GNG 1105

ENGINEERING MECHANICS

FINAL EXAM

SOLUTIONS

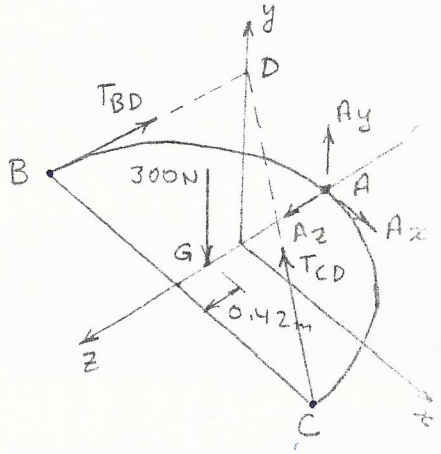
Dec. 10, 2015

1.

a) FBD – Sec Diagram.

b) Center of gravity G:

$$\bar{z} = \frac{4r}{3\pi} = \frac{4\times 1}{3\pi} = 0.42\,m.$$



$\bar{BD} = +1.0\bar{i} + 2.0\bar{j} - 0.7\bar{k}$; $BD = 2.34m$

$\bar{CD} = -1.0\bar{i} + 2.0\bar{j} - 0.7\bar{k}$; $CD = 2.34m$

$\bar{W} = -300N\,\bar{j}$

$$\bar{T}_{BD} = T_{BD}\,\bar{\lambda}_{BD} = T_{BD}\,\frac{\bar{BD}}{BD} = \frac{T_{BD}}{2.34}(1.0\bar{i} + 2.0\bar{j} - 0.7\bar{k})$$

$$\bar{T}_{CD} = T_{CD}\,\bar{\lambda}_{CD} = T_{CD}\,\frac{\bar{CD}}{CD} = \frac{T_{CD}}{2.34}(-1.0\bar{i} + 2.0\bar{j} - 0.7\bar{k})$$

$\bar{W} = -300N\,\bar{j}$

c) $\Sigma \bar{M}_A = 0$

$$\Sigma \bar{M}_A = \bar{r}_{B/A}\times\bar{T}_{BD} + \bar{r}_{C/A}\times\bar{T}_{CD} - \bar{r}_{G/A}\times(300N)\bar{j} = 0$$

where, $\bar{r}_{B/A} = -1.0\bar{i} + 1.0\bar{k}$; $\bar{r}_{C/A} = +1.0\bar{i} + 1.0\bar{k}$); $\bar{r}_{G/A} = +(1.0 - 0.42)\bar{k}$

$= +0.58\bar{k}$

$$\therefore \Sigma \bar{M}_A = (-1.0\bar{i} + 1.0\bar{k})\times\frac{T_{BD}}{2.34}(1.0\bar{i} + 2.0\bar{j} - 0.7\bar{k}) + (1.0\bar{i} + 1.0\bar{k})\times\frac{T_{CD}}{2.34}(-1.0\bar{i} + 2.0\bar{j} - 0.7\bar{k}) - 0.58\bar{k}\times 300\bar{j} = 0$$

$$\Sigma \bar{M}_A = -2\times\frac{T_{BD}}{2.34}\bar{k} - 0.7\times\frac{T_{BD}}{2.34}\bar{j} + 1\times\frac{T_{BD}}{2.34}\bar{j} - 2\times\frac{T_{BD}}{2.34}\bar{i} + 2\times\frac{T_{CD}}{2.34}\bar{k} + 0.7\frac{T_{CD}}{2.34}\bar{j} - 1\frac{T_{CD}}{2.34}\bar{j} - 2\times\frac{T_{CD}}{2.34}\bar{i} + 174\bar{i} = 0$$

i.e. $$\Sigma M_A = -0.85\,T_{BD}\bar{k} - 0.3\,T_{BD}\bar{j} + 0.43\,T_{BD}\bar{j} - 0.85\,T_{BD}\bar{i} + 0.85\,T_{CD}\bar{k} + 0.3\,T_{CD}\bar{j} - 0.43\,T_{CD}\bar{j} - 0.85\,T_{CD}\bar{i} + 174\bar{i} = 0$$

1. (Cont'd)

Equate the Coefficients of $\bar{i}$, $\bar{j}$ & $\bar{k}$ to zero:

$(\bar{i})$: $-0.85\,T_{BD} - 0.85\,T_{CD} + 174 = 0$; Because of Symmetry $T_{BD} = T_{CD}$

$\therefore 1.70\,T_{BD} = 174$

Hence, $T_{BD} = T_{CD} = \frac{174}{1.70} = \underline{\underline{102.35\,N}}$ ANS.

