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Concordia University
Department of Mathematics and Statistics

Course	Number	Section	
MATH	208	All	
Examination	Date	Time	Page
Mid Term	FEB. 2013.	1 Hour 30 minutes	2
Instructor	Course Examiner	Marks	
D. Dryanov, L. Dube & U. Tiwari	D. Sen	60	
Special Instructions:	Answer ALL questions.		

Formulae:

$$A = P(1 + i)^n, A = Pe^{rt}, FV = PMT \frac{(1 + i)^n - 1}{i}, PV = PMT \frac{1 - (1 + i)^{-n}}{i}$$

[10] Q 1. A small company manufactures picnic tables. The weekly fixed cost is \$1,200 and the variable cost is \$45 per table. Find the total daily cost of producing x picnic tables. How many picnic tables can be produced for a total weekly cost of \$4,800?

[10] Q 2. Solve for x in the following equations:

(a) $4^{5x-x^2} = 4^{-6}$

(b) $16^{x-1} = 4^{1+x}$

(c) $3 \log_b 2 + \frac{1}{2} \log_b 25 - \log_b 20 = \log_b x$

(d) $\log_a x + \log_a (x + 1) = \log_a 6$

[10] Q 3. A person borrows \$3,600 and agrees to repay the loan in monthly installments over a period of 3 years. The agreement is to pay 1% of the unpaid balance each month for using the money and \$100 each month to reduce the loan. What is the total cost of the loan over the 3 years?

[10] Q 4. Parents have set up a sinking fund in order to have \$140,000 in 15 years for their children's college education. How much should be paid semiannually into an account paying 6.8% compounded semiannually?

[10] Q 5. American Express's online banking division offered a money market account with an annual percentage yield (APY) of 5.65%.

- (a) If interest is compounded monthly, what is the equivalent annual nominal rate?
- (b) If a company wishes to have \$1,000,000 in this account after 8 years, what equal deposit should be made each month?

[10] Q 6. If you buy a television set for \$800 and agree to pay for it in 18 equal monthly payments at 1.5% interest per month on the unpaid balance.

- (a) How much are your payments?
- (b) How much interest will you pay?

Good Luck!

Solutions to Math208 Midterm, May 2011

Course: 2012 Math208H/4; Instructor: Dr. V. Kalvin

February 11, 2013

- [10] Q 1. The total daily cost of producing x picnic tables is $C(x) = \$45 \cdot x + \$1,200/7$.
The total weekly cost of producing x picnic tables is $C(x) = \$45 \cdot x + \$1,200$. If x tables can be produced for a total weekly cost of \$4,800, then

$$C(x) = \$4,800.$$

Therefore we need to solve the equation $\$45 \cdot x + \$1,200 = \$4,800$, which gives $x = 80$. Thus 80 picnic tables can be produced for a total weekly cost of \$4,800.

- [10] Q 2. (a)

$$4^{5x-x^2} = 4^{-6},$$

$$5x - x^2 = -6,$$

$$x^2 - 5x - 6 = 0,$$

$$(x+1)(x-6) = 0$$

$$x = -1; x = 6.$$

Answer: $x \in \{-1, 6\}$

- (b)

$$16^{x-1} = 4^{1+x},$$

$$(4^2)^{x-1} = 4^{1+x},$$

$$4^{2x-2} = 4^{1+x},$$

$$2x - 2 = 1 + x,$$

$$x = 3.$$

Answer: $x \in \{3\}$.

- (c)

$$3 \log_b 2 + \frac{1}{2} \log_b 25 - \log_b 20 = \log_b x, \quad b > 0,$$

$$\log_b 2^3 + \log_b 25^{1/2} - \log_b 20 = \log_b x,$$

$$\log_b 8 + \log_b 5 - \log_b 20 = \log_b x,$$

$$\log_b(8 \cdot 5 : 20) = \log_b x,$$

$$\log_b 2 = \log_b x,$$

$$x = 2.$$

Check:

$$3 \log_b 2 + \frac{1}{2} \log_b 25 - \log_b 20 = \log_b 2.$$

As it was found before, the left hand side of the latter equality is equal to $\log_b 2$. Thus $x = 2$ satisfies the initial equation.

Answer: $x \in \{2\}$.

