.MATH 1300D-MIDTERM # 2 Fall-2016.

Termeh Kousha

Last Name: ________________ **First Name:** ________________

ID# Solutions and Marking Scheme.

Instructions: This midterm exam consists of 4 multiple choice questions and 3 long answer questions. The multiple choice questions are worth 5 points each, and the long answer questions are as indicated. The total value of the exam is 50 points.
Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. You may use the backs of pages.

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For long answer questions, YOU MUST SHOW YOUR WORK.

NO CALCULATORS. NO BOOKS. NO NOTES.

Multiple Choice Answers:

B	F	A	E
#1	#2	#3	#4

Multiple Choice Questions (1-4)

Question 1. *Find the interval(s) on which the following function is concave down:*

$$f(x) = \frac{8}{x} - 8x^2 - 16$$

A) $(-\infty, -1)$ **B)** $(-\infty, 0) \cup (1, \infty)$ **C)** $(0, 1)$
D) $(-1, \infty)$ **E)** $(-1, 1)$ **F)** $(-\infty, -1) \cup (0, \infty)$

$$f'(x) = -\frac{8}{x^2} - 16x$$

$$f''(x) = \frac{16}{x^3} - 16 = 0 \Rightarrow 16x^3 = 16 \quad x = 1$$

$D_f = \mathbb{R} - \{0\}$

f	$\cap$	0	$\cup$	1	$\cap$
f''	$-$		$+$		$-$

Question 2. *Suppose* $f'(x) = 3e^x + x^2$, *and that* $f(0) = 2$. *Find* $f(1)$.

A) $\ln 3 - 2$ **B)** $3e + 4$ **C)** $e + 1$ **D)** $3e + \frac{4}{3}$ **E)** $\ln 3 + e$ **F)** $3e - \frac{2}{3}$

$$f(x) = \int (3e^x + x^2)\,dx = 3e^x + \frac{x^3}{3} + C$$

$$f(0) = 3e^0 + C = 2$$

$$3 + C = 2 \Rightarrow C = -1$$

$$f(1) = 3e + \frac{1}{3} - 1 = 3e - 2/3$$

Question 3. *Suppose that the demand function for a product is given by $p = 21 - 6\sqrt{x}$. What is the elasticity of demand when $x = 9$? Is demand elastic or inelastic?*

A) $\eta = -\frac{1}{3}$, inelastic B) $\eta = -\frac{4}{5}$, inelastic C) $\eta = -\frac{4}{5}$, elastic
D) $\eta = -\frac{1}{3}$, elastic E) $\eta = -1$, unit elasticity F) $\eta = -\frac{2}{5}$, inelastic

$$\eta = \frac{p(x)}{x\,p'(x)} = \frac{21 - 6\sqrt{9}}{9\left(\frac{-3}{\sqrt{9}}\right)} = \frac{21-18}{-9} = \frac{-3}{9} = \frac{-1}{3}$$

$$|\eta| = \frac{1}{3} < 1$$

Question 4. *Calculate the following definite integral.*

$$\int_1^3 \left(\frac{2}{x} + \frac{x}{2}\right) dx = 2\ln|x| + \frac{x^2}{4}\Big|_1^3$$

$$= \left(2\ln 3 + \frac{9}{4}\right) - \left(2\ln 1 + \frac{1}{4}\right)$$

$$= \ln 9 + \frac{9}{4} - \frac{1}{4} = \ln 9 + \frac{8}{4}$$

$$= 2 + \ln 9$$

A) $\frac{4}{3} + \ln 9$

B) $e^3 + 4$

C) $5 - \ln 9$

D) $\frac{2\sqrt{2}-2}{3} + e^3$

E) $\ln 9 + 2$

F) $\frac{1}{e^3} + 5$

Long Answer Section Questions (5-7)

Question 5. (14 points) *For the following function find the appropriate information (listed next page) to sketch the graph of the following function.*

$$f(x) = \frac{4}{x+3}$$

$\boxed{0.5 \text{ point}}$ ① $D_f = \mathbb{R} - \{-3\}$

$\boxed{0.5}$ ② $x = 0 \rightarrow y = 4/3$ $\quad (0, 4/3)$ y-intercept

$\boxed{0.5}$ ③ $y \neq 0 \rightarrow$ No x-intercept

$\boxed{1 \text{ point}}$ ④ $\lim_{x \to \infty} f(x) = 0$ $\quad y = 0$ H.A $\quad$ [0.5 for limit, 0.5 for H.A]

$\boxed{0.5}$ ⑤ $x = -3$ V.A

$\boxed{1 \text{ point}}$ ⑥ $f'(x) = \dfrac{-4}{(x+3)^2} < 0$

$\boxed{1 \text{ point}}$ ⑦ $f'(x) < 0 \rightarrow f$ decreases everywhere.

