permises, mais ne doivent pas être pré-programmables.
- Ne pas oublier de préciser les unités.
- Écrire lisiblement toute supposition que vous faites.

Choix Multiple Choice	**/ 5**
Q1	**/ 10**
Q2	**/ 5**
Q3	**/ 10**
Q4	**/ 10**
Q5	**/ 10**

Final Mark / Note finale:

/ 50

Questions à Choix Multiples // Multiple Choice Questions
(1 point each/1 point chacun)

1. The primary parameters of a transmission line are:

   Les paramètres principaux d'une ligne de transmission sont :

   (a) I, V, γ
   (b) R', G', L', C'
   (c) α, β, Z_0
   (d) **E**, **B**, **J**, **D**

2. A uniform plane wave is:

   Une onde plane uniforme est :

   (a) longitudinal in nature
   (b) transverse in nature
   (c) neither transverse nor longitudinal
   (d) x-directed

   (a) de nature longitudinale
   (b) de nature transverse
   (c) ni transverse, ni longitudinale
   (d) va dans la direction de l'axe des x

3. A wave with a propagation constant given by $0.1\pi + j0.2\pi$ has a wavelength of:

   Une onde ayant une constante de propagation donnée par $0.1\pi + j0.2\pi$ a une longueur d'onde de :

   (a) 10 m
   (b) 20 m
   (c) 30 m
   (d) 25 m

4. The direction of propagation of an electromagnetic wave is given by the direction of:

   La direction de propagation d'une onde électromagnétique est obtenue à partir de :

   (a) $\mathbf{E}$
   (b) $\mathbf{H}$
   (c) $\mathbf{E}\times\mathbf{H}$
   (d) $\mathbf{E}\cdot\mathbf{H}$

5. The intrinsic impedance of a medium with $\sigma = 0$, $\varepsilon_r = 9$ and $\mu_r = 1$ is

   L'impédance intrinsèque d'un milieu ayant $\sigma = 0$, $\varepsilon_r = 9$ et $\mu_r = 1$ est

   (a) $40\pi\,\Omega$
   (b) $9\,\Omega$
   (c) $120\pi\,\Omega$
   (d) $60\pi\,\Omega$

Long Answer Questions // Questions à réponse longue

Question 1

This question must be done using the Smith chart.
Cette question doit être résolue en utilisant l'abaque de Smith.

An antenna with an impedance of 40 + *j*30 Ω is to be matched in parallel to a 100 Ω lossless line with a shorted stub. Determine for a positive stub admittance:

Une antenne ayant une impédance de 40 + *j*30 Ω doit être adaptée à une ligne de transmission de 100 Ω, sans perte, avec un stub court-circuité. Déterminez pour un stub ayant une admittance positive :

(a) The required stub admittance;

La valeur de l'admittance du stub requis ;

$$z_L = \frac{Z_L}{Z_0} = \frac{40 + j30}{100} = 0.4 + j0.3$$

Locate z_L on the Smith chart and from this draw the *s*-circle so that y_L can be located diametrically opposite z_L. Thus y_L = 1.6 – *j*1.2. We want a positive stub admittance so where the *SWR*-circle intersects the *g* = 1 circle, we choose the point in the lower half-plane, point B at $1 - j1.05$. That is where we place the stub. Canceling the susceptance gives $y_s = +j1.05$. Thus the required stub admittance is

$$Y_s = Y_o y_s = \frac{j1.05}{100} = j0.0105 \text{ S}$$

(b) The distance between the stub and the antenna;

La distance entre le stub et l'antenne ;

From y_L to B: $\ell_B = 0.337\lambda - 0.304\lambda = 0.033\lambda$

(c) The stub length;

