

# Tutorial 6

MATH 1104 B • November 30, 2016 • Jonathan Nilsson

Work alone or in small groups with the following problems during the tutorial. Your TA is available to help you both during the tutorial and during their weekly office hours. Problems marked by ★ can be tricky and should probably be saved for last. Suggested solutions will be posted after the tutorial.

## Dot product and Orthogonality

- Let  $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ ,  $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ , and  $\mathbf{w} = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$ .
  - Calculate  $(\mathbf{u} + 2\mathbf{v}) \bullet \mathbf{w}$ .
  - Calculate  $\|\mathbf{w} + 3\mathbf{u} - \mathbf{v}\|$ .
  - Is  $\mathbf{u} + \mathbf{v}$  orthogonal to  $\mathbf{w}$ ?
  - Normalize the vector  $\mathbf{w}$ .
  - Find the angle between  $\mathbf{u}$  and  $\mathbf{v}$ .
- Is  $\{(1, 2, -1), (1, 0, 1), (3, 2, 3)\}$  an orthogonal set?
- Find a vector of  $\mathbb{R}^3$  that is orthogonal to both  $(3, 0, 1)$  and  $(2, 1, 1)$ .
- Find an equation for the plane in  $\mathbb{R}^3$  of all vectors that are orthogonal to  $(3, -5, 2)$ .
- Show that for any angle  $\alpha$ , the set  $\left\{ \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{bmatrix}, \begin{bmatrix} -\sin(\alpha) \\ \cos(\alpha) \end{bmatrix} \right\}$  is orthonormal.
- ★ Show that the four points  $(1, 0, 0)$ ,  $(0, 1, 0)$ ,  $(0, 0, 1)$ , and  $(1, 1, 1)$  are the corners of a tetrahedron in  $\mathbb{R}^3$ .

## Projections

- Find the projection of  $(1, 2, -5)$  onto  $(3, 1, 2)$ . Also find the projection of  $(2, 3, 8, 6)$  onto  $(1, 2, 1, -1)$ .
- Let  $L = \{t(1, 2, -2) \mid t \in \mathbb{R}\}$ . Then  $L$  is a line in  $\mathbb{R}^3$ . Find the point on  $L$  closest to  $(3, -2, 1)$ . Also find the closest distance.
- Let  $\mathbf{w} = (1, 2, 3)$ . Write the vector  $(1, 4, 2)$  as the sum of two vectors, one of which is parallel to  $\mathbf{w}$  and one which is orthogonal to  $\mathbf{w}$ .
- Let  $S = \{(1, 1, -1, -1, 0), (2, -2, 0, 0, 1), (1, 1, 1, 1, 0)\}$  and let  $\mathbf{v} = (1, 0, 1, 1, 1)$ .
  - Show that  $S$  is an orthogonal set in  $\mathbb{R}^5$ .
  - Let  $W = \text{span}(S)$ . Find  $\text{proj}_W \mathbf{v}$ , the projection of  $\mathbf{v}$  onto  $W$ .
  - Find the distance between  $\mathbf{v}$  and  $W$ .
- ★ Find the standard matrix for the linear map  $\mathbb{R}^2 \rightarrow \mathbb{R}^2$  that projects vectors onto the line with equation  $y = 2x$ .