CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Section(s)
Mathematics	209	All except EC
Examination	**Date**	**Pages**
Final	December 2009	3
Instructors		**Course Examiner**
M. Amir, V. Enolski, H. Greenspan, T. Koulis, R. Mearns, R. Raphael, B. Rhodes, F. Romanelli, J. Ruddy,		R. Raphael

Special Instructions

- ▷ Ruled booklets to be used.
- ▷ **Only approved calculators are allowed.**

MARKS

[11] **1.** (i) Find $\lim_{x\to\infty} \dfrac{-3x^3+5x^2-7}{5-x^2}$

(ii) Given $\lim_{x\to 6} f(x) = -5$ and $\lim_{x\to 6} g(x) = 4$, find

(a) $\lim_{x\to 6} [-3g(x)]$ (b) $\lim_{x\to 6} \sqrt{g(x)}$ (c) $\lim_{x\to 6} [g(x)/2f(x)]$

(iii) Find the value of each of the following:

(a) $\lim_{x\to 4} \dfrac{x^2-16}{(x-4)}$ (b) $\lim_{x\to 4} \dfrac{x^2-3x+2}{(x-1)}$

[5] **2.** Using the definition of the derivative $\lim_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$, find the derivative if $f(x) = 3 - x^3$.

[21] **3.** (a) If $f(x) = 2e - 5x^{16} - 3x^4$, find $f'(x)$.

(b) If $f(x) = \frac{x^2-9}{x^3+2x+5}$ find $f'(x)$.

(c) If $f(x) = \frac{x^2-3}{7ln(x)+3}$ find $f'(x)$.

(d) If $y = e^{(5-x^2)}$, then $y' = ?$

3. (Continued)

(e) If $y = \ln(5x^3 - 5)^3$, then $\dfrac{dy}{dx} = ?$

(f) If $y = \dfrac{1}{\sqrt[6]{x-7}}$, then $\dfrac{dy}{dx} = ?$

(g) Find y' given $x^2y = e^{2y} + 7$.

[8] **4.** Does the line tangent to the graph of $f(x) = e^x$ at $x = 1$ pass through the origin? Are there any other lines tangent to the graph of f that pass through the origin? Explain.

[6] **5.** A cube with a side of 12 centimeters is coated with ice 0.2 centimeter thick. Use differentials to estimate the volume of the ice.

[6] **6.** Profit analysis. The total profit (in dollars) from the sale of x charcoal grills is

$$P(x) = 20x - 0.02x^2 - 320\,, \quad 0 \le x \le 1{,}000$$

(a) Find the average profit per grill if 40 grills are produced.

(b) Find the marginal average profit at a production level of 40 grills, and interpret the results.

(c) Use the results from parts (a) and (b) to estimate the average profit per grill if 41 grills are produced.

[9] **7.** Suppose a point is moving along the graph of $x^2 + y^2 = 8$. When the point is at $(2, 2)$, its x coordinate is increasing at the rate of 0.3 units per second. How fast is the y coordinate changing at that moment?

[12] **8.** Compute the following:

(a) $\int (4x^2 - 7x)\, x\, dx$

(b) $\int \frac{1}{3+x^2}\, x\, dx$

(c) $\int (2x^3 + 7)^{18}\, x^2\, dx$

(d) $\int \frac{x}{\sqrt{x+7}}\, dx$

(e) $\int e^{3-5x}\, dx$

(f) $\int (x-3)^{-4}\, dx$

[7] **9.** Evaluate the following integrals [accurate to 2 decimals].

(a) $\int_1^2 e^{x^2}\, x\, dx$

(b) $\int_0^4 (t^2 + 3)\, dt$

[4] **10.** A note will pay \$25,000 at maturity 10 years from now. How much should you be willing to pay for the note now if money is worth 2%, compounded continuously?

[5] **11.** Find the area bounded by the graphs of $y = x^2 - 3x$; $y = 0, -2 \leq x \leq 2$.