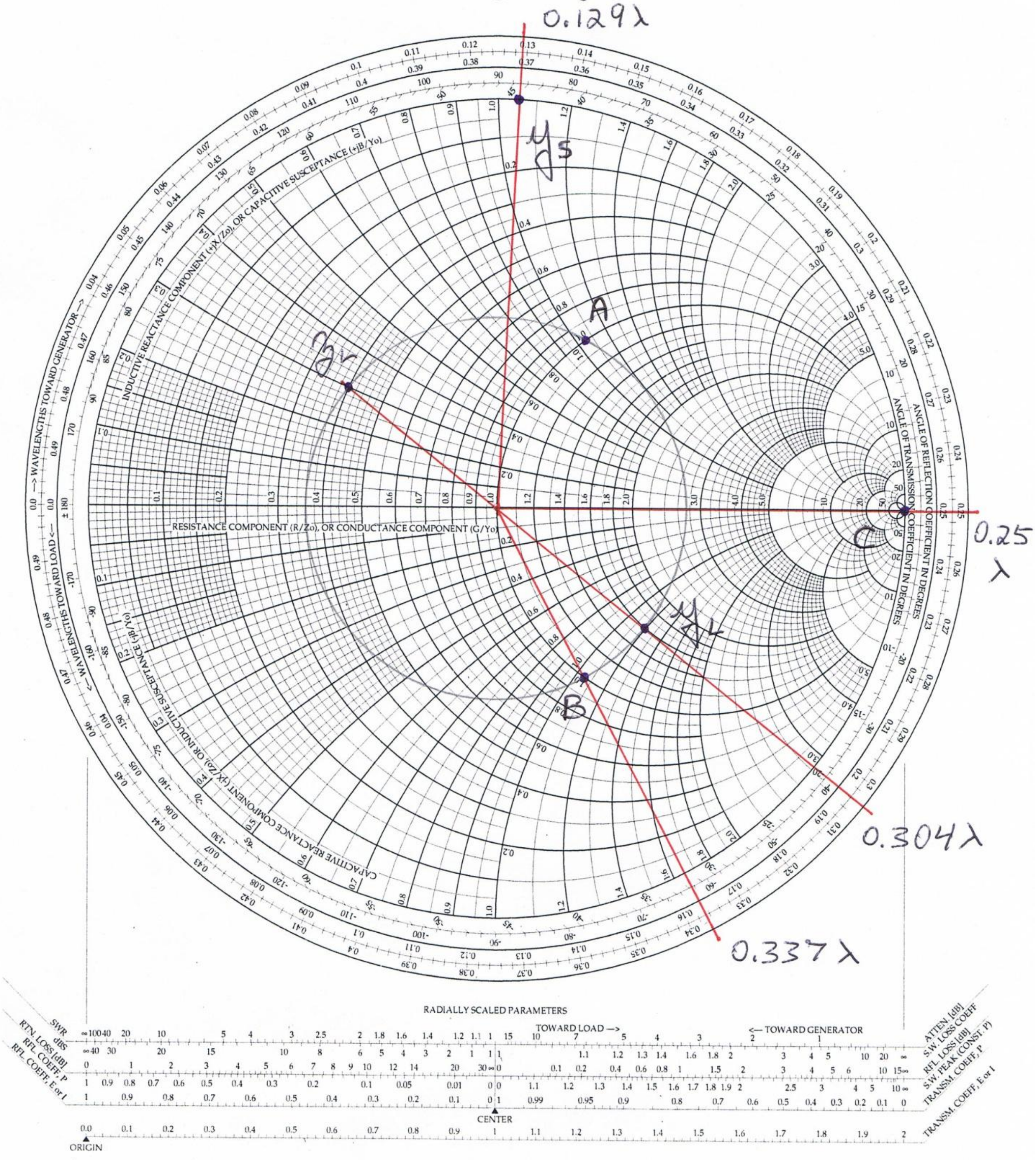
La longueur du stub ;

