Solution Assignment #6
Summer 2016

13.37 / 13.39 / 10.5 / 10.16 / 10.31

13.37

Experimental rocket $M = 25\ kg$

Tair in tank = $125°C = T_0$

A converging section with exit area $25\ mm^2$ is attached to tank, $\dot{m} = 0.05\ kg/s$

a) P_{tank} ? Air ⇒ $k = 1.4$, $R = 287\ J/kg\,K$

b) P_e ? T_e ? V_e ?

c) Initial acceleration ?

b) Since back pressure = 0 ⇒ the flow is choked ⇒ $P_e = P^*$, $T_e = T^*$

T_e ⇒ we know that $\frac{T_0}{T^*} = \frac{k+1}{2}$ ⇒ $\boxed{T^* = T_e = 332\ K}$

V_e ⇒ $M_e = \frac{V_e}{c_e}$ where $M_e = 1$ since flow choked

⇒ $V_e = c_e = \sqrt{kRT_e} = \sqrt{1.4 \times 287 \times 332} = \boxed{365\ m/s = V_e}$

P_e ⇒ we know for ideal gas ⇒ $P_e = \overset{?}{\rho_e} R T_e$

ρ_e ? we know $\dot{m} = 0.05\ kg/s = \rho_e V_e A_e$

⇒ $\rho_e = \frac{\dot{m}}{V_e A_e} = \frac{0.05}{365 \times 25} = 0.0548\ kg/m^3$

⇒ $P_e = 0.0548 \times 287 \times 332$

⇒ $\boxed{P_e = 5.21\ kPa}$

a) $\frac{P_0}{P^*} = \frac{P_0}{P_e} = \left(\frac{k+1}{2}\right)^{\frac{k}{k-1}} = \left(\frac{1.4+1}{2}\right)^{\frac{1.4}{1.4-1}}$ ⇒ $\boxed{9.87\ kPa = P_0}$

c) $\underset{\text{surface }P}{\vec{F}} - \int_{CV} \vec{a}_{rf}\, \rho\, dV = \cancel{\frac{\partial}{\partial t}\int_{VC} \vec{V}_{xyz}\, \rho\, dV} + \int_{CS} \vec{V}_{xyz}\, \rho\, \vec{V}_{xyz} \cdot d\vec{A}$ (momentum conservation)

⇒ $P_e A - M a_x = \dot{m} V_e$

⇒ $a_x = \frac{\dot{m} V_e + P_e A}{M} = \boxed{1.25\ m/s^2 = a_x}$

13.39

Hydrogen ⇒ adiabatically without friction from :

$P_0 = 100$ psi ⇒ converging diverging Nozzle ⇒ $P_e = 20$ psia

$T_0 = 540°F$

$V_0 = 0$ M_e ?

For Hydrogen $k = 1{,}41$ $T_0 = 540°F = 1000°R$

$R = 766{,}5 \dfrac{lbf\, ft}{lbm\, R°}$ ⇒ convert T_0

⇒ we know that $\dfrac{P_0}{P} = \left(1 + \dfrac{k-1}{2} M^2\right)^{\frac{k}{k-1}}$

$$M_e = \sqrt{\frac{2}{k-1}\left(\left(\frac{P_0}{P}\right)^{\frac{k-1}{k}} - 1\right)} \Rightarrow M_e = 1{,}71$$

10,5

Dimensions of centrifugal pump impeller :

	①	②
r	15 in	45 in
b.	4,75 in	3,25 in
β	40°	60°

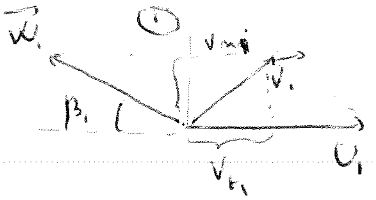
ω = 575 rpm , water

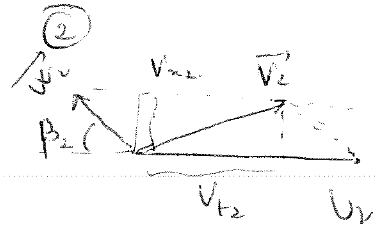
calculate H? and $\dot{W}_m$? if Q = 80000 gpm

we know : $\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m}$

$$H = \frac{\dot{W}_m}{\dot{m} g} = \frac{1}{g}\left(U_2 V_{t_2} - U_1 V_{t_1}\right)$$

Using the velocity diagrams :





$$V_n = \frac{Q}{2\pi rb} \quad (\text{continuity}) \Rightarrow Q = 2\pi rb \times V_n$$

geometry : $\vec{W_1} = \vec{V_1} - \vec{U_1}$

$$\Rightarrow U - V_t = W\cos\beta \qquad V_n = W\sin\beta \quad \Bigg\} \Rightarrow \frac{U - V_t}{V_n} = \cot\beta$$

$$\Rightarrow V_t = -\cot\beta \times V_n + U$$

$$V_t = \frac{-Q}{2\pi rb}\cot\beta + U$$

we know $U = r\omega$ where $\omega = 575\,rpm = 60{,}21\,rd/s$

$U_1 = 75{,}3\,ft/s \qquad U_2 = 226\,ft/s$

$Q = 178\,ft^3/s$ = from $(80000\,gpm)$

$\Rightarrow V_{t_1} = 6{,}94\,ft/s$

$V_{t_2} = 210\,ft/s$

$\Rightarrow \dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\rho Q \quad \rho_{water} = 1{,}94\,slug/ft^3$

$\Rightarrow \boxed{\dot{W}_m = 1{,}62\,10^7\,\frac{ft\,lbf}{s} = 2{,}94\,10^4\,hp}$

$H = \frac{\dot{W}_m}{\dot{m}g} = 1455\,ft$

10.16.