$\boxed{1}$ ⑧ $f' \neq 0 \rightarrow$ No critical point / no local max or min

$\boxed{1 \text{ point}}$ ⑨ $f''(x) = \dfrac{8}{(x+3)^3}$

		-3	
f	∩		∪
f''	—		+

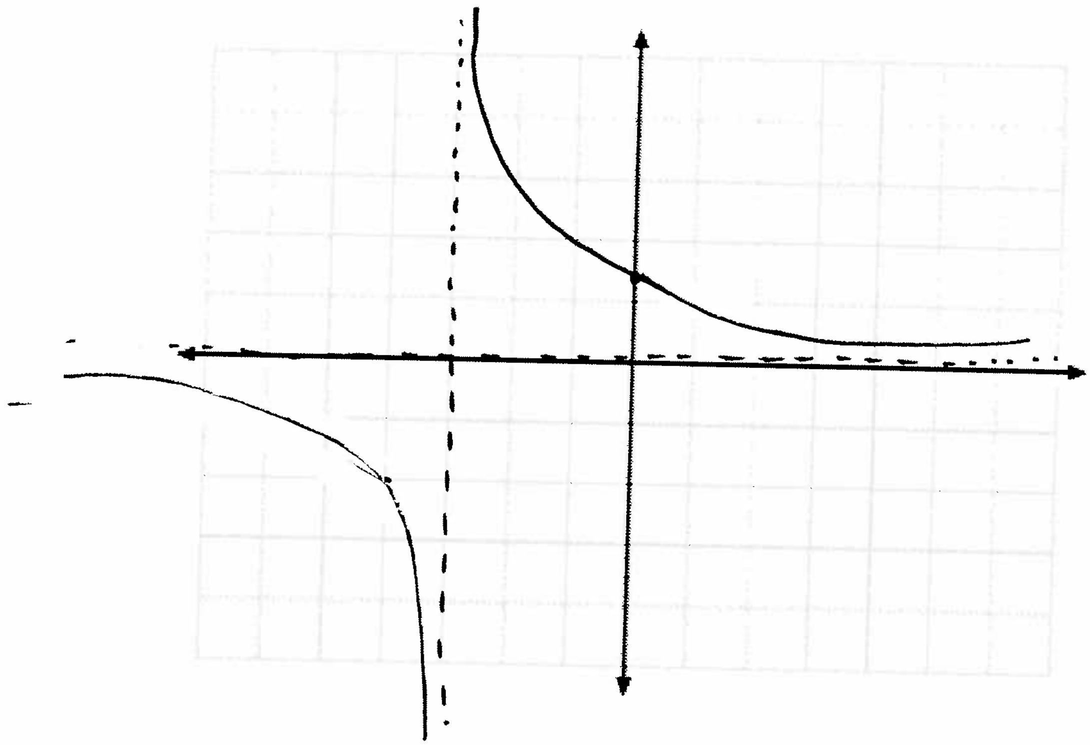
$\rightarrow \boxed{1 \text{ point}}$

$\boxed{1 \text{ point}}$ ⑩ $f''(x) \neq 0$ $\quad$ NO P.O.I

9 points

5 points for graph
(2 points for sketching Asymptotes)

Graph of $f(x)$



1. Find the domain of the function
2. Find the y-intercept and plot it
3. Find the x-intercepts and plot them
4. Find the horizontal asymptotes and plot them
5. Find the vertical asymptotes and plot them
6. Find the critical numbers
7. Find the intervals of increase and decrease
8. Identify the relative extrema and plot them
9. Find the intervals of concave up and concave down
10. Identify the points of inflection and plot them
11. Fill in the rest of the graph using (7) and (9)

Question 6. (8 points) *Use the* <u>*second derivative test*</u> *to find and classify the critical points for the following function. Don't forget to find the Y-coordinate of the local extremas.*

$$f(x) = e^{-x^3+3x^2+8}.$$

2 points $F'(x) = (-3x^2+6x)\, e^{-x^3+3x^2+8} = 0$

$\Rightarrow -3x^2+6x = -3x(x-2) = 0$

$\underbrace{x=0}_{1 \text{ point}} \qquad \underbrace{x=2}_{1 \text{ point}}$

2 point $F''(x) = (-6x+6)\, e^{-x^3+3x^2+8} + (-3x^2+6x)^2 e^{-x^3+3x^2+8}$

$\Rightarrow$

1 point $F''(0) = 6e^8 > 0 \quad (0, e^8) \rightarrow$ local min

1 point $F''(2) = (-6)e^{12} < 0 \quad (2, e^{12})$ local max

(0.5 points for finding the Y coordinate)