This is a shorted stub, so it has infinite admittance, point C. From C to y_s is

$$d_B = (0.129\lambda - 0.25\lambda) + 0.5\lambda = 0.379\lambda$$

The Complete Smith Chart

Black Magic Design



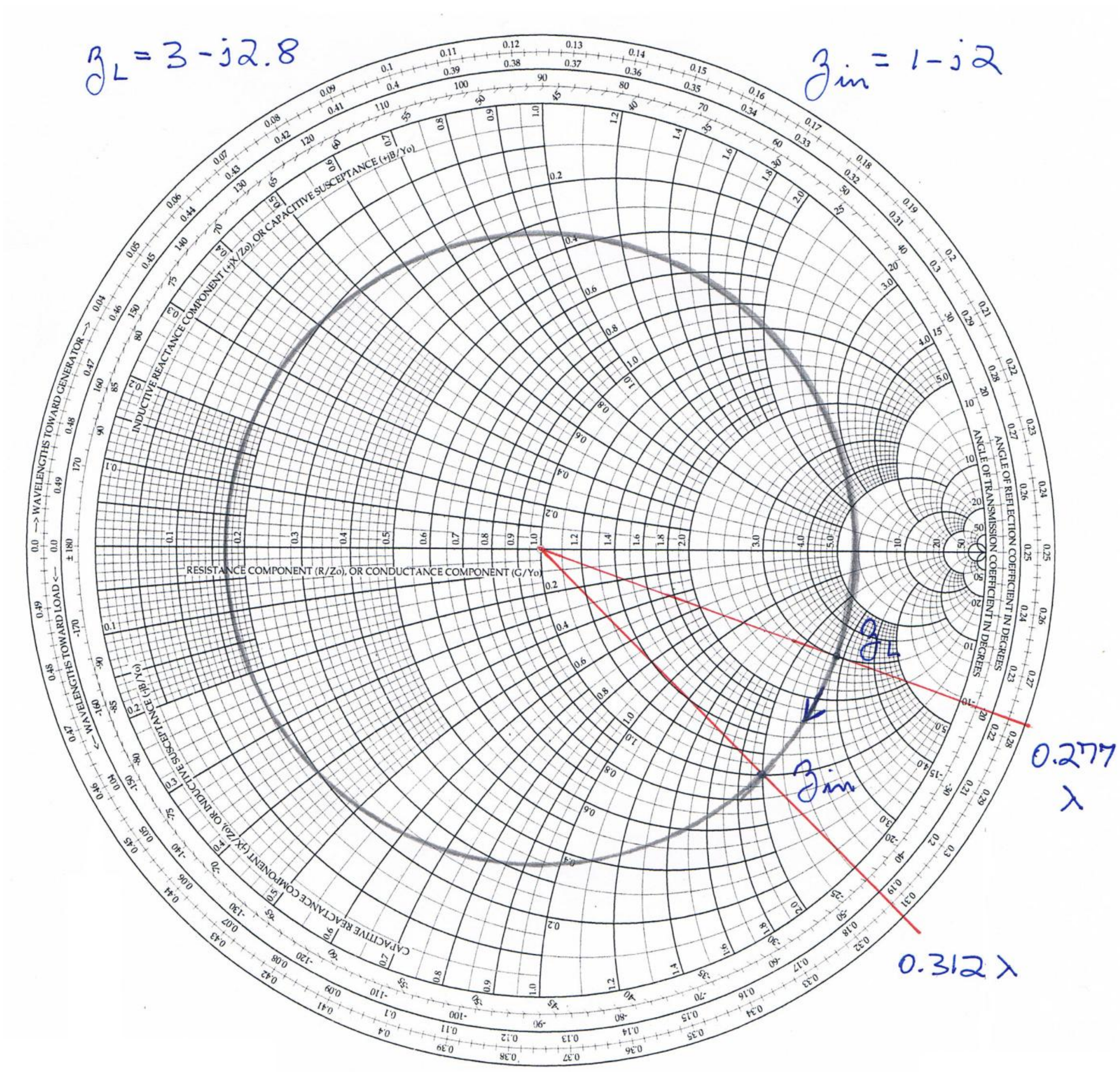
Question 2

This question must be done using the Smith chart.
Cette question doit être résolue en utilisant l'abaque de Smith.

A 100-Ω lossless transmission line with a load impedance of $Z_L = (300 - j280)$ Ω has an input impedance of $Z_{in} = (100 - j200)$ Ω. Assuming the line length to be less than 0.5λ, use the Smith chart to find the line length.

Une ligne de transmission de 100 Ω ayant une impédance de charge de $Z_L = (300 - j280)$ Ω à une impédance d'entrée de $Z_{in} = (100 - j200)$ Ω. Supposons que la longueur de la ligne est moins de 0.5λ, utiliser l'abaque de Smith pour trouver la longueur de la ligne.

Answer: $l = 0.312\lambda - 0.277\lambda = 0.035\lambda$



Question 3

This question must be done using the Smith chart.
Cette question doit être résolue en utilisant l'abaque de Smith.

A lossless transmission line of length 0.434λ and characteristic impedance 100 Ω is terminated in an impedance of (260 + *j* 180) Ω. Find the following quantities:

Une ligne de transmission, sans perte, de longueur 0.434λ et d'impédance caractéristique 100 Ω est terminée par une impédance de (260 + *j* 180) Ω. Déterminer les grandeurs suivantes :

(a) The voltage reflection coefficient;

Le coefficient de réflexion en tension ;

$$z_L = \frac{260 + j180}{100} = 2.6 + j1.8$$

From the θ scale, we find $\theta_\Gamma = 22^\circ$; from the bottom grid we find $|\Gamma| = 0.595$. Hence: $\Gamma = 0.60 e^{j22^\circ}$

(b) The standing wave ratio;

Le rapport d'ondes stationnaire ;

To find the standing wave ratio, we need to locate P_{max} on the SC. This is S=3.95

(c) The input impedance;

L'impédance d'entrée;

To find z_{in}, start at z_L and move clockwise $(0.219 + 0.434)\lambda = 0.653\lambda$. Reduce by 0.5λ yields $(0.653 - 0.5)\lambda = 0.153\lambda$

Drawing a straight line from this position to the origin intersects the constant $|\Gamma|$ circle at $z_{in} = 0.69 + j1.18$

Hence: $Z_{in} = z_{in}.Z_0 = (0.69 + j1.18)(100) = (69 + j118)\Omega$

(d) The location of the voltage maximum closest to the load.