$(\bar{j})$: $-0.3\,T_{BD} + 0.43\,T_{BD} + 0.3\,T_{CD} - 0.43\,T_{CD} = 0$

$0.13\,T_{BD} - 0.13\,T_{CD} = 0$

$\therefore T_{BD} = T_{CD}$ ✓ (check)

$(\bar{k})$: $-0.85\,T_{BD} + 0.85\,T_{CD} = 0$

$\therefore T_{BD} = T_{CD}$ ✓ (check)

Components of reaction at A:

$\Sigma F_x = 0$

$(\bar{i})$: $A_x + \frac{1.0}{2.34} \times T_{BD} - \frac{1.0}{2.34} \times T_{CD} = 0$

$A_x + \frac{102.35}{2.34} - \frac{102.35}{2.34} = 0$; $\therefore \underline{\underline{A_x = 0}}$ ANS.

$(\bar{j})$: $A_y + \frac{2.0}{2.34} \times T_{BD} + \frac{2.0}{2.34} \times T_{CD} - 300\,N = 0$

$A_y + \frac{2.0}{2.34} \times 102.35 + \frac{2.0}{2.34} \times 102.35 - 300\,N = 0$

$A_y + 87.48 + 87.48 - 300\,N = 0$; $\therefore \underline{\underline{A_y = +125.04\,N}}$ ANS

$(\bar{k})$: $A_z - \frac{0.7}{2.34} \times T_{BD} - \frac{0.7}{2.34} \times T_{CD} = 0$

$A_z - \frac{0.7}{2.34} \times 102.35 - \frac{0.7}{2.34} \times 102.35 = 0$

$A_z - 30.62 - 30.62 = 0$; $\therefore \underline{\underline{A_z = +61.24\,N}}$ ANS.

1.c –

Another Method (part c)

$$\Sigma M_A = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -1 & 0 & +1 \\ +1 & +2 & -0.7 \end{vmatrix} \times \frac{T_{BD}}{2.34} + \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ +1 & 0 & +1 \\ -1 & +2 & -0.7 \end{vmatrix} \times \frac{T_{CD}}{2.34} - 0.58\bar{k} \times 300\bar{j} = 0$$

Coeff $\bar{i}$: $\frac{-2T_{BD}}{2.34} - \frac{2T_{CD}}{2.34} + 174 = 0$

$$-0.85T_{BD} - 0.85T_{CD} + 174 = 0$$

$T_{BD} = T_{CD}$; $\therefore -1.70T_{BD} = 174$

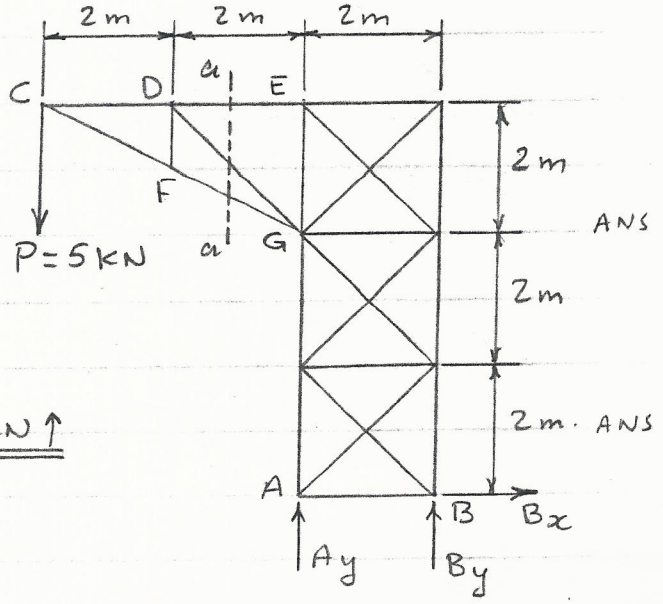
Hence, $T_{BD} = T_{CD} = \frac{174}{1.70} = 102.35\ N$ ANS.