Question 7. (8 points) *School bags are sold to students for \$30 each, 200 students are willing to buy them at that price. For every \$2 increase in price, there are 8 fewer students willing to buy the bag. (i.e. for the price of \$32, 192 students are willing to buy) What selling price will produce the maximum revenue and what is the maximum revenue?*
Be sure to explain why your answer is an absolute maximum.

x	P
200	30
192	32

$m = \frac{\Delta P}{\Delta x} = \frac{-2}{8} = \frac{-1}{4}$

$D(x) = P = mx + b$

$30 = \frac{-1}{4}(200) + b$

$b = 30 + 50 = 80$

2 points $\leftarrow$ $P = \frac{-1}{4}x + 80$

1 point $\leftarrow$ $R(x) = x \cdot P = \frac{-1}{4}x^2 + 80x$

2 point $\leftarrow$ $R'(x) = \frac{-1}{2}x + 80 = 0 \quad x = 160$

1 point $\leftarrow$ $R''(x) = \frac{-1}{8} < 0 \rightarrow x = 160$ is a local max.

1point $\leftarrow$ Since it is also the only local max $\rightarrow$ it is a an abs max

1 point $\leftarrow$ $R(160) = \frac{-1}{4}(160)^2 + 80(160) =$

Full point if they only sub "160"

$160(-40+80) = 160(40)$

$= 6400$

They don't need to Simplify

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Multiple Choice Answers:

C	D	D	A
#1	#2	#3	#4

Multiple Choice Questions (1-4)

Question 1. *Find the interval(s) on which the following function is concave up:*

$$f(x) = \frac{8}{x} - 8x^2 + 2$$

A) $(-\infty, -1)$ **B)** $(-\infty, 0) \cup (1, \infty)$ **C)** $(0, 1)$
D) $(-1, \infty)$ **E)** $(-1, 1)$ **F)** $(-\infty, -1) \cup (0, \infty)$

$$f'(x) = \frac{-8}{x^2} - 16x$$

$$f''(x) = \frac{16}{x^3} - 16 = 0 \qquad 16x^3 = 16 \qquad x = 1$$

$$D_f = \mathbb{R} - \{0\}$$

		0		1	
f			$\cup$		
f''	$-$		$+$		$-$

Question 2. *Suppose* $f'(x) = 3e^x + x^2$, *and that* $f(0) = 4$. *Find* $f(1)$.

A) $\ln 3 - 2$ **B)** $3e + 4$ **C)** $e + 1$ **D)** $3e + \frac{4}{3}$ **E)** $\ln 3 + e$ **F)** $3e - \frac{2}{3}$

$$f(x) = \int (3e^x + x^2)\, dx = 3e^x + \frac{x^3}{3} + C$$

$$f(0) = 3 + C = 4 \qquad C = 1$$

$$f(1) = 3e + \frac{1}{3} + 1 = 3e + 4/3$$

Question 3. *Suppose that the demand function for a product is given by $p = 21 - 5\sqrt{x}$. What is the elasticity of demand when $x = 4$? Is demand elastic or inelastic?*

A) $\eta = -\frac{11}{5}$, inelastic
B) $\eta = -\frac{12}{15}$, inelastic
C) $\eta = -\frac{12}{15}$, elastic
D) $\eta = -\frac{11}{5}$, elastic
E) $\eta = -1$, unit elasticity
F) $\eta = -\frac{2}{5}$, inelastic

$$\eta = \frac{21 - 5(2)}{4\left(\frac{-5}{2\sqrt{4}}\right)} = \frac{11}{-5}$$

$|\eta| > 1 \rightarrow$ elastic

Question 4. *Calculate the following definite integral.*

$$\int_1^3 \left(\frac{2}{x} + \frac{x}{3}\right) dx = 2\ln|x| + \frac{x^2}{6} \Big|_1^3$$

$$= \left(\ln 9 + \frac{9}{6}\right) - \left(2\ln 1 + \frac{1}{6}\right)$$

$$= \ln 9 + \frac{8}{6}$$

$$= \ln 9 + 4/3.$$

A) $\frac{4}{3} + \ln 9$

B) $e^3 + 4$

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$$f(x) = \frac{2}{x+3}$$

1. $D_f = \mathbb{R} - \{-3\}$

2. $x = 0 \Rightarrow y = 2/3$ y-intercept

3. $y \neq 0 \rightarrow$ No x-intercept.