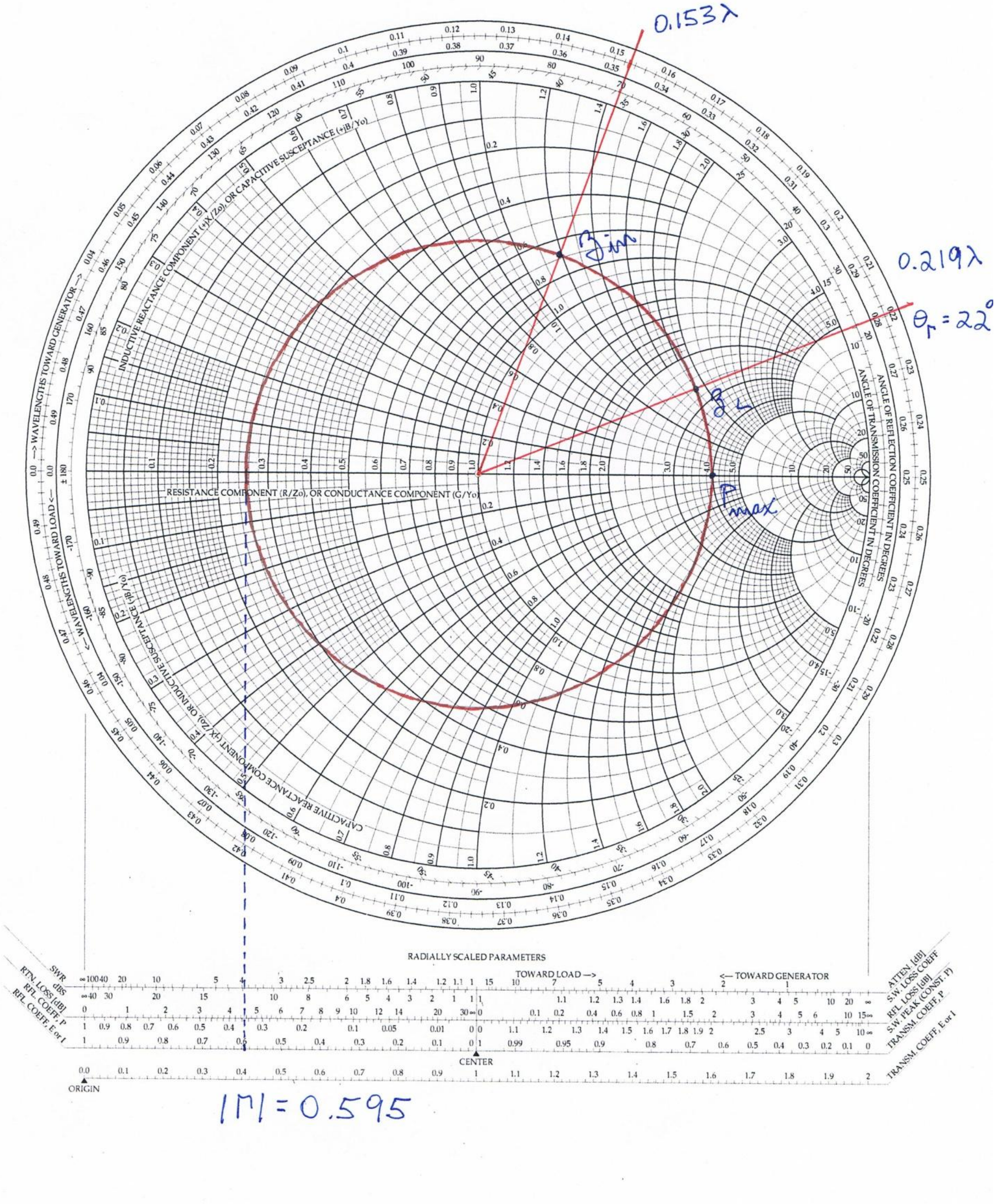
La localisation du maximum de tension le plus proche de la charge.

