Solutions for assignment #2

Section 2.1

14. The integrating factor is $\mu(t) = e^{2t}$. After multiplying both sides by $\mu(t)$, the equation can be written as $(e^{2t}\,y)' = t$. Integrating both sides of the equation results in the general solution $y(t) = t^2 e^{-2t}/2 + c\,e^{-2t}$. Invoking the specified condition, we require that $e^{-2}/2 + c\,e^{-2} = 0$. Hence $c = -1/2$, and the solution to the initial value problem is $y(t) = (t^2 - 1)e^{-2t}/2$.

16. The integrating factor is $\mu(t) = e^{\int \frac{2}{t}dt} = t^2$. Multiplying both sides by $\mu(t)$, the equation can be written as $(t^2\,y)' = \cos t$. Integrating both sides of the equation results in the general solution $y(t) = \sin t/t^2 + c\,t^{-2}$. Substituting $t = \pi$ and setting the value equal to zero gives $c = 0$. Hence the specific solution is $y(t) = \sin t/t^2$.

19. After writing the equation in standard form, we find that the integrating factor is $\mu(t) = e^{\int \frac{4}{t}dt} = t^4$. Multiplying both sides by $\mu(t)$, the equation can be written as $(t^4\,y)' = t\,e^{-t}$. Integrating both sides results in $t^4 y(t) = -(t+1)e^{-t} + c$. Letting $t = -1$ and setting the value equal to zero gives $c = 0$. Hence the specific solution of the initial value problem is $y(t) = -(t^{-3} + t^{-4})e^{-t}$.

Section 2.2

2. For $x \neq -1$, the differential equation may be written as

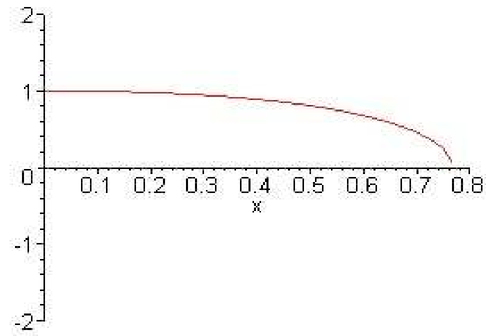
$$y\,dy = \left[x^2/(1+x^3)\right]dx\,.$$

Integrating both sides, with respect to the appropriate variables, we obtain the relation $y^2/2 = \frac{1}{3}\ln\left|1+x^3\right| + c$. That is, $y(x) = \pm\sqrt{\frac{2}{3}\ln\left|1+x^3\right| + c}$.

8. Write the differential equation as $(1+y^2)dy = x^2\,dx$. Integrating both sides of the equation, we obtain the relation $y + y^3/3 = x^3/3 + c$.

11.(a) Rewrite the differential equation as $x\,e^x dx = -y\,dy$. Integrating both sides of the equation results in $x\,e^x - e^x = -y^2/2 + c$. Invoking the initial condition, we obtain $c = -1/2$. Hence $y^2 = 2e^x - 2x\,e^x - 1$. The explicit form of the solution is $y(x) = \sqrt{2e^x - 2x\,e^x - 1}$. The positive sign is chosen, since $y(0) = 1$.

(b)



(c) The function under the radical becomes negative near $x \approx -1.7$ and $x \approx 0.77$.

32.(a) Observe that $(x^2 + 3y^2)/2xy = \frac{1}{2}(y/x)^{-1} + \frac{3}{2}(y/x)$. Hence the differential equation is homogeneous.

(b) The substitution $y = x\,v$ results in $v + x\,v' = (x^2 + 3x^2v^2)/2x^2v$. The transformed equation is $v' = (1 + v^2)/2xv$. This equation is separable, with general solution $v^2 + 1 = c\,x$. In terms of the original dependent variable, the solution is $x^2 + y^2 = c\,x^3$.

34.(a) Observe that $-(4x + 3y)/(2x + y) = -2 - \frac{y}{x}\left[2 + \frac{y}{x}\right]^{-1}$. Hence the differential equation is homogeneous.

(b) The substitution $y = x\,v$ results in $v + x\,v' = -2 - v/(2 + v)$. The transformed equation is $v' = -(v^2 + 5v + 4)/(2 + v)x$. This equation is separable, with general solution $(v + 4)^2\,|v + 1| = c/x^3$. In terms of the original dependent variable, the solution is $(4x + y)^2\,|x + y| = c$.