Important Solution Steps

MAT1320

1. Chain Rule

Question Type: Find the derivative of a function $y = f(x)$.

Applicability: Look at the "last operation" when use the value of x to calculate the value of y. If this operation is not one of the arithmetic operations (+, −, *, or /), chain rule is to be used.

Solving Steps:

(1) Decompose $f(x)$ into the composition of two functions. Let $u = g(x)$, then $y = f(x) = h(u)$, where $h(u)$ has only one operation, i.e., the "last operation".

(2) Find $y_u = h'(u)$, and $u'_x = g(x)$.

(3) $f'(x) = h'(u)g'(x) = h'(g(x))g'(x)$. (or, equivalently, $y_x' = y_u' u_x'$).

2. Related Rates

Question Type: Find the derivative of a function from the derivative(s) of other functions at a given point, which is given by particular values of some functions.

Solving Steps:

(1) Find a relation that relates functions of time t that are involved in the problem. A function is involved in this relation if it has a known derivative or its derivative is to be found.

(2) Take the derivative of this relation on both sides with respect to time t. This gives you a relation that relates these functions and their derivatives.

(3) Solve this relation for the unknown derivative.

(4) Use given particular value and know derivatives to find the unknown derivative.

3. Optimization

Question Type: Find the maximum or minimum value of a quantity.

Solving Steps:

(1) Identify the quantity to be optimized.

(2) Express this quantity as a function (the *objective function*) with one or more variables.

(3) If the number of variables is more than one, use a condition, or conditions, given in the problem, which a relation among the variables, to eliminate extra variable(s), so that the objective function becomes a one-variable function. Identify the domain of this function.

(4) Use the derivative of the objective function to find the absolute maximum or minimum value of the objective function.

4. Graph Sketching

Question Type: Use the derivative to find properties, such as the interval(s) of increasing / decreasing, interval(s) of concave up/down, relative maximum / minimum, and / or inflection points, of a function $y = f(x)$. Then sketch the graph of the function.

Solving Steps:

(1) First derivative analysis:

(i) Find the first derivative $f'(x)$.

(ii) Find critical numbers of this function. A number $x = a$ is a critical number if a is in the domain of $f(x)$ and either $f'(a) = 0$ or $f'(a)$ does not exist.

(iii) The critical numbers subdivide the domain of $f(x)$ into a number of subintervals. Use a particular value in each subinterval to determine the sign of $f'(x)$ in this interval.

Then $f(x)$ increases / decreases in a subinterval if $f'(x) > 0$ / $f'(x) < 0$.

(iv) Use the first derivative test to determine if a critical number corresponds to a relative maximum or minimum or neither.

(2) Second derivative analysis:

(i) Find the second derivative $f''(x)$.

(ii) Find values a that is in the domain of $f(x)$ and either $f''(a) = 0$ or $f''(a)$ does not exist.

(iii) The values subdivide the domain of $f(x)$ into a number of subintervals. Use a particular value in each subinterval to determine the sign of $f''(x)$ in this interval.

Then $f(x)$ is concave up / down in a subinterval if $f''(x) > 0$ / $f''(x) < 0$.

(iv) If $f''(x)$ has different sings on two sides of a value $x = a$, then $f(x)$ has an inflection point at $x = a$.

(3) Sketch the graph:

(i) Draw the real axis and mark the domain of the function.

(ii) Mark all critical points and values found in second derivative analysis.

(iii) Use the monotonicity and concavity of the function in each of the intervals to determine the shape of the graph in each interval.

(iv) Determine vertical/horizontal asymptote(s) if there is any.

(v) Plot some points on the graph by calculating some values of the function, and use these points and information in (iii) and (iv) to sketch the graph.

5. Integration by Variable Substitution

Question Type: Find integral $\int f(x)dx$ and basic formulas cannot be used directly.

Solving Steps:

(1) Define an intermediate variable $u = g(x)$ and find $u' = g'(x)$.

(2) Divide $f(x)$ by $g'(x)$ and change dx to du: $\int f(x)dx = \int f(x)\frac{1}{g'(x)}du$.

(3) Write the integrand as a function of u: $\frac{f(x)}{g'(x)} = h(u)$. Then the integral becomes $\int h(u)du$.

(4) Integrate $h(u)$: $\int h(u)du = H(u) + C = H(g(x)) + C$.

6. Integration by Parts

(1) The integrand is a product of two functions. (One of them may be constant 1).

(2) Let one function be u and the other function be the derivative of v.

(3) Find u' by taking the derivative of u, and find v by integrating v'.

(4) Use the formula $\int uv'dx = uv - \int u'vdx$. The integral on the right-hand side should be easier to find. If it is not the case, you may have used wrong functions as u and v', or this method does not work for this problem.