4. $\lim\limits_{x \to \infty} f(x) = 0$ $y = 0$ H.A

5. $x = -3$ V.A

6. $\frac{-4}{(x+3)^2} < 0 \rightarrow$ 7 $f'(x) < 0 \rightarrow$ f decreases everywhere.

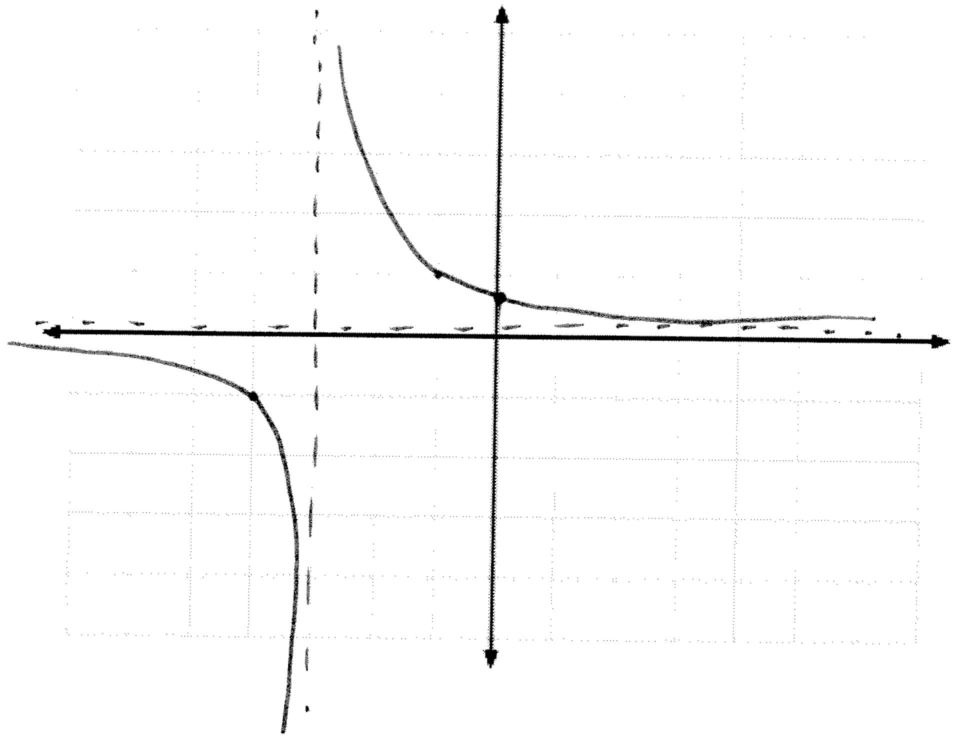
8. $f'(x) \neq 0 \rightarrow$ NO critical point / no local max/min

9. $\frac{8}{(x+3)^2}$

		-3	
f	$\cap$		$\cup$
f''	$-$		$+$

10. $f''(x) \neq 0$ NO P.O.I

Graph of $f(x)$



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$$f(x) = e^{-x^3+3x^2+9}.$$

$$f'(x) = (-3x^2+6x)\, e^{-x^3+3x^2+9}$$

$$=0 \implies (-3x^2+6x)=0$$

$$-3x(x-2)=0$$

$$x=0 \qquad x=2$$

$$f''(x) = (-6x+6)\, e^{-x^3+3x^2+9} + (-3x^2+6x)^2 e^{-x^3+3x^2+9}$$

$$f''(0) = 6e^{-9} > 0 \rightarrow (0, e^{9}) \text{ local min.}$$

$$f''(2) = -6e^{13} < 0 \quad (2, e^{13}) \text{ local max.}$$

Question 7. (8 points) *School bags are sold to students for \$20 each, 200 students are willing to buy them at that price. For every \$2 increase in price, there are 8 fewer students willing to buy the bag. (i.e. for the price of \$22, 192 students are willing to buy) What selling price will produce the maximum revenue and what is the maximum revenue?*
Be sure to explain why your answer is an absolute maximum.

p	x
20	200
22	192

$p = mx + b$

$\Delta p = \frac{-2}{8} = -\frac{1}{4}$

$p = D(x) = mx + b$

$20 = -\frac{1}{4}(200) + b$

$b = 20 + 50 = 70$

$p = -\frac{1}{4}x + 70$

$R(x) = -\frac{1}{4}x^2 + 70x \Rightarrow R'(x) = -\frac{1}{2}x + 70 = 0$

$x = 140.$

$R''(x) = -\frac{1}{2} < 0 \rightarrow x = 140$ is a local max

Since it's the only local max, it is also an abs max.

$R(140) = -\frac{1}{4}(140)^2 + 70(140) =$

$140\left(-\frac{1}{4}(140) + 70\right)$

$140(35) = 4900$