P_{max} is the location of the voltage maximum. Hence:

$$\ell_{max,0} = (0.25 - 0.219)\lambda = 0.031\lambda$$

The Complete Smith Chart

Black Magic Design



Question 4

In a non-conducting medium with $\varepsilon = 16\varepsilon_0$ and $\mu = \mu_0$, the electric field intensity of an electromagnetic wave is

$$\mathbf{E}(z,t) = \hat{\mathbf{x}}10\sin\left(10^{10}t - kz\right) \text{ V/m}.$$

Dans un milieu non-conducteur ayant $\varepsilon = 16\varepsilon_0$ et $\mu = \mu_0$, l'intensité du champ électrique d'une onde électromagnétique est

$$\mathbf{E}(z,t) = \hat{\mathbf{x}}10\sin\left(10^{10}t - kz\right) \text{ V/m}.$$

(a) Find the phasor $\tilde{\mathbf{E}}(z)$;
Trouvez le phaseur $\tilde{\mathbf{E}}(z)$;

$$\mathbf{E}(z,t) = \hat{\mathbf{x}}\,10\cos(10^{10}t - kz - \pi/2) = \text{Re}[\tilde{\mathbf{E}}(z)e^{j\omega t}] \text{ (V/m)}$$

Therefore

$$\tilde{\mathbf{E}}(z) = \hat{\mathbf{x}}10e^{-jkz}e^{-j\pi/2} = -\hat{\mathbf{x}}\,j10e^{-jkz}.$$

(b) Find both $\tilde{\mathbf{H}}(z)$ and k;
Trouvez $\tilde{\mathbf{H}}(z)$ et k ;

$$\tilde{\mathbf{H}}(z) = \eta^{-1}\hat{\mathbf{k}}\times\tilde{\mathbf{E}} = \left(\sqrt{\frac{\mu}{\varepsilon}}\right)^{-1}\hat{\mathbf{z}}\times\left(-\hat{\mathbf{x}}\,j10e^{-jkz}\right) = \left(\frac{120\pi}{\sqrt{16}}\right)^{-1}\hat{\mathbf{y}}\left(-j10e^{-jkz}\right) = -\hat{\mathbf{y}}0.106\,je^{-jkz}$$

$$k = \omega\sqrt{\mu\varepsilon} = \frac{\omega\sqrt{\mu_r\varepsilon_r}}{c} = \frac{\left(10^{10}\right)\sqrt{(1)(16)}}{3\times10^{8}} = \frac{400}{3} = 133.3 \quad \text{(rad/m)}.$$

(c) Find $\mathbf{H}(z,t)$;
Trouvez $\mathbf{H}(z,t)$;

$$\mathbf{H}(z,t) = \text{Re}[\tilde{\mathbf{H}}(z)e^{j\omega t}] = \text{Re}\left[-\hat{\mathbf{y}}0.106\,je^{-jkz}e^{j\omega t}\right] = \hat{\mathbf{y}}\;0.106\sin(10^{10}t - 133.3z) \quad \text{(A/m)}.$$

Question 5

Consider a metallic rectangular waveguide filled with a lossless dielectric of refractive index 1.5 and whose interior dimensions are a = 1.75 cm and b = 1.0 cm.

Considérons un guide d'onde rectangulaire métallique creux rempli d'un diélectrique sans perte ayant un index de réfraction d'1.5. Les dimensions intérieures du guide d'onde sont a = 1.75 cm et b = 1.0 cm.

(a) How many modes are capable of excitation at an operating frequency of f = 17.5 GHz? Which ones are they? You may use a graph to solve the problem.

Combien de modes peuvent être propagés pour une fréquence d'opération de f = 17.5 GHz ? Quels modes sont-ils ? Vous pouvez utiliser un graphique pour résoudre la question.

$b = 1.0\,\text{cm}$

$a = 1.75\,\text{cm}$

$$f = 17.5\,\text{GHz}$$

$$u = \frac{3\times10^{10}\ \mathbf{cm/s}}{1.5} = 2\times10^{10}\ \mathbf{cm/s}$$

$$\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 < \left(\frac{2f}{u}\right)^2, \text{ where } \frac{2f}{u} = \frac{2\left(17.5\times10^9\right)}{2\times10^{10}} = 1.75\,\text{cm}^{-1}$$

