

**CONCORDIA UNIVERSITY**  
**Department of Economics**

**ECON 221/2 SECTIONS A, B, C and DD**  
*STATISTICAL METHODS I*  
**FALL 2016 – MIDTERM 1 (SOLUTIONS)**  
Sunday, October 23, 2016, 1pm – 3pm

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**TOTAL: 50 points**

1. **(10 points)** World-renowned Jazzoleon was a headline performer at this summer’s Calgary Stampede. He announced on his Facebook page that he would be performing seven of his 20 best songs.

a. **(2 points)** Calculate how many different song combinations Jazzoleon could create.

$$C_7^{20} = \frac{20!}{7!13!} = 77520$$

Once Jazzoleon selected the seven songs he wanted, he then had to decide the order in which to perform them.

b. **(2 points)** Calculate how many different ways he could arrange the seven songs.

$$7! = 5040$$

Tootie and Michelle know all 20 of Jazzoleon’s songs and plan to attend the concert. They are particularly fans of his song *UHUM!*

c. **(2 points)** If all 20 songs were equally likely to be played, calculate the probability that *UHUM!* would be performed at the concert.

$$\frac{C_6^{19}}{C_7^{20}} = \frac{\frac{19!}{6!13!}}{\frac{20!}{7!13!}} = \frac{7}{20} = 0.35. \text{ Combinatorics are unnecessary. If each song is equally likely}$$

*and he is performing 7 of his 20 songs, then there is a 35 percent chance that UHUM! is on the set list.*

It turns out that when Jazzoleon performs *UHUM!*, there is an 80 percent probability that he

also performs the song *la Corde du roi*.

- d. (2 points) Using your answer from part (c), calculate the probability that Jazzoleon would perform both *UHUM!* and *la Corde du roi* at the concert.

$$\Pr(U \cap C) = \Pr(C|U) \Pr(U) = 0.8 \cdot 0.35 = 0.28$$

After a concert, Jazzoleon eats a burger or a pizza. For every 130 times he performs *UHUM!*, he eats a hamburger 117 times and for every 130 times he does not perform *UHUM!*, he eats a pizza 93 times. Roger and Zoë were unable to attend the concert with Tootie and Michelle, but they did see Jazzoleon eating a hamburger at a local restaurant after the concert.

- e. (2 points) Using your answer from part (c), calculate the probability that Jazzoleon performed *UHUM!* at the concert if the only information Roger and Zoë had was seeing him eating a hamburger.

From part (c),  $\Pr(U) = 0.35$  and  $\Pr(\bar{U}) = 0.65$ . From the previous statement,

$$\Pr(B|U) = \frac{117}{130} = 0.9 \text{ and } \Pr(B|\bar{U}) = 1 - \Pr(P|\bar{U}) = 1 - \frac{93}{130} = 0.2846153. \text{ By Bayes' theorem,}$$

$$\Pr(U|B) = \frac{\Pr(B|U) \Pr(U)}{\Pr(B)} = \frac{\Pr(B|U) \Pr(U)}{\Pr(B|U) \Pr(U) + \Pr(B|\bar{U}) \Pr(\bar{U})} = \frac{0.9 \cdot 0.35}{0.9 \cdot 0.35 + 0.2846153 \cdot 0.65} = 0.63$$

2. (8 points) Every day since 1857, gunners of the Royal Brigade 3<sup>rd</sup> Artillery fire one of four cannons at noon at the Halifax Citadel. Unfortunately, the warmth and shelter inside the cannons make it an attractive spot for Tootie birds to sleep. Tourists watching the noon performance are often surprised to see Tootie birds launched deep into Halifax harbour. The probability of each cannon being fired and the average number of Tootie birds sleeping in each cannon are as follows:

Cannon	1	2	3	4
Probability of being fired	0.1	0.2	0.3	0.4
Average number of Tooties sleeping inside	120	60	40	30

- a. (2 points) Calculate the expected number of Tootie birds to be launched on a given day.

$$E(X) = \sum_{x=1}^4 x \Pr(x) = 120 \cdot 0.1 + 60 \cdot 0.2 + 40 \cdot 0.3 + 30 \cdot 0.4 = 48$$

- b. (2 points) Calculate the variance in the number of Tootie birds to be launched on a given day.

$$V(X) = \sum_{x=1}^4 x^2 \Pr(x) = 120^2 \cdot 0.1 + 60^2 \cdot 0.2 + 40^2 \cdot 0.3 + 30^2 \cdot 0.4 - 48^2 = 696$$

Each Tootie bird launched traumatises tourists and costs the city \$1000 in lost tourist revenue.

