Tutorial 5

MATH 1104 B • November 16, 2016 • Jonathan Nilsson

Work alone or in small groups with the following problems during the tutorial. Your TA is available to help you both during the tutorial and during their weekly office hours. Problems marked by ★ can be tricky and should probably be saved for last. Suggested solutions will be posted after the tutorial.

Complex Numbers

1. Write the following complex numbers in standard form $a + bi$ where a and b are real numbers.

$$i(1+i)(3+i) \qquad \overline{(2+5i)(i-3)} \qquad (1+i)^4 \qquad \frac{2+5i}{i-4} \qquad |2i+5|$$

2. Solve each of the following quadratic equations.

$$z^2 = 3 \qquad z^2 = -3 \qquad z^2 - 2z + 2iz + 3 - 6i = 0 \qquad 2z^2 - 5iz + 15 = -iz^2 + 10z - 5i$$

Eigenvalues, Eigenvectors, and Diagonalization

From here on you may assume that we are working over the complex numbers unless otherwise stated.

3. Which of the following matrices are diagonalizable? Diagonalize the ones that are! In other words, for each matrix A below, try to find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

$$\begin{bmatrix} 2 & 5 \\ 5 & 2 \end{bmatrix} \qquad \begin{bmatrix} 6 & 9 \\ -1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 2 & 1 & -1 & 0 \\ 0 & -3 & 0 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 5 & 1 & -7 \\ 8 & 1 & -10 \\ 2 & 1 & -4 \end{bmatrix}$$

4. Find the eigenvalues of the matrices $B = \begin{bmatrix} 0 & -4 \\ 9 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -2 \\ 5 & 3 \end{bmatrix}$.

5. Show that both $\begin{bmatrix} i \\ 1 \end{bmatrix}$ and $\begin{bmatrix} i \\ -1 \end{bmatrix}$ are eigenvectors for the matrix $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. What are the corresponding eigenvalues? Use your result to diagonalize the matrix A.

6. ★ Suppose that v is an eigenvector for A with eigenvalue 3. Show that v is also an eigenvector for the matrix $(A^2 + 5A - 2I)$. What is the corresponding eigenvalue?

7. ★★ *(Just for fun).* A completed Sudoku-puzzle is a 9×9-matrix. Show that 45 always is an eigenvalue of such a matrix.