

ASSIGNMENT 1

PAGE 1

1.(a) Figure 0.1.1, page 2

Math model is a way to describe an applied problem using mathematics. It consists of the following stages:

- * observation, question or problem
- * mathematical formulation, using dynamical systems or differential equations or other objects from math
- * solution
- * interpretation in terms of the application
- * if needed: improvement of the model, adjustment, refinement, etc.

(b) discrete-time: values given at equally spaced intervals (graph is a collection of dots)

continuous time: values given for all values of the variable (continuous curve)

page 4

table (c)
page 6 $(1, 4)$ open $[1, 4]$ closed $(0, \infty)$ open $(-\infty, 7]$ closed $(-\infty, 7)$ open

(d)

one order of magnitude is one power of 10

37 \rightarrow one order of magn. is 370

2 orders 3700

3 orders 37000

4 orders 370000

table
page 5

2(a) Assume 1 year \approx 365 days

$$365 \cdot 24 \cdot 60 \cdot 60 = 31,536,000 \text{ seconds}$$

if 1 year \approx 365.25 days

$$365.25 \cdot 24 \cdot 60 \cdot 60 = 31,557,600 \text{ seconds}$$

(b) $72 \cdot 60 \cdot 24 \cdot 365 \cdot 50 = 1,892,160,000$

one hour one day one year

(c) volume $V = \frac{4\pi}{3} \text{ radius}^3$

diameter = 15 inches \rightarrow radius = 7.5 inches
 $= 7.5 \cdot 2.54 = 19.05 \text{ cm}$

$$V = \frac{4\pi}{3} (19.05)^3 \approx 28,958.33 \text{ cm}^3$$

mass = density \cdot volume = $3.4 \frac{\text{g}}{\text{cm}^3} \cdot 28,958.33 \text{ cm}^3$
 $= 98,458.32 \text{ g} \approx 98.4 \text{ kg}$

(d) 650 kg = 650,000 g

650 kg = $650 \cdot 2.204 \text{ lb} = 1432.6 \text{ lb}$

(e) $95 \frac{\text{km}}{\text{h}} = 95 \cdot \frac{1}{1.609} \frac{\text{miles}}{\text{h}}$ \nwarrow table on page 7

$= 59.04 \text{ mph}$

3 (a) A parameter is a variable which changes from experiment to experiment, but is constant within an experiment

exercise 1: dependent: weight density
 independent: altitude
 parameter: rainfall

(b) A relation is a set of all pairs of values of independent and dependent variables
A function is a unique assignment of a number y (or $f(x)$) to a number x



(c) To identify those curves that are the graphs of functions

(d) Natural domain: largest set of values where f is defined. Given domain: subset of the natural domain that has to be explicitly stated

(e) Set of all points $(x, f(x))$, where x is in the domain of f

4. (a) $f_1, f_2, f_3, f_4, f_5, f_9, f_{10}$

(b) f_6, f_7

(c) f_6, f_7

(d) f_1, f_5, f_9 keep in mind that it says increasing for all real numbers
(\sqrt{x} is increasing, but only for $x \geq 0$
 x^2 is increasing only for $x > 0$)

(e) f_4

(f) linear means $f(x) = mx + b$: f_1, f_2, f_3

textbook look at pages 16, 17

f_{10} is not linear, but consists of two linear pieces:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

(g) positive means > 0 for all x (so if a function touches x -axis it is not called positive; it's called non-negative)

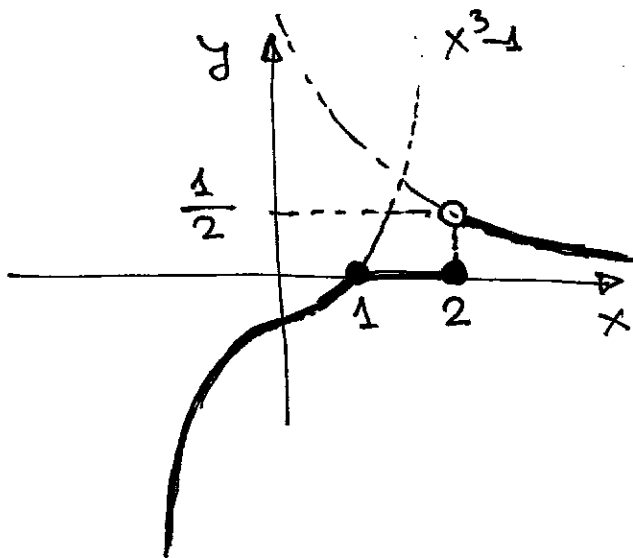
f_3, f_7

(h) f_2, f_5, f_6, f_9

(i) f_3, f_4, f_7, f_{10}

(j) f_3

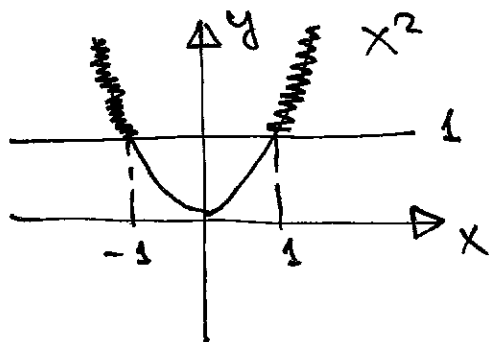
5.



domain: all x
 range: $(-\infty, 0]$
 and $(0, 1/2)$
 combine:
 $(-\infty, 1/2)$

6.