- c. **(2 points)** Calculate the expected loss in tourist revenue.

$$Y = 1000X \Rightarrow E(Y) = E(1000X) = 1000E(X) = 1000 \cdot 48 = 48000$$

- d. **(2 points)** Calculate the variance of the loss in tourist revenue.

$$Y = 1000X \Rightarrow V(Y) = V(1000X) = 1000^2 \cdot V(X) = 1000000 \cdot 696 = 696000000$$

3. **(6 points)** I make 20 independent claims in a day and I lie 25 percent of the time.

- a. **(2 points)** Calculate the probability that I lie at least 6 but not more than 9 times in a day.

*This question involves using a binomial distribution. The cumulative probability distribution is found in Table 3. From there,*

$$\Pr(6 \leq X \leq 9) = \Pr(X \leq 9) - \Pr(X \leq 5) = 0.986 - 0.617 = 0.369.$$

- b. **(2 points)** Calculate the expected number of times I lie in a day.

$$E(X) = nP = 20 \cdot 0.25 = 5$$

- c. **(2 points)** Calculate the variance in the number of times I lie in a day.

$$V(X) = nP(1 - P) = 20 \cdot 0.25 \cdot 0.75 = 3.75$$

4. **(4 points)** On average, two Hapolonians arrive every 5 minutes at an all-you-can-eat restaurant. Assume that the arrivals are independent, constant and follow a Poisson distribution.

- a. **(2 points)** Calculate the probability that more than two Hapolonians arrive in a given 5-minute interval.

*This question involves using a Poisson distribution. The cumulative probability distribution is found in Table 6. From there,  $\Pr(X > 2) = 1 - \Pr(X \leq 2) = 1 - 0.6767 = 0.3233$ .*

- b. (2 points) Calculate the standard deviation in the number of Hapolonians arriving in a given 5-minute interval.

$$\sigma_X = \sqrt{\sigma_X^2} = \sqrt{\lambda} = \sqrt{2} = 1.4142$$

5. (10 points) Hapibeh ( $B$ ) and Hapideh ( $D$ ) are disagreeable sisters. When one says “yes”, the other instinctively says “no”. Let “Yes” = 1 and “No” = 0.

		$B$	
		1	0
$D$	1	0.0	0.7
	0	0.3	0.0

- a. (2 points) Calculate the marginal probability distributions for  $B$  and  $D$ . **Briefly** interpret the results.

$$\Pr(B=0) = \Pr(B=0 \cap D=0) + \Pr(B=0 \cap D=1) = 0.0 + 0.7 = 0.7$$

$$\Pr(B=1) = \Pr(B=1 \cap D=0) + \Pr(B=1 \cap D=1) = 0.3 + 0.0 = 0.3$$

*The probability that Hapibeh says no is 70 percent and the probability that she says yes is 30 percent.*

$$\Pr(D=0) = \Pr(D=0 \cap B=0) + \Pr(D=0 \cap B=1) = 0.0 + 0.3 = 0.3$$

$$\Pr(D=1) = \Pr(D=1 \cap B=0) + \Pr(D=1 \cap B=1) = 0.7 + 0.0 = 0.7$$

*The probability that Hapideh says no is 30 percent and the probability that she says yes is 70 percent.*

- b. (2 points) Calculate the expected values of  $B$  and  $D$ .

$$E(B) = \sum_{b=0}^1 b \Pr(b) = 0 \cdot 0.7 + 1 \cdot 0.3 = 0.3. \quad E(D) = \sum_{d=0}^1 d \Pr(d) = 0 \cdot 0.3 + 1 \cdot 0.7 = 0.7$$

- c. (2 points) Calculate the variances of  $B$  and  $D$ .

$$V(B) = \sum_{b=0}^1 b^2 \Pr(b) - E(B)^2 = 0^2 \cdot 0.7 + 1^2 \cdot 0.3 - 0.3^2 = 0.21.$$

$$V(D) = \sum_{d=0}^1 d^2 \Pr(d) - E(D)^2 = 0^2 \cdot 0.3 + 1^2 \cdot 0.7 - 0.7^2 = 0.21$$

- d. (2 points) Calculate the covariance of  $B$  and  $D$ .

$$\text{cov}(B, D) = \sum_{b=0}^1 \sum_{d=0}^1 bd \Pr(b, d) - E(B)E(D) = 0 \cdot 0 \cdot 0.0 + 0 \cdot 1 \cdot 0.7 + 1 \cdot 0 \cdot 0.3 + 1 \cdot 1 \cdot 0.0 - 0.3 \cdot 0.7 = -0.21$$

e. (2 points) Calculate the correlation of  $B$  and  $D$ .

$$\text{cor}(B, D) = \frac{\text{cov}(B, D)}{\sqrt{V(B)V(D)}} = \frac{-0.21}{\sqrt{0.21 \cdot 0.21}} = -1$$

6. (10 points) The intelligence quotient (IQ) is a measure of intelligence and is assumed to be normally distributed with a mean of 100 and a variance of 225.

