Section 2.3

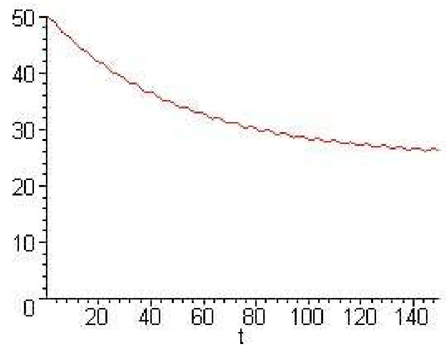
5.(a) Let Q be the amount of salt in the tank. Salt enters the tank of water at a rate of $2\,\frac{1}{4}(1+\frac{1}{2}\sin t) = \frac{1}{2}+\frac{1}{4}\sin t$ oz/min. It leaves the tank at a rate of $2\,Q/100$ oz/min. Hence the differential equation governing the amount of salt at any time is

$$\frac{dQ}{dt} = \frac{1}{2} + \frac{1}{4}\sin t - Q/50\,.$$

The initial amount of salt is $Q_0 = 50$ oz. The governing ODE is linear, with integrating factor $\mu(t) = e^{t/50}$. Write the equation as $(e^{t/50}Q)' = e^{t/50}(\frac{1}{2}+\frac{1}{4}\sin t)$. The specific solution is

$$Q(t) = 25 + (12.5\sin t - 625\cos t + 63150\,e^{-t/50})/2501 \quad \text{oz.}$$

(b)



(c) The amount of salt approaches a steady state, which is an oscillation of approximate amplitude 1/4 about a level of 25 oz.

13. Let $P(t)$ be the population size of mosquitoes at any time t. The rate of increase of the mosquito population is rP. The population decreases by $20,000$ per day. Hence the equation that models the population is given by

$$dP/dt = rP - 20,000.$$

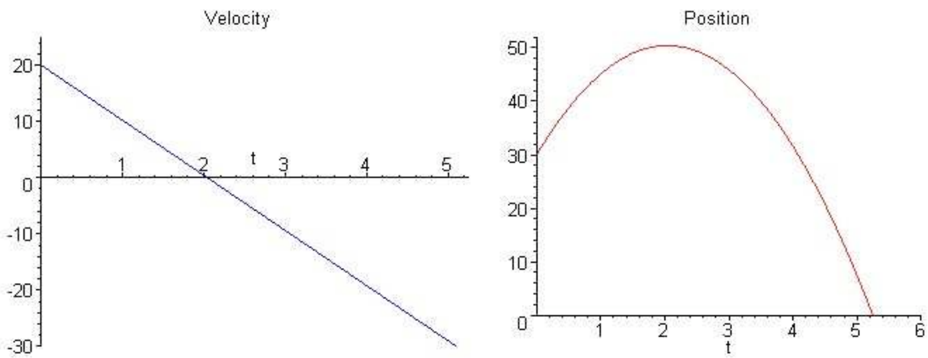
Note that the variable t represents days. The solution is

$$P(t) = P_0e^{rt} - \frac{20,000}{r}(e^{rt}-1).$$

In the absence of predators, the governing equation is $dP_1/dt = rP_1$, with solution $P_1(t) = P_0e^{rt}$. Based on the data, set $P_1(7) = 2P_0$, that is, $2P_0 = P_0e^{7r}$. The growth rate is determined as $r = \ln(2)/7 = .09902$ per day. Therefore the population, including the predation by birds, is

$$P(t) = 2\times 10^5 e^{.099t} - 201,997(e^{.099t}-1) = 201,997.3 - 1977.3\,e^{.099t}.$$

20.(c)



22.(a) The differential equation for the upward motion is $mdv/dt = -\mu v^2 - mg$, in which $\mu = 1/1325$. This equation is separable, with $\frac{m}{\mu\, v^2+mg}\,dv = -dt$. Integrating both sides and invoking the initial condition, $v(t) = 44.133\,\tan(.425 - .222\,t)$. Setting $v(t_1) = 0$, the ball reaches the maximum height at $t_1 = 1.916$ s. Integrating $v(t)$, the position is given by $x(t) = 198.75\,\ln\left[\cos(0.222\,t - 0.425)\right] + 48.57$. Therefore the maximum height is $x(t_1) = 48.56$ m.

(b) The differential equation for the downward motion is $m\,dv/dt = +\mu v^2 - mg$. This equation is also separable, with $\frac{m}{mg-\mu\, v^2}\,dv = -dt$. For convenience, set $t = 0$ at the top of the trajectory. The new initial condition becomes $v(0) = 0$. Integrating both sides and invoking the initial condition, we obtain

$$\ln((44.13 - v)/(44.13 + v)) = t/2.25.$$

Solving for the velocity, $v(t) = 44.13(1 - e^{t/2.25})/(1 + e^{t/2.25})$. Integrating $v(t)$, the position is given by $x(t) = 99.29\ln(e^{t/2.25}/(1 + e^{t/2.25})^2) + 186.2$. To estimate the duration of the downward motion, set $x(t_2) = 0$, resulting in $t_2 = 3.276$ s. Hence the total time that the ball remains in the air is $t_1 + t_2 = 5.192$ s.

(c)