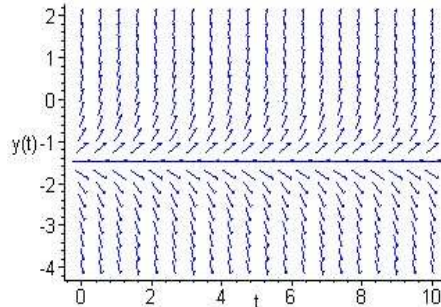


## Solutions for assignment #1

### Section 1.1

3.



For  $y > -1.5$ , the slopes are positive, and hence the solutions increase. For  $y < -1.5$ , the slopes are negative, and hence the solutions decrease. All solutions appear to diverge away from the equilibrium solution  $y(t) = -1.5$ .

### Section 1.3

4. The differential equation is first order, since the only derivative is of order one. The dependent variable is squared, hence the equation is nonlinear.

9.  $y(t) = 3t + t^2 \Rightarrow y'(t) = 3 + 2t$ . Substituting into the differential equation, we have  $t(3 + 2t) - (3t + t^2) = 3t + 2t^2 - 3t - t^2 = t^2$ . Hence the given function is a solution.

11.  $y_1(t) = t^{1/2} \Rightarrow y_1'(t) = t^{-1/2}/2$  and  $y_1''(t) = -t^{-3/2}/4$ . Substituting into the left hand side of the equation, we have

$$\begin{aligned} 2t^2(-t^{-3/2}/4) + 3t(t^{-1/2}/2) - t^{1/2} &= -t^{1/2}/2 + 3t^{1/2}/2 - t^{1/2} \\ &= 0 \end{aligned}$$

Likewise,  $y_2(t) = t^{-1} \Rightarrow y_2'(t) = -t^{-2}$  and  $y_2''(t) = 2t^{-3}$ . Substituting into the left hand side of the differential equation, we have

$$2t^2(2t^{-3}) + 3t(-t^{-2}) - t^{-1} = 4t^{-1} - 3t^{-1} - t^{-1} = 0.$$

Hence both functions are solutions of the differential equation.

20.  $y(t) = t^r \Rightarrow y'(t) = r t^{r-1}$  and  $y''(t) = r(r-1)t^{r-2}$ . Substituting the derivatives into the differential equation, we have  $t^2 [r(r-1)t^{r-2}] - 4t(r t^{r-1}) + 4t^r = 0$ . After some algebra, it follows that  $r(r-1)t^r - 4r t^r + 4t^r = 0$ . For  $t \neq 0$ , we obtain the algebraic equation  $r^2 - 5r + 4 = 0$ . The roots of this equation are  $r_1 = 1$  and  $r_2 = 4$ .