[6] **12.** Find the interval(s) on which the graph of $f(x) = x^3 - 6x^2 + 9x + 1$ is concave upward, the interval(s) on which the graph of f is concave downward, and the inflection point(s).

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Course	Number	Section(s)
Mathematics	209	All
Examination	**Date**	**Pages**
Final	April 2009	3

Instructors	Course Examiner
S.T. Ali, F. Balogh, L. Dube, E. Duma, H. Greenspan, A. Kokotov, R. Mearns, M. Padamadan, J. Park, C. Santana	E. Cohen

Special Instructions

- ▷ Ruled booklets to be used.
- ▷ Calculators **not** allowed.

MARKS

[6] **1.** (a) Find the following limits

(i) $\lim_{x\to 2} \dfrac{3x^2+2x-1}{x^2+3x+2}$

(ii) $\lim_{x\to 2} \dfrac{2x^2-3x-2}{x^2+x-6}$

(b) Where is the function $n(x) = \dfrac{x-2}{(x-3)(x+1)}$ continuous?

[5] **2.** Find the derivative $f'(x)$ of the functions $f(x)$: (Do not simplify)

(a) f(x)= $3x^4 - 4x^3 + x - 2$

(b) f(x)= $\dfrac{x^{-7}}{7} + \sqrt[3]{x}$

[9] **3.** Find $\frac{dy}{dx}$ (do not simplify):

(a) $y = \frac{e^{2x}}{x^2 - 4}$

(b) $y = ln(3x^2 + 5)$

(c) $y = (2x^2 + 1)^3(4x + 6)^2$

(d) $y = (4 + x^2 lnx)^3$

[7] **4.** Let $f(x) = 3x^4 - 6x^2 - 7$

(a) Find the slope of the tangent line to the curve when $x = 2$

(b) Find the equation of the tangent line to the curve when $x = 2$

[13] **5.** Let $f(x) = (x - 2)(x^2 - 4x - 8)$

Find

(a) the critical and inflection points of $f(x)$

(b) the intervals where $f(x)$ is increasing and where it is decreasing

(c) the intervals on which $f(x)$ is concave up and on which it is concave down

(d) use the above to sketch the graph

[8] **6.** A student center sells 1600 cups of coffee per day at the price of \$2.40 per cup. A market survey shows that for every \$0.05 reduction in price per cup, 50 more cups of coffee will be sold.

How much should the student center charge for a cup of coffee in order to maximize revenue?

[7] **7.** Find the absolute extrema of the function $f(x) = x^3 - 6x^2 + 9x - 6$ on the interval $[-1, 5]$.

[3] **8.** If interest is compounded continuously and the interest rate is 6.4%, how long will it take for money invested to double?

[10] **9.** Find the equation(s) of the tangent line(s)to the graph of $y^2 - xy - 6 = 0$ at the point(s) with $x = 1$.

[10] **10.** Compute these antiderivatives:

(a) $\int (3x^5 - 2x^3 - 7)\, dx$

(b) $\int \dfrac{e^{-7x}}{2 + e^{-7x}}\, dx$

(c) $\int \dfrac{x^2}{\sqrt{x-3}}\, dx$

[10] **11.** Evaluate the integrals:

(a) $\displaystyle\int_0^1 (x^3 - 4)\, dx$

(b) $\displaystyle\int_2^7 \frac{1}{x-5}\, dx$

(c) $\displaystyle\int_4^6 \sqrt{x-3}\, dx$

[10] **12.** Find the area bounded by the graphs of $f(x) = x^2 - 1$ and $y = x - 2$ over the interval $-2 \leq x \leq 1$.

