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CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	203	All
Examination	Date	Pages
Final	April 2015	3
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Special Instructions:	Only approved calculators are allowed. Show all your work for full marks.	

MARKS

- [11] 1. (a) Let $f(x) = x^2 - 4$ and $g(x) = \sqrt{4 - x}$. Find $g \circ f$ and $f \circ g$ and determine the domain of each of these composite functions.
(b) Find the range of the function $f = e^{2x} + 2e^x$, the inverse function f^{-1} , and the range of f^{-1} . (HINT: assume $e^x = u$ to see how to find f^{-1})

- [10] 2. Evaluate the limits:

(a) $\lim_{x \rightarrow -2} \frac{x^2 - x - 6}{4 - x^2}$ (b) $\lim_{x \rightarrow 0} \frac{\sqrt{x+9} - 3}{x}$

Do not use l'Hôpital rule.

- [6] 3. Find all horizontal and vertical asymptotes of the function

$$f(x) = \frac{|x|\sqrt{4x^2 + 1} - 2x^2}{x^2 - 3}$$

- [15] 4. Find the derivatives of the following functions:

(a) $f(x) = \frac{2\sqrt{x^5} - x^{3/2}}{x^2}$

(b) $f(x) = \ln \frac{x^4}{x-3}$

(c) $f(x) = e^3 + \arctan(e^x - e^{-x})$

(d) $f(x) = \frac{e^x}{1 + \cos(x^2)}$

(e) $f(x) = (1 + x^2)^{2x}$ (use logarithmic differentiation)

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- [15] 5. (a) Verify that the point $(2,0)$ belongs to the curve defined by the equation $y + x\sqrt{1+y^2} + 2 = x^2$, and find the equation of the tangent line to the curve at this point.
- (b) A particle is moving along a circle with radius $r = 5$ m described by the equation $x^2 + y^2 = 25$ in the (x, y) plane. At the point $(-4, 3)$ the x -coordinate changes at the rate $\frac{dx}{dt} = 15 \frac{\text{m}}{\text{sec}}$. How fast is the y coordinate changing at that instant?
- (c) Use the l'Hôpital's rule to evaluate the $\lim_{x \rightarrow 0} \frac{e^x - x - 1}{x^2 + x^3}$.
- [6] 6. Let $f(x) = \frac{x}{3x - 1}$.
- (a) Find the slope m of the secant line joining the points $(1, f(1))$ and $(3, f(3))$.
- (b) Find all points $x = c$ (if any) on the interval $[1, 3]$ such that $f'(c) = m$.
- [9] 7. The volume of a sphere with radius r is given by the formula $V(r) = \frac{4\pi}{3}r^3$.
- (a) Use the definition of the derivative to show that $\frac{dV}{dr} = 4\pi r^2$.
- (b) If a is a given fixed value for r , write the formula for the linearization of the volume function $V(r)$ at a .
- (c) Use this linearization to calculate the thickness Δr (in centimeters) of a layer of paint on the surface of a spherical ball with radius $r = 52$ cm if the total volume of paint used is 340 cm^3 .
- [12] 8. (a) Find the absolute extrema of $f(x) = xe^{-x^2}$ on the interval $[-\frac{1}{2}, 1]$.
- (b) Find the radius r and the height h of the a cylindrical can that is open at the top and has a volume 1000 cm^3 , but has the smallest possible surface area.

- [16] 9. Given the function $f(x) = 2x^2 - x^4$.
- (a) Find the domain of f and check for symmetry. Find asymptotes of f (if any).
 - (b) Calculate $f'(x)$ and use it to determine intervals where the function is increasing, intervals where it is decreasing, and the local extrema (if any).
 - (c) Calculate $f''(x)$ and use it to determine intervals where the function is concave upward, intervals where the function is concave downward, and the inflection points (if any).
 - (d) Sketch the graph of the function $f(x)$ using the information obtained above.

[5] **Bonus Question**

We know that a function f is differentiable on the interval $[0,2]$ and has values $f(0) = 0$, $f(1) = 1$ and $f(2) = -1$. Is this information sufficient to claim, using the Mean Value theorem, that the tangent line to the graph of $f(x)$ must be horizontal at least at one point x in the interval $(0,2)$? Explain why yes or why not.