Coeff $\bar{j}$: $-\frac{0.7T_{BD}}{2.34} + \frac{1T_{BD}}{2.34} + 0.7\frac{T_{CD}}{2.34} - \frac{1T_{CD}}{2.34} = 0$

$$\frac{0.3T_{BD}}{2.34} - \frac{0.3T_{CD}}{2.34} = 0 \; ; \; \therefore T_{BD} = T_{CD} \checkmark \text{ check.}$$

Coeff $\bar{k}$: $-\frac{2T_{BD}}{2.34} + \frac{2T_{CD}}{2.34} = 0$; $\therefore T_{BD} = T_{CD} \checkmark$ check.

2. a)



FBD- Entire truss

$\pm \Sigma F_x = 0$

$B_x = 0$; $\underline{B_x = 0}$

$+\circlearrowleft \Sigma M_B = 0$

$5KN \times 6m - A_y \times 2m = 0$

$2A_y = 30$; $\therefore A_y = \frac{30}{2} = \underline{15KN \uparrow}$

$+\uparrow \Sigma F_y = 0$

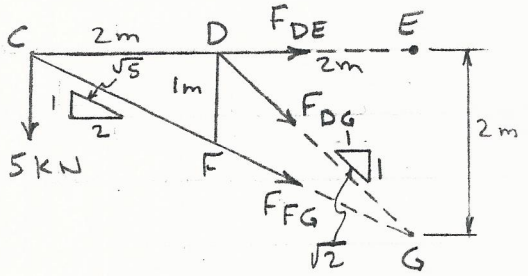
$A_y + B_y - 5\,KN = 0$

$15KN + B_y = 5KN$; $\therefore B_y = -10\,KN$

i.e $\underline{B_y = 10\,KN \downarrow}$ ANS.

b)

FBD- Left of a-a



$+\circlearrowleft \Sigma M_D = 0$

$5KN \times 2m + F_{FG} \times \frac{2}{\sqrt{5}} \times 1m = 0$

$10 + 0.89\,F_{FG} = 0$

$\therefore F_{FG} = -\frac{10}{0.89} = -11.24KN$

i.e $\underline{F_{FG} = 11.24KN\,(C)}$ ANS.

$+\circlearrowleft \Sigma M_G = 0$

$5KN \times 4m - F_{DE} \times 2m = 0$

$2F_{DE} = 20$; $\therefore F_{DE} = \frac{20}{2} = 10KN\,(T)$; i.e. $\underline{F_{DE} = 10KN\,(T)}$ ANS

$+\circlearrowleft \Sigma M_C = 0$

$F_{DG} = 0$ $\therefore \underline{F_{DG} = 0}$ ANS

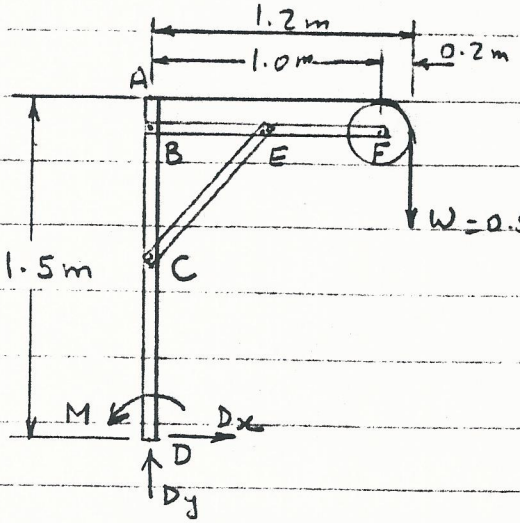
Check: $+\uparrow \Sigma F_y = 0$

$11.24 \times \frac{1}{\sqrt{5}} - 5KN - 0 = 0$

$5 - 5 = 0$ ✓ (Check)

3.

a) FBD – Entire frame



$+\circlearrowleft \Sigma M_D = 0$

$M - 0.5\,KN \times 1.2\,m = 0$

$\therefore M = 0.6\,KN.m \circlearrowleft$ ANS.

$\pm \Sigma F_x = 0$

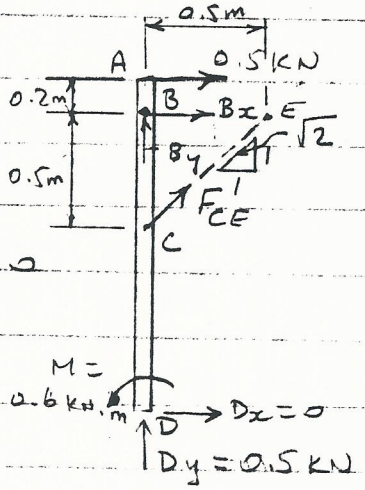
$D_x = 0$ ANS.