(d)

$$\log_a x + \log_a(x+1) = \log_a 6, \quad a > 0,$$

$$\log_a(x(x+1)) = \log_a 6,$$

$$x(x+1) = 6,$$

$$x^2 + x - 6 = 0,$$

$$(x+3)(x-2) = 0,$$

$$x = -3; x = 2.$$

Check: $x = -3$ does not satisfy the initial equation as $\log_a(-3)$ has no sense as well as $\log_a(-3+1)$.

Next, $x = 2$ gives $\log_a 2 + \log_a 3 = \log_a 6$, which is true (since $\log_a 2 + \log_a 3 = \log_a(2 \cdot 3)$).

Answer: $x \in \{2\}$.

[10] Q 3. In this problem the periodic payments are unequal, so it is not about an ordinary annuity!!! Here every payment is a sum of \$100 (to reduce the balance) and the interest (which is the balance multiplied by the interest rate $i = 1\%$ per period). The balance before the first payment is \$3,600. Therefore the interest for the first month is $I_1 = \$3,600 \cdot i = \$3,600 \cdot 0.01 = \$36$ (and the first payment is $\$100 + I_1 = \136). The balance for the second month is $B_2 = \$3,600 - \100 and, in general, the balance for the n -th month is $B_n = \$3,600 - (n-1)\100 , $n \leq 36$. Therefore the interest for the second month is $i \cdot B_2$ and, in general, the interest for the n -th month is

$$I_n = i \cdot B_n = 0.01 \cdot (\$3,600 - (n-1)\$100) = \$37 - n;$$

and the n -th payment is $\$100 + I_n$, $n \leq 36$, so you can see that the payments are unequal. The total cost of the loan is the total interest plus \$3,600. The total interest is the sum

$$\sum_{n=1}^{36} I_n = I_1 + I_2 + \cdots + I_{35} + I_{36} = \$36 + \$35 + \cdots + \$2 + \$1.$$

Observe that this is the sum of first 36 elements of an arithmetic sequence with the common difference $d = -\$1$, $a_1 = \$36$, and $a_{36} = \$1$. Thus $\sum_{n=1}^{36} I_n = (a_1 + a_{36}) \cdot 36/2 = \$37 \cdot 18 = \$666$. The total cost of the loan is the total interest \$666 plus \$3,600, which is \$4,266.

[10] Q 4. It is about a sinking fund, so we should find PMT from the formula

$$FV = PMT \frac{(1+i)^n - 1}{i}$$

Thus we have

$$PMT = FV \frac{i}{(1+i)^n - 1},$$

where $FV = \$140,000$, $i = \frac{6.8\%}{2} = 0.034$, and

$$n = 2(\text{number of payments per year}) \cdot 15(\text{ years}) = 30.$$

Therefore

$$PMT = \$140,000 \frac{0.034}{(1 + 0.034)^{30} - 1} \simeq \$2,756.92.$$

[10] Q 5. (a) As we know, $APY = (1 + \frac{r}{m})^m - 1$, where $APY = 0.0565$, $m = 12$ (compounded monthly means 12 times per year), and r is the equivalent annual interest rate. Hence we need to solve

$$0.0565 = \left(1 + \frac{r}{12}\right)^{12} - 1$$

for r . We have

$$1.0565 = \left(1 + \frac{r}{12}\right)^{12},$$

$$(1.0565)^{1/12} = 1 + \frac{r}{12},$$

$$r = 12(1.0565)^{1/12} - 12 \simeq 0.055 = 5.5\%.$$

(b) It is a sinking fund problem. From the formula

$$FV = PMT \frac{(1+i)^n - 1}{i}$$

we find that

$$PMT = FV \frac{i}{(1+i)^n - 1}.$$

Here $FV = \$1,000,000$, $i = \frac{r}{12} \simeq 0.0046$, and

$$n = 8(\text{years}) \cdot 12(\text{payments per year}) = 96.$$

Thus we have

$$PMT = \$1,000,000 \frac{0.0046}{(1 + 0.0046)^{96} - 1} \simeq \$8,308.96.$$

[10] Q 6. (a) It is about an amortization. From

$$PV = PMT \frac{1 - (1 + i)^{-n}}{i}$$

we find that

$$PMT = PV \frac{i}{1 - (1 + i)^{-n}}$$

Here $PV = \$800$, $i = 1.5\% = 0.015$ per period (per month), and $n = 18$.

Thus we get

$$PMT = \$800 \frac{0.015}{1 - (1 + 0.015)^{-18}} \simeq \$51.05.$$

(b) The total interest paid is

$$PMT \cdot 18 - \$800 \simeq \$51.05 \cdot 18 - \$800 = \$118.90.$$