Tutorial 1

MATH 1104 B • September 21, 2016 • Jonathan Nilsson

Work alone or in small groups with the following problems during the first tutorial. Your TA is available to help you both during the tutorial and during their weekly office hours. You are not expected to be able to solve all problems in one hour, but you may want to complete the exercises at home. Suggested solutions will be posted by the end of the week.

Linear systems and Echelon forms

1. For each of the following linear systems, do the following:
   (a) Write down the augmented matrix for the system.
   (b) Reduce the matrix to row echelon form. Circle the pivots and determine how many solutions the system has.
   (c) Reduce the matrix to reduced row echelon form, and write down the solutions.

$$\begin{cases} 4x + y = -5 \\ 3x + 4y = 6 \\ -x + 2y = 8 \end{cases} \qquad \begin{cases} x - y + z = 2 \\ x + y - 2z = 7 \\ 3x - 7y + 9z = -1 \end{cases} \qquad \begin{cases} x - y + 3z = -5 \\ -2x + 3y - 2z = 7 \end{cases}$$

2. The following matrix is an augmented matrix for a linear system with variables $x_1, \dots, x_6$. Reduce the matrix to row echelon form. Circle the pivot positions and determine how many free variables there are. Write down the solutions to the system. Also express the solutions in vector form.

$$\begin{bmatrix} 1 & 3 & 2 & 3 & -1 & 1 & 4 \\ 0 & 0 & 1 & 1 & 0 & 0 & 2 \\ 2 & 6 & 0 & 2 & -2 & 1 & 1 \\ 1 & 3 & -1 & 0 & -1 & 0 & -1 \end{bmatrix}$$

3. For each value of the real parameter b, determine *how many* solutions the following system has.

$$\begin{cases} x - by = -1 \\ bx + 2by = 2 \end{cases}$$

   If you have time, solve the system for each $b \in \mathbb{R}$.

4. Find the reduced row echelon form of the following matrices.

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ 2 & -2 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 1 & 2 & 3 \end{bmatrix} \qquad \begin{bmatrix} 3 & 1 & 2 & 0 \\ 2 & 1 & 2 & 3 \end{bmatrix}$$

5. (In case you have time) Let a, b, c, d, e, f be the last six digits of your phone number. Solve the linear system

$$\begin{cases} (a-4)x_1 + (b-4)x_2 & = c-4 \\ (d-4)x_1 + (e-4)x_2 & = f-4 \end{cases}$$

How many solutions did your system have?

Vectors and linear dependence

6. On an old exam I asked if two given vectors $\mathbf{v_1}$ and $\mathbf{v_2}$ were linearly dependent. One student answered *"v_1 is, but v_2 isn't"*. Explain what's wrong with this answer.

7. Let

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{v_2} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \quad \mathbf{v_3} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \quad \mathbf{v_4} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \mathbf{v_5} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

   (a) Can $\mathbf{v_4}$ be expressed as a linear combination of $\mathbf{v_1}$, $\mathbf{v_2}$, and $\mathbf{v_3}$?

   (b) Show that the set $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}, \mathbf{v_5}\}$ is linearly dependent.

   (c) Does $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ span $\mathbb{R}^3$?

   (d) Is the set $\{\mathbf{v_2}, \mathbf{v_3}, \mathbf{v_5}\}$ linearly dependent or independent?

   (e) Is the set $\{\mathbf{v_2}, \mathbf{v_4}, \mathbf{v_5}\}$ linearly dependent or independent?

8. For what values of c are the vectors $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ c \\ -3 \end{bmatrix}$ linearly independent?