$\omega = 1200\,rpm$ centrifugal pump

	①	②
radius r	90 mm	150 mm
h	10 mm	7,5 mm
β	25°	45°

a) Q? when $V_{t_1} = 0$

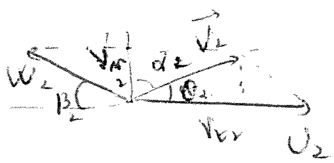
b) Draw outlet velocity diagram ? outlet absolute flow angle ?

c) $\dot{W}_h$ if $\eta = 0{,}7$? and H ?

$$\dot{W}_m = \left(U_2 V_{t_2} - U_1 V_{t_1}\right)\dot{m}$$

$$H = \frac{\dot{W}_m}{\dot{m}\, g} = \frac{1}{g}\left(U_2 V_{t_2} - U_1 V_{t_1}\right)$$

a) ②



continuity: $Q = V_n \times 2\pi r b \Rightarrow V_n = \dfrac{Q}{2\pi r b}$

$$\left.\begin{aligned} U - V_t &= W\cos\beta \\ V_n &= W\sin\beta \end{aligned}\right\} \Rightarrow V_t = U - V_n \cot\beta$$

For ① $\Rightarrow V_{t_1} = 0 \Rightarrow U_1 - V_{n_1}\cot\beta_1 = 0$

$$\Rightarrow \underset{r_1\omega}{U_1} - \frac{Q}{2\pi r_1 b_1}\cot\beta_1 = 0 \Rightarrow \cancel{Q = 2\pi r_1^2 b_1 \omega}$$

$$Q = \frac{U_1 \times 2\pi r_1 b_1}{\cot(\beta_1)} = r_1\omega \times 2\pi r_1 b \times tg(\beta_1)$$

$\omega = 1200\,rpm = 125{,}66\ rad/s$

$$\Rightarrow \boxed{Q = 2{,}98\,10^{-2}\ m^3/s}$$

b) $\alpha_2 = tg^{-1}\left(\dfrac{V_{t2}}{V_{n2}}\right)$

$$V_{n2} = \frac{Q}{2\pi r_2 b_2} = 4{,}22\ m/s$$

$$V_{t_2} = U_2 - V_{n_2}\cot(\beta_2) = r_2\omega - V_{n_2}\cot(\beta_2) = 14{,}6\ m/s$$

$$\Rightarrow \alpha_2 = tg^{-1}\left(\frac{14{,}6}{4{,}22}\right) = \boxed{73{,}9^\circ = \alpha_2}$$

$/m^3$ for water

$$\dot{W}_m = \left(U_2 V_{t_2} - \cancel{U_1 V_{t_1}}\right) \rho Q = \underbrace{U_2}_{r_2\omega} V_{t_2} \rho Q$$

$$\Rightarrow \boxed{\dot{W}_m = 8{,}22\,10^3\ W}$$

we know $\dfrac{\dot{W}_h}{\dot{W}_m} = \eta \Rightarrow \dot{W}_h = \eta\, \dot{W}_m = 0{,}7 \times 8{,}22\,10^3 = \boxed{5{,}75\,10^3\ W = \dot{W}_h}$

$$H = \frac{1}{g}\left(\underbrace{U_2}_{r_2\omega} V_{t_2}\right) = 28{,}1\ m$$

$$\eta = \frac{H_p}{H} \Rightarrow H_p = H \times \eta = 28{,}1 \times 0{,}7 \Rightarrow \boxed{H_p = 19{,}7\ m}$$

<u>10,31</u>

small centrifugal pump $\underset{\omega}{\dot{N}} = 2875\ rpm$ ($\Rightarrow$ rad/s) with water

$Q = 0{,}016\ m^3/s$ and $H = 40\ m$ at its best efficiency $\eta = 0{,}7$

a) N_s? Sketch?

b) $\dot{W}_m$?

a) $N_s = \dfrac{\omega\, Q^{1/2}}{h^{3/4}}$ where $h = gH$ $\quad \omega = 2875\ rpm = 301{,}07\ rd/s$

$$N_s = \frac{\omega\, Q^{1/2}}{gH} = \boxed{0{,}432 = N_s}$$

$\Rightarrow$ impeller will be centrifugal

b) $\dot{W}_m$?

$$\eta = \frac{\dot{W}_h}{\dot{W}_m} \Rightarrow \dot{W}_m = \frac{\dot{W}_h}{\eta} = \frac{\rho Q g H}{\eta} \qquad \rho_{water} = 1000\ kg/m^3$$

$$\Rightarrow \boxed{\dot{W}_m = 8{,}97\,10^3\ W}$$