There are 8 allowed modes:
6 TE and 2 TM

TE_{10}, $\frac{2f_c^{1,0}}{u} = 0.571\,\text{cm}^{-1}$

TE_{20}, $\frac{2f_c^{2,0}}{u} = 1.143\,\text{cm}^{-1}$

TE_{30}, $\frac{2f_c^{3,0}}{u} = 1.714\,\text{cm}^{-1}$

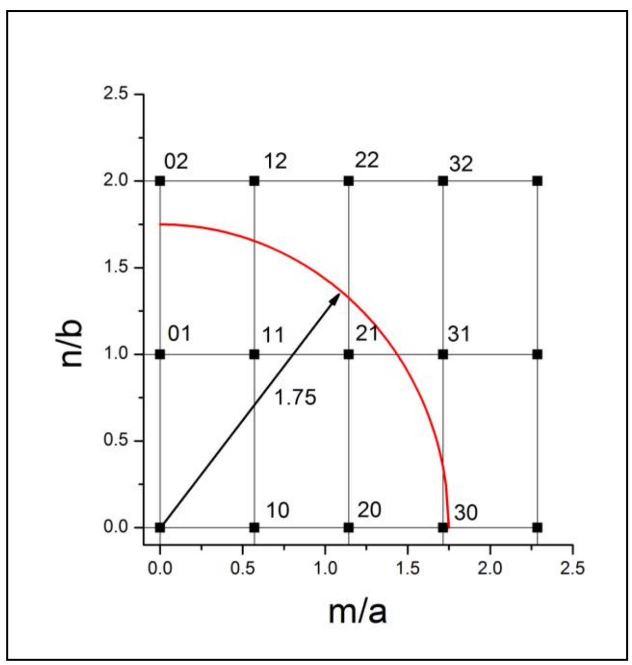
TE_{01}, $\frac{2f_c^{0,1}}{u} = 1.000\,\text{cm}^{-1}$

TE_{11}, $\frac{2f_c^{1,1}}{u} = 1.152\,\text{cm}^{-1}$

TM_{11}, $\frac{2f_c^{1,1}}{u} = 1.152\,\text{cm}^{-1}$

TE_{21}, $\frac{2f_c^{2,1}}{u} = 1.519\,\text{cm}^{-1}$

TM_{21}, $\frac{2f_c^{2,1}}{u} = 1.519\,\text{cm}^{-1}$



(b) What is the cutoff frequency of the dominant (or, fundamental) mode?

Quel est la fréquence de coupure du mode dominant (ou fundamental)?

The dominant mode is TE_{10}, and $\frac{2f_c^{1,0}}{u} = 0.571\,\text{cm}^{-1} \Rightarrow f_c^{1,0} = \frac{(0.571)(2\times10^{10})}{2} = 5.71\text{ GHz}$

(c) Can this waveguide support a mode, at any frequency, whose guided wavelength is 3.75 cm? Justify your answer.

Est-ce que ce guide d'onde peut supporter un mode, à n'importe quelle fréquence, avec une longueur d'onde guidée de 3.75 cm? Justifier votre réponse.

$$\lambda_c = \frac{u}{f_c^{1,0}} = \frac{2\times10^{10}}{5.71\times10^9} = 3.503\text{ cm}$$

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2} = \frac{1}{(3.75)^2} + \frac{1}{(3.503)^2} = 0.0711 + 0.0815 = 0.1526$$

$$\Rightarrow \quad \lambda = 2.56\,\text{cm}$$

$$f = \frac{u}{\lambda} = \frac{2\times10^{10}}{2.56} = 7.81\,\text{GHz} > 5.71\,\text{GHz}$$

Therefore, YES – the mode is supported.

(d) If a TE mode is supported at an operating frequency 50% greater than its cutoff frequency, what is its waveguide impedance and phase velocity?

Si le mode TE est supporté à une fréquence d'opération 50% plus grande que sa fréquence de coupure, quelle est l'impédance et la vitesse de phase du guide d'onde.

$$f = 1.5 f_c\,, \quad \eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{120\pi}{1.5} = 80\pi$$

$$Z_{TE} = \frac{\eta}{\sqrt{1-\left(f_c/f\right)^2}} = \frac{80\pi}{0.745} = 337\,\Omega$$

$$u_p = \frac{u}{\sqrt{1-\left(f_c/f\right)^2}} = \frac{2\times10^8}{0.745} = 2.68\times10^8\text{ m/s}$$