$$x^2 - 1 \geq 0 \rightarrow x^2 \geq 1$$



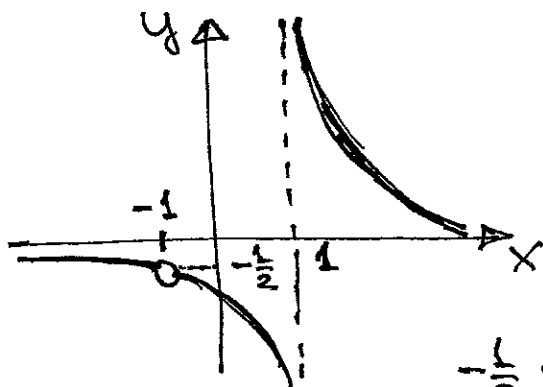
(identify x where x^2 lies above, or touches (horizontal line) 1)
 $x \leq -1$ or $x \geq 1$

7 (a) $x \neq 1$

(b) $x^2 - 1 = 0 \rightarrow x = \pm 1$... domain: all $x \neq \pm 1$

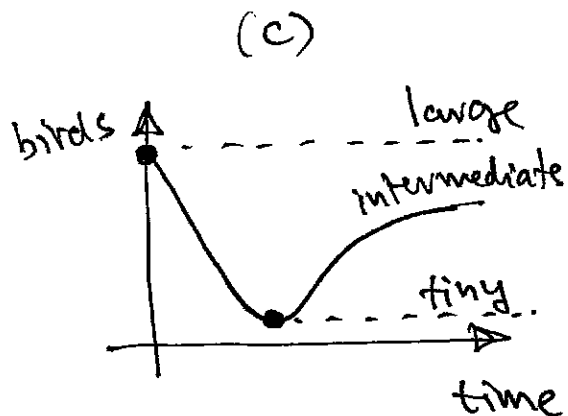
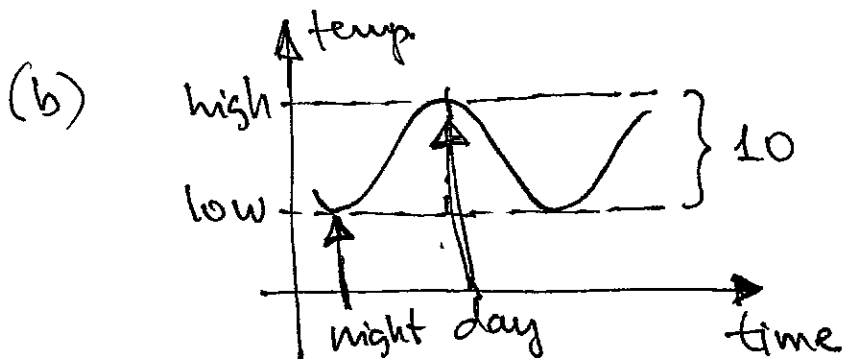
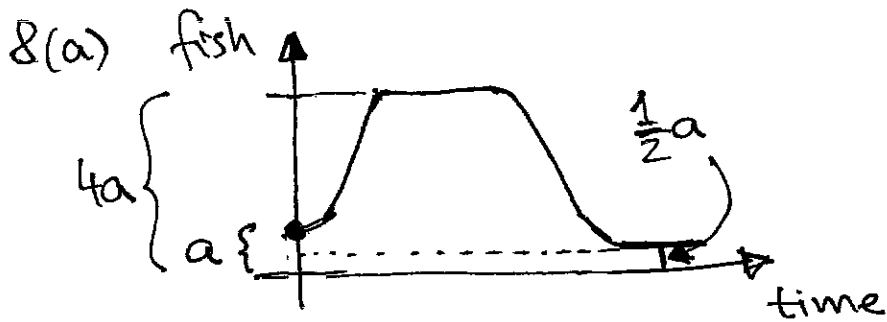
(c) $x^2 + 1 = 0$ has no solutions ... domain: all x

(d) $f_2(x) = \frac{x+1}{(x+1)(x-1)} = \begin{cases} \frac{1}{x-1} & \text{if } x \neq -1 \\ \text{not defined} & x = -1 \end{cases}$



range: all real numbers except $-\frac{1}{2}$ and 0

$-\frac{1}{2}$ is what we get when we substitute -1 into $\frac{1}{x-1}$



9. Average height increases with the age of a tree until tree is about 30 years old. Then, the height starts decreasing.

$$10.(a) \quad f(a+1) = 3(a+1)^2 - (a+1) + 2 = 3a^2 + 5a + 4$$

$$(b) \quad f(a) + f(1) = (3a^2 - a + 2) + (4) = 3a^2 - a + 6$$

$$(c) \quad f(1+h) - f(1) = (3(1+h)^2 - (1+h) + 2) - (4) \\ = 3h^2 + 5h$$

$$11.(a) \quad f\left(\frac{a}{4}\right) = \frac{2}{\frac{4}{a}} = 2 \cdot \frac{4}{a} = \frac{8}{a}$$

$$(b) \quad f\left(\frac{22}{a}\right) = \frac{2}{\frac{22}{a}} = 2 \cdot \frac{a}{22} = \frac{a}{11}$$

$$(c) \quad \frac{f(3+h) - f(3)}{h} = \frac{\frac{2}{3+h} - \frac{2}{3}}{h} \\ = \frac{2(3) - 2(3+h)}{(3+h)(3)} \cdot \frac{1}{h} = \frac{-2h}{3(3+h)} \cdot \frac{1}{h} \\ = \frac{-2}{3(3+h)}$$