1 Ed. 10

Solutions to Ass't #8

Sec 3.5

2. The characteristic equation for the homogeneous problem is $r^2+2r+5=0$, with complex roots $r=-1\pm 2i$. Hence $y_c(t)=c_1e^{-t}\cos 2t+c_2e^{-t}\sin 2t$. Since the function $g(t)=3\sin 2t$ is not proportional to the solutions of the homogeneous equation, set $Y=A\cos 2t+B\sin 2t$. Substitution into the given ODE, and comparing the coefficients, results in the system of equations $B-4A=3$ and $A+4B=0$. Hence $Y=-\frac{12}{17}\cos 2t+\frac{3}{17}\sin 2t$. The general solution is $y(t)=y_c(t)+Y$.

#7 ⟷ # 9. The characteristic equation for the homogeneous problem is $2r^2+3r+1=0$, with roots $r=-1,\ -1/2$. Hence $y_c(t)=c_1e^{-t}+c_2e^{-t/2}$. To simplify the analysis, set $g_1(t)=t^2$ and $g_2(t)=3\sin t$. Based on the form of g_1, set $Y_1=A+Bt+Ct^2$. Substitution into the differential equation, and comparing the coefficients, results in the system of equations $A+3B+4C=0,\ B+6C=0$, and $C=1$. Hence we obtain $Y_1=14-6t+t^2$. On the other hand, set $Y_2=D\cos t+E\sin t$. After substitution into the ODE, we find that $D=-9/10$ and $E=-3/10$. The general solution is $y(t)=y_c(t)+Y_1+Y_2$.

#15 → #17 The characteristic equation for the homogeneous problem is $r^2-2r+1=0$, with a double root $r=1$. Hence $y_c(t)=c_1e^t+c_2t\,e^t$. Consider $g_1(t)=t\,e^t$. Note that g_1 is a solution of the homogeneous problem. Set $Y_1=At^2e^t+Bt^3e^t$ (the first term is not sufficient for a match). Upon substitution, we obtain $Y_1=t^3e^t/6$. By inspection, $Y_2=4$. Hence the general solution is $y(t)=c_1e^t+c_2t\,e^t+t^3e^t/6+4$. Invoking the initial conditions, we require that $c_1+4=1$ and $c_1+c_2=1$. Hence $c_1=-3$ and $c_2=4$.

#18 # 20 The characteristic equation for the homogeneous problem is $r^2+2r+5=0$, with complex roots $r=-1\pm 2i$. Hence $y_c(t)=c_1e^{-t}\cos 2t+c_2e^{-t}\sin 2t$. Based on the form of $g(t)$, set $Y=At\,e^{-t}\cos 2t+Bt\,e^{-t}\sin 2t$. After comparing coefficients, we obtain $Y=t\,e^{-t}\sin 2t$. Hence the general solution is

$$y(t)=c_1e^{-t}\cos 2t+c_2e^{-t}\sin 2t+t\,e^{-t}\sin 2t.$$

Invoking the initial conditions, we require that $c_1=1$ and $-c_1+2c_2=0$. Hence $c_1=1$ and $c_2=1/2$.

#21 # 23 The characteristic equation for the homogeneous problem is $r^2-5r+6=0$, with roots $r=2,3$. Hence $y_c(t)=c_1e^{2t}+c_2e^{3t}$. Consider $g_1(t)=e^{2t}(3t+4)\sin t$, and $g_2(t)=e^t\cos 2t$. Based on the form of these functions on the right hand side of the ODE, set $Y_2(t)=e^t(A_1\cos 2t+A_2\sin 2t)$ and $Y_1(t)=(B_1+B_2t)e^{2t}\sin t+(C_1+C_2t)e^{2t}\cos t$. Substitution into the equation and comparing the coefficients results in

$$Y(t)=-\frac{1}{20}(e^t\cos 2t+3e^t\sin 2t)+\frac{3}{2}te^{2t}(\cos t-\sin t)+e^{2t}(\frac{1}{2}\cos t-5\sin t).$$