*By converting  $X$  into  $Z$ , we can use the cumulative distribution function data in Table 1.*

a. (2 points) Calculate the probability of having an IQ greater than 115.

$$\Pr(X > 115) = \Pr\left(\frac{X - \mu}{\sigma} > \frac{115 - 100}{\sqrt{225}}\right) = \Pr(Z > 1) = 1 - \Pr(Z < 1) = 1 - 0.8413 = 0.1587$$

A person is cognitively impaired if he/she has an IQ between 70 and 80.

b. (2 points) Calculate the probability of being cognitively impaired.

$$\begin{aligned} \Pr(70 < X < 80) &= \Pr\left(\frac{70 - 100}{\sqrt{225}} < \frac{X - \mu}{\sigma} < \frac{80 - 100}{\sqrt{225}}\right) = \Pr(-2 < Z < -1.\dot{3}) \\ &= \Pr(Z < -1.\dot{3}) - \Pr(Z < -2) = \Pr(Z > 1.\dot{3}) - \Pr(Z > 2) = (1 - \Pr(Z < 1.\dot{3})) - (1 - \Pr(Z < 2)) \\ &= \Pr(Z < 2) - \Pr(Z < 1.\dot{3}) = 0.9772 - 0.9082 = 0.0690 \end{aligned}$$

c. (2 points) Calculate the IQ level that 20 percent of the population should exceed.

$$\begin{aligned} \Pr(X > x) = 0.2 &\Rightarrow \Pr\left(\frac{X - \mu}{\sigma} > \frac{x - 100}{\sqrt{225}}\right) = 0.2 \Rightarrow \Pr(Z > z) = 0.2 \Rightarrow 1 - \Pr(Z < z) = 0.8 \\ &\Rightarrow z = 0.842 \Rightarrow \frac{x - 100}{15} = 0.842 \Rightarrow x = 112.63 \end{aligned}$$

Noted physicist Stephen Hawking has an IQ of 150.

d. (2 points) Calculate the probability of having an IQ higher than Stephen Hawking.

$$\Pr(X > 150) = \Pr\left(\frac{X - \mu}{\sigma} > \frac{150 - 100}{\sqrt{225}}\right) = \Pr(Z > 3.\dot{3}) = 1 - \Pr(Z < 3.\dot{3}) = 1 - 0.9996 = 0.0004$$

- e. **(2 points)** Calculate the shortest range of IQs that includes 34 percent of humanity.

*The shortest range is centred around the mean. Let  $x_0$  and  $x_1$  be equidistant values from the mean such that  $\Pr(x_0 < X < x_1) = 0.34$ .*

$$\Pr(x_0 < X < x_1) = \Pr\left(\frac{x_0 - 100}{\sqrt{225}} < \frac{X - \mu}{\sigma} < \frac{x_1 - 100}{\sqrt{225}}\right) = \Pr(-z < Z < z) = \Pr(Z < z) - \Pr(Z < -z) =$$

$$\Pr(Z < z) - \Pr(Z > z) = \Pr(Z < z) - (1 - \Pr(Z < z)) = 2\Pr(Z < z) - 1 = 0.34 \Rightarrow \Pr(Z < z) = 0.67$$

$$\Rightarrow z = \pm 0.44 \Rightarrow \begin{cases} \frac{x_0 - 100}{\sqrt{225}} = -0.44 \Rightarrow x = 93.4 \\ \frac{x_1 - 100}{\sqrt{225}} = 0.44 \Rightarrow x = 106.6 \end{cases}$$

7. **(4 points)** The average time to complete this exam follows an exponential distribution with a mean of 90 minutes.

- a. **(2 points)** Calculate the probability that a given student does not complete this exam in the allotted 120 minutes.

$$\Pr(T > 120) = 1 - \Pr(T < 120) = 1 - (1 - e^{-120/90}) = e^{-1.\dot{3}} = 0.2636$$

- b. **(2 points)** Calculate the probability that a given student completes this exam in the first hour.

$$P(T < 60) = 1 - e^{-60/120} = 1 - e^{-0.5} = 0.4866$$