Equation sheet / Page de formules

$$\Gamma = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o} = |\Gamma|\, e^{j\theta_\Gamma} \qquad S = \frac{|V_{\max}|}{|V_{\min}|} = \frac{1+|\Gamma|}{1-|\Gamma|} \qquad Z(l) = \frac{V(l)}{I(l)} = Z_o \frac{Z_L + Z_o \tanh(\gamma l)}{Z_o + Z_L \tanh(\gamma l)}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \qquad \nabla \cdot \mathbf{D} = \rho_v \qquad \nabla \cdot \mathbf{B} = 0$$

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{k_1}{k_2} \qquad \eta = \sqrt{\frac{\mu}{\varepsilon}} \qquad k = \omega\sqrt{\mu\varepsilon} \qquad \beta^2 = k^2 - h^2 \qquad h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\Gamma_\perp = \Gamma_{TE} = \frac{E_o^r}{E_o^i} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \qquad \tau_\perp = \tau_{TE} = \frac{E_o^{tr}}{E_o^i} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \qquad \varepsilon_0 = 8.85\times10^{-12}\ \text{F/m}$$

$$\Gamma_{||} = \Gamma_{TM} = \frac{E_o^r}{E_o^i} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \qquad \tau_{||} = \tau_{TM} = \frac{E_o^{tr}}{E_o^i} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \qquad \mu_0 = 4\pi\times10^{-7}\ \text{H/m}$$

Dans l'air/ in air $\eta = 120\,\pi = 377\ \Omega$ $\qquad \varepsilon = \varepsilon_r\varepsilon_0 \qquad \beta = \dfrac{2\pi}{\lambda}$

$$n = \sqrt{\mu_r\varepsilon_r} = \sqrt{\frac{\mu\,\varepsilon}{\mu_0\varepsilon_0}} = \frac{c}{u_p}$$

$$\mathbf{D} = \varepsilon\mathbf{E} \qquad \mathbf{B} = \mu\mathbf{H} \qquad \mathbf{J} = \sigma\mathbf{E}$$

$$\nabla^2\mathbf{E} + k_c{}^2\mathbf{E} = 0 \qquad \nabla^2\mathbf{H} + k_c{}^2\mathbf{H} = 0 \qquad k_c = \omega\sqrt{\mu\varepsilon_c} \qquad \eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}$$

	General Case *Cas général*	*Lossless* *Milieu sans pertes*	*Low loss* *Milieu à faibles pertes*	*Good conductor* *Bon conducteur*
α (Np/m)	$\omega\left\{\frac{1}{2}\mu\varepsilon'\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}-1\right]\right\}^{1/2}$	0	$\frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$	$\sqrt{\pi f\mu\sigma}$
β (rad/m)	$\omega\left\{\frac{1}{2}\mu\varepsilon'\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}+1\right]\right\}^{1/2}$	$\omega\sqrt{\mu\varepsilon}$	$\omega\sqrt{\mu\varepsilon}$	$\sqrt{\pi f\mu\sigma}$
η_c (Ω)	$\sqrt{\frac{\mu}{\varepsilon'}}\left(1-j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2}$	$\sqrt{\frac{\mu}{\varepsilon}}$	$\sqrt{\frac{\mu}{\varepsilon}}$	$(1+j)\frac{\alpha}{\sigma}$
u_p (m/s)	$\frac{\omega}{\beta}$	$\frac{1}{\sqrt{\mu\varepsilon}}$	$\frac{1}{\sqrt{\mu\varepsilon}}$	$\sqrt{\frac{4\pi f}{\mu\sigma}}$

$$\mathbf{H} = \frac{1}{\eta}\hat{\mathbf{a}}_{\mathbf{k}} \times \mathbf{E} \qquad f = \frac{u}{\lambda} \qquad \omega = 2\pi f \qquad \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\nabla \times \tilde{\mathbf{E}} = -j\omega\mu\tilde{\mathbf{H}} \qquad \nabla \times \tilde{\mathbf{H}} = +j\omega\varepsilon_c\tilde{\mathbf{E}} \qquad \nabla \bullet \tilde{\mathbf{E}} = 0 \qquad \nabla \bullet \tilde{\mathbf{B}} = 0$$

$$\mathbf{S} = \mathbf{E} \times \mathbf{H} \qquad Z(l) = \frac{V(l)}{I(l)} = Z_o \frac{Z_L + jZ_o \tan(\beta l)}{Z_o + jZ_L \tan(\beta l)}$$

$$\tilde{\mathbf{S}}_{av} = \tfrac{1}{2}\mathrm{Re}[\tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^*] \ (\mathrm{W/m^2}) \qquad \tilde{\mathbf{S}}^i_{av}(z) = \hat{\mathbf{z}}\frac{|E_{i0}|^2}{2\eta_1^*}$$

