Tutorial 4

MATH 1104 B • November 2, 2016 • Jonathan Nilsson

Work alone or in small groups with the following problems during the tutorial. Your TA is available to help you both during the tutorial and during their weekly office hours. Problems marked by ★ can be tricky and should probably be saved for last. Suggested solutions will be posted after the tutorial. *This is the only tutorial before the third test - focus on the first page in the tutorial and try to solve the second page in preparation for the test!*

Determinants

1. Compute the following four determinants.

$$\begin{vmatrix} 2 & 5 \\ 8 & 7 \end{vmatrix} \qquad \begin{vmatrix} 1 & 1 & 0 \\ 0 & 5 & 3 \\ 3 & 0 & -2 \end{vmatrix} \qquad \begin{vmatrix} 1 & 2 & -1 \\ 1 & 1 & -3 \\ -2 & 1 & 4 \end{vmatrix} \qquad \begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & -3 \\ 3 & 0 & 4 & 0 \\ 1 & 1 & 5 & -2 \end{vmatrix}$$

2. Use determinants to answer the following questions.

   (a) For what values of the number m does the matrix $\begin{bmatrix} 4 & 3m \\ m & 2m \end{bmatrix}$ have an inverse?

   (b) Does the system $\begin{cases} x + y + z = 137 \\ 2x \qquad - z = 129 \\ x - y + z = 385 \end{cases}$ have a unique solution?

   (c) Is $\{(1, 2, 1), (1, 0, 1), (1, 1, 1)\}$ a linearly dependent set of vectors?

3. Solve the equation $\begin{vmatrix} x & 1 & x & 1 \\ 2 & x & -3 & 1 \\ 1 & x & 1 & x \\ x & 2x & 0 & 0 \end{vmatrix} = 0.$

4. Solve the following system using Cramer's rule.

$$\begin{cases} x + 2y - z = 0 \\ x + y - 3z = 1 \\ -2x + y + 4z = -1 \end{cases}$$

   Hint: A calculation from problem 1 *can be re-used here!*

5. Use the adjugate matrix formula to find the inverse of the matrix $\begin{bmatrix} 1 & -3 & 4 \\ 0 & 2 & 3 \\ 5 & 7 & 4 \end{bmatrix}$.

6. Suppose that A, B, and C are 8×8-matrices such that $det(A) = 2$, $det(B) = 3$, and $det(C) = 5$. Compute $det(AB^{-1}C)$ and $det(BA^3C^T)$.

7. ★★ An $n \times n$ matrix A is called *skew-symmetric* if $A^T = -A$. Show that such a matrix can not have an inverse when n is odd.

Introduction to Eigenvectors and Eigenvalues

Recall that if $A\mathbf{v} = \lambda\mathbf{v}$, for some nonzero vector $\mathbf{v}$ and for some number λ, then $\mathbf{v}$ is called an *eigenvector* for A, and λ is called the corresponding *eigenvalue*. Geometrically this means that A stretches $\mathbf{v}$ but preserves its direction.

8. Let $A = \begin{bmatrix} 8 & 5 \\ -10 & -7 \end{bmatrix}$.

   (a) Show that $\mathbf{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is an eigenvector for A. What is the corresponding eigenvalue?

   (b) The number $\lambda = -2$ is another eigenvalue for A. Find all eigenvectors corresponding to this eigenvalue!

9. Find all eigenvalues and corresponding eigenvectors of the matrix $B = \begin{bmatrix} 15 & -30 \\ 4 & -7 \end{bmatrix}$.

10. Find all eigenvalues and corresponding eigenvectors of the matrix $C = \begin{bmatrix} 2 & 1 & 1 \\ 0 & -5 & -2 \\ 0 & 8 & 3 \end{bmatrix}$.

11. ★ Here follows descriptions of four different linear maps. In each case, describe all the eigenvectors and eigenvalues of the map.

    (a) The map that projects vectors of $\mathbb{R}^3$ onto the xy-plane.

    (b) The map that reflects vectors of $\mathbb{R}^2$ in the x-axis.

    (c) The map that rotates vectors of $\mathbb{R}^3$ by an angle $\frac{\pi}{2}$ around the z-axis.

    (d) The map that does nothing to the vectors of $\mathbb{R}^3$ (the identity map).

    Hint: Reason geometrically. You don't need to find any standard matrices.