$+\uparrow \Sigma F_y = 0$

$D_y - 0.5\,KN = 0$, $\therefore D_y = 0.5\,KN \uparrow$ ANS.

b) FBD – member ABCD

CE is a 2-force member.



$+\circlearrowleft \Sigma M_B = 0$

$-0.5\,KN \times 0.2\,m + F_{CE} \times \frac{1}{\sqrt{2}} \times 0.5\,m + 0.6\,KN.m = 0$

$-0.1\,KN.m + 0.35\,F_{CE} + 0.6\,KN.m = 0$

$0.35\,F_{CE} = -0.5$

$\therefore F_{CE} = -\frac{0.5}{0.35} = \underline{\underline{-1.43\,KN}}$ ANS.

Since CE is a 2-force member, $\therefore F_{EC} = \underline{\underline{1.43\,KN \nearrow}}$ ANS.

$\pm \Sigma F_x = 0$

$B_x + 0.5\,KN - 1.43 \times \frac{1}{\sqrt{2}} = 0$

$B_x + 0.5 - 1.01 = 0$; $\therefore B_x = \underline{\underline{0.51\,KN \rightarrow}}$ ANS.

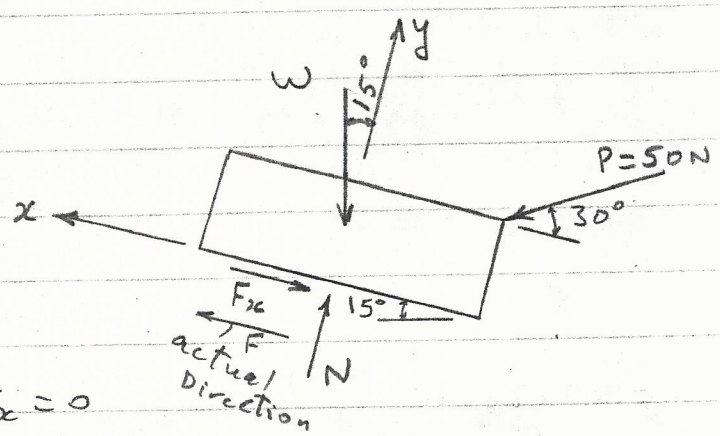
$+\uparrow \Sigma F_y = 0$

$B_y + 0.5\,KN - 1.43 \times \frac{1}{\sqrt{2}} = 0$

$B_y + 0.5 - 1.01 = 0$; $\therefore B_y = \underline{\underline{0.51\,KN \uparrow}}$ ANS.

4. a)

FBD. Block



*- Suppose motion is up the incline.

$W = 35 \times 9.81 = 343.4\ N$

$\mu_s = 0.1$; $\mu_k = 0.05$

$\leftarrow^{+} \Sigma F_x = 0$

$50\cos 30^\circ - 343.4 \sin 15^\circ - F_x = 0$

$43.30 - 88.88 - F_x = 0$

$\therefore F_x = 45.58\ N$

$+\uparrow \Sigma F_y = 0$

$-50\sin 30^\circ - 343.4\cos 15^\circ + N = 0$

$-25 - 331.7 + N = 0$

$\therefore N = 356.7$ Newtons

$F_m = \mu_s N = 0.1 \times 356.7 = 35.67\ N$

Since $F_m < F_x$, $\therefore$ Motion is down the incline. ↘ ANS.

b) Actual Friction Force:

$F = \mu_k N = 0.05 \times 356.7 = 17.8\ N$ ANS.

c) $\nearrow^{+} \Sigma F_x = ma_x$

$\Sigma F_x = 35\ kg \times a_x = +50\cos 30^\circ - 343.4 \sin 15^\circ$

$35 a_x = 43.30 - 88.88 + 17.8 = -27.78$

$\therefore a_x = \frac{-27.78}{35} = -0.79\ m/s^2$

i.e. $a_x = 0.79\ m/s^2$ → ANS.

d) $x = \cancel{x_0}^{0} + \cancel{v_0}^{0} t + \frac{1}{2} a t^2$

$3m = \frac{1}{2} \times 0.79 t^2$; $\therefore t^2 = 7.6$, $\therefore t = 2.76$ secs ANS.

$V = \cancel{v_0}^{0} + at$

$V = 0.79 \times 2.76 = 2.18\ m/s$ ↘ ANS.

END