Tutorial 3

MATH 1104 B • October 12, 2016 • Jonathan Nilsson

Work alone or in small groups with the following problems during the tutorial. Your TA is available to help you both during the tutorial and during their weekly office hours. You are not expected to be able to solve all problems in one hour, but you may want to complete the exercises at home. Problems marked by ★ can be tricky and should probably be saved for last. Suggested solutions will be posted after the tutorial.

Matrix Inverse

1. Find the inverse (if it exists) of each of the following four matrices.

$$A_1 = \begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad A_3 = \begin{bmatrix} 2 & 2 & -1 \\ 2 & 3 & -1 \\ -2 & 3 & 4 \end{bmatrix} \qquad A_4 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & -3 \\ 3 & 0 & 4 & 0 \\ 1 & 1 & 5 & -2 \end{bmatrix}$$

2. Let

$$A = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

   In each case, find all matrices X satisfying the given equation

   (a) $AX = B - 3X$
   (b) $AXB^{-1} = C$
   (c) $XC = B^T + XA - 2I$

3. Solve the following linear system by first writing it as a matrix equation $A\mathbf{x} = \mathbf{b}$ and then multiplying by A^{-1} from the left.

$$\begin{cases} 2x + 2y - z = 0 \\ 2x + 3y - z = 1 \\ -2x + 3y + 4z = -1 \end{cases}$$

   Hint: You have already computed A^{-1} in problem 1!

4. For what values of the parameter c does the matrix $\begin{bmatrix} -1 & 3 \\ c & 5 \end{bmatrix}$ have an inverse? Find the inverse for all such values of c!

5. ★ For a certain square matrix A we know that $A^2 + 3A - 2I = 0$. Show that A must be invertible!

6. ★ Here follows three descriptions of linear maps $\mathbb{R}^2 \to \mathbb{R}^2$. Each such map has a standard matrix. Determine in what cases this matrix has an inverse, and describe the inverse map geometrically!

   (a) R=Rotation by an angle $\frac{2\pi}{5}$ counter-clockwise.

   (b) P=Projection onto the x-axis.

   (c) S=Reflection in the line $y = x$.

   Hint: Think about what it means geometrically that such an inverse exists. You don't need to find any matrices.

Subspaces, rank, dimension

We will talk more about this kind of problems on the lecture this week, so you may want to save them until after the tutorial.

7. Let S be the subspace of $\mathbb{R}^4$ spanned by the following four vectors

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 3 \end{bmatrix} \quad \mathbf{v_2} = \begin{bmatrix} 3 \\ 1 \\ -1 \\ 2 \end{bmatrix} \quad \mathbf{v_3} = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} \quad \mathbf{v_4} = \begin{bmatrix} 5 \\ -1 \\ 5 \\ 6 \end{bmatrix}$$

   (a) Find a basis for S.

   (b) What is the dimension of S?

   (c) Does the vector $\mathbf{w} = (1, 2, 0, 0)$ belong to S?

   (d) Does $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}\}$ span $\mathbb{R}^4$?

8. Let

$$M = \begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 0 & 4 & 2 \\ 3 & 1 & 4 & 4 \end{bmatrix}$$

   (a) Find a basis for $Col(M)$. What is the rank of M?

   (b) Find a basis for $Nul(M)$. What is the dimension of $Nul(M)$?