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CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections	
Mathematics	203	All	
Examination	Date	Pages	
Alternate	December 2014	3	
Instructors:	Z. Ben Salah, A. Boyarsky, J. Brody, I. Gorelyshev, T. Hughes, P. Moore	Course Examiner	A. Atoyan
Special Instructions:	Only approved calculators are allowed Show all your work for full marks.		

MARKS

- [11] 1. (a) Solve for x : $2\log_2(x) - \log_2(x+3) = 5^{\log_5(2)}$
(b) Let $f(x) = \frac{1}{x^2 - 1}$ and $g(x) = \sqrt{1+x}$. Find $f \circ g$ and determine its domain.
(c) Given the function $f(x) = (8 + 2^x)^{1/3}$, find the inverse function f^{-1} , the range of f and the range of f^{-1} .

- [7] 2. Find the limit if it exists (do not use l'Hôpital's rule.):

(a) $\lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{|x - 1|}$ (b) $\lim_{x \rightarrow -2} \frac{3 - \sqrt{x^2 - 3x - 1}}{x^2 - 4}$

- [6] 3. Find all horizontal and vertical asymptotes of the function

$$f(x) = \frac{\sqrt{9x^4 + 2x^2 + 1}}{x^2 + 4x}$$

- [15] 4. Find the derivatives of the following functions (you don't need to simplify your final answer, but you must show how you calculate it):

(a) $f(x) = (x^{3/2} + 1)(x^{3/2} - 1) \tan x$

(b) $f(x) = (x + e^{-x}) \ln^2(x)$

(c) $f(x) = \sqrt{x \sin(x^3) + \sin(x^3 - x)}$

(d) $f(x) = \frac{3^x}{3^x + 3^{-x}} + 3^2$

(e) $f(x) = (\cos x + x^2)^{5x}$ (use logarithmic differentiation)

- [12] 5. Consider the function $y = 3x + x^{-1}$.
- (a) Use the definition of derivative to find the formula for dy/dx .
 - (b) Find the linearization $L(x)$ of the function $y(x)$ at $a = 2$.
 - (c) Find the differential dy and evaluate it for the values $x = 2$ and $dx = 0.1$.
- [7] 6. Let $f(x) = \frac{x+1}{x+3}$.
- (a) Find the slope m of the secant line joining the points $(-1, f(-1))$ and $(2, f(2))$.
 - (b) Find all points $x = c$ (if any) on the interval $[-1, 2]$ such that $f'(c) = m$.
- [17] 7. (a) Verify that the point $(1, 2)$ belongs to the curve defined by the equation $y^3 - xy - 2\sqrt{3+x^2} = 2$, and find an equation of the tangent line to the curve at this point.
- (b) At 1 PM, ship A is 5 kilometers strictly to the west of ship B. Ship A is sailing west at speed 20 km/hour and ship B is sailing north at 30 km/hour. How fast (in km/hour) is the distance between ships changing at 3 PM?
- (c) Use l'Hôpital's rule to evaluate the $\lim_{x \rightarrow 0} \frac{\sin(x) - x}{e^{x^3} - 1}$.
- [11] 8. (a) Find the point (x_0, y_0) on the curve $y = 2\sqrt{x}$ that is closest to the point $(3, 0)$.
- (b) A box with a square base is to be constructed with a volume of 50 m^3 . The material for the bottom and the sides of the box costs $\$2/\text{m}^2$, and the material for the top costs $\$5/\text{m}^2$. Find the dimensions that minimize the cost of the box.

[14] 9. The function $f(x)$ and its derivatives are given:

$$f(x) = \frac{(1+x)}{x^2}, \quad f'(x) = \frac{-(x+2)}{x^3}, \quad f''(x) = \frac{2(x+3)}{x^4}.$$

- (a) Find the domain of $f(x)$, check for symmetry, and also find asymptotes (if any).
- (b) Find intervals where the function $f(x)$ is increasing, intervals where it is decreasing, and the local maxima and local minima (if any) of $f(x)$.
- (c) Determine intervals where the function $f(x)$ is concave upward, intervals where the function is concave downward, and the inflection points of $f(x)$.
- (d) Sketch the graph of the function $f(x)$ using the information obtained above.

[5] **Bonus Question.** Let $p(x) = x^4 + a^2x^2 - 2a^2x$, where a is any real number. Prove that the graph $y = p(x)$ has at least one point of local minimum on the interval $(-1, 1)$.

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