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Course	Number	Section(s)
Mathematics	209	All
Examination	**Date**	**Pages**
Final	December 2008	3
Instructors		**Course Examiner**
M. Amir, A. Atoyan, F. Balogh, L. Dube T. Hughes, R. Mearns, R. Raphael, J. Ruddy		R. Raphael

Special Instructions

- ▷ Ruled booklets to be used.
- ▷ **Only approved calculators are allowed.**

MARKS

[11] **1.** (i) Find $\lim\limits_{x\to\infty} \dfrac{4x^3-2x+5}{7x^3+5x^2+2}$

(ii) Given $\lim\limits_{x\to 3} f(x) = -5$ and $\lim\limits_{x\to 3} g(x) = 4$, find

(a) $\lim\limits_{x\to 3} [-2g(x)]$ (b) $\lim\limits_{x\to 3} [-3f(x)]$ (c) $\lim\limits_{x\to 3} [g(x) * f(x)]$

(iii) Find the value of each of the following:

(a) $\lim\limits_{x\to 5} \dfrac{x^2-25}{(x-5)}$ (b) $\lim\limits_{x\to 3} \dfrac{x^2-3x+2}{(x-1)}$

[5] **2.** Using the definition of the derivative $\lim\limits_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$, find the derivative if $f(x) = -2x^3 + 5$.

[14] **3.** Do not simplify answers.

(a) If $f(x) = 3x^{15} - 4x^3 + e$, find $f'(x)$.

(b) If $f(x) = (x^3 - 2x^2 + 1)(3x^2 - 7)$, find $f'(x)$.

(c) If $f(x) = \dfrac{x^2-1}{x^3+3}$, find $f'(x)$.

3. (Continued)

(d) If $y = e^{(x^3+7)}$, then $y' = ?$

(e) If $y = \ln[(3x^2+5)^4]$, then $\dfrac{dy}{dx} = ?$

(f) If $y = \dfrac{1}{\sqrt[5]{2x-1}}$, then $\dfrac{dy}{dx} = ?$

[3] **4.** Find y' given $x^2y^3 = 4x^2e^y + 5$.

[11] **5.** The total profit (in dollars) from the sale of x lawn mowers is

$$P(x) = 30x - 0.03x^2 - 750, \quad 0 \leq x \leq 1{,}000$$

(a) Find the average profit per mower if 50 mowers are produced.

(b) Find the marginal average profit at a production level of 50 mowers, and interpret the results.

(c) Use the results from parts (a) and (b) to estimate the average profit per mower if 51 mowers are produced.

[11] **6.** Suppose that for a company manufacturing transistor radios, the cost, revenue and profit equations are given by

$$C = 5{,}000 + 2x$$

$$R = 10x - 0.001x^2$$

where the production output in 1 week is x radios. If production is increasing at the rate of 500 radios per week when production is 2,000 radios, find the rate of increase per week in

(a) Cost;

(b) Revenue;

(c) Profit.

[11] **7.** Compute the following:

(a) $\int 3x^3(x^4+5)^3\,dx$

(b) $\int \sqrt[4]{x^3}\,dx$

(c) $\int \frac{5}{\sqrt{1-6x}}\,dx$

(d) $\int \frac{\ln(5x)}{x}\,dx$

(e) $\int (2x+x^2)\,e^{(3x^2+x^3)}\,dx$

[7] **8.** Use the price-demand equation to determine whether demand is elastic, is inelastic, or has unit elasticity at the indicated values of p for

$$x = f(p) = 1{,}875 - p^2$$

(a) $p = 15$ (b) $p = 25$ (c) $p = 40$

[4] **9.** How long will it take money to triple if it is invested at 5.5% compounded continuously?

[11] **10.** Find the area bounded by $f(x) = x^2 - x$ and $g(x) = 2x$, for $-2 \le x \le 3$.

[12] **11.** Let $f(x) = x^4 - 2x^3$.

(a) Find where $f(x)$ is increasing, is decreasing, and has local extrema.

(b) Find where $f(x)$ is concave up, is concave down, and has points of inflection.

(c) Graph $f(x)$.

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(c) If $f(x) = \dfrac{x^2-1}{x^3+3}$, find $f'(x)$.

3. (Continued)

(d) If $y = e^{(x^3+7)}$, then $y' = ?$

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