$$\tilde{\mathbf{S}}_{1av}(z) = \hat{\mathbf{z}}\frac{|E_{i0}|^2}{2\eta_1^*}\left(1 - |\Gamma|^2\right) \qquad \tilde{\mathbf{S}}_{2av}(z) = \hat{\mathbf{z}}\frac{|\tau|^2|E_{i0}|^2}{2\eta_2^*}$$

$$S = \frac{|\tilde{\mathbf{E}}_i|_{\max}}{|\tilde{\mathbf{E}}_i|_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|} \qquad \tau = 1 + \Gamma$$

$$l_{\max} = \frac{\theta_r\lambda_1}{4\pi} + n\frac{\lambda_1}{2} \quad (n = 0,1,2\ldots) \quad \text{where/où } \theta_r = 0 \text{ if/si } \eta_1 > \eta_2 \text{ and/et } \theta_r = \pi \text{ if/si } \eta_1 < \eta_2$$

$$f_c^{m,n} = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(m/a\right)^2 + \left(n/b\right)^2} \qquad \beta = k\sqrt{1 - \left(f_c^{m,n}/f\right)^2} \qquad u_p = \frac{u}{\sqrt{1 - \left(f_c^{m,n}/f\right)^2}}$$

$$Z_{TM} = \frac{\tilde{E}_x}{\tilde{H}_y} = -\frac{\tilde{E}_y}{\tilde{H}_x} = \eta\sqrt{1 - \left(\frac{f_c^{m,n}}{f}\right)^2}, \quad Z_{TE} = \frac{\tilde{E}_x}{\tilde{H}_y} = -\frac{\tilde{E}_y}{\tilde{H}_x} = \frac{\eta}{\sqrt{1 - \left(\frac{f_c^{m,n}}{f}\right)^2}}, \quad \frac{1}{\lambda^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2}$$

Table 6-2: Boundary conditions for the electric and magnetic fields.

Field Components	General Form	Medium 1 Dielectric / Medium 2 Dielectric	Medium 1 Dielectric / Medium 2 Conductor	
Tangential E	$\hat{\mathbf{n}}_2 \times (\mathbf{E}_1 - \mathbf{E}_2) = 0$	$E_{1t} = E_{2t}$	$E_{1t} = E_{2t} = 0$	
Normal D	$\hat{\mathbf{n}}_2 \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_s$	$D_{1n} - D_{2n} = \rho_s$	$D_{1n} = \rho_s$	$D_{2n} = 0$
Tangential H	$\hat{\mathbf{n}}_2 \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s$	$H_{1t} = H_{2t}$	$H_{1t} = J_s$	$H_{2t} = 0$
Normal B	$\hat{\mathbf{n}}_2 \cdot (\mathbf{B}_1 - \mathbf{B}_2) = 0$	$B_{1n} = B_{2n}$	$B_{1n} = B_{2n} = 0$	

Notes: (1) ρ_s is the surface charge density at the boundary; (2) $\mathbf{J}_s$ is the surface current density at the boundary; (3) normal components of all fields are along $\hat{\mathbf{n}}_2$, the outward unit vector of medium 2; (4) $E_{1t} = E_{2t}$ implies that the tangential components are equal in magnitude and parallel in direction; (5) direction of $\mathbf{J}_s$ is orthogonal to $(\mathbf{H}_1 - \mathbf{H}_2)$.

$c \cong 3 \times 10^8$ m/s

Paramètre (Parameters)	*Coaxiale (Coaxial)*	*Bifilaire (Twin lead)*	*Plaques parallèles (Parallel Plate)*
R' (Ω/m)	$\frac{R_S}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{R_S}{\pi a}$	$\frac{2R_S}{w}$
L' (H/m)	$\frac{\mu}{2\pi}\ln\left(b/a\right)$	$\frac{\mu}{\pi}\ln(D)$	$\frac{\mu d}{w}$
G' (S/m)	$\frac{2\pi\sigma}{\ln\left(b/a\right)}$	$\frac{\pi\sigma}{\ln(D)}$	$\frac{\sigma w}{d}$
C' (F/m)	$\frac{2\pi\varepsilon}{\ln\left(b/a\right)}$	$\frac{\pi\varepsilon}{\ln(D)}$	$\frac{\varepsilon w}{d}$

Where/où $R_S = \sqrt{\frac{\pi f \mu_c}{\sigma_c}}$, $D = \frac{d}{2a} + \sqrt{\left(\frac{d}{2a}\right)^2 - 1}$