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Solution Assignment #5
Summer 2016

9.10 / 9.11 / 9.16 / 9.17 / 9.18 / 9.19 / 9.40 / 9.78 / 9.81 / 9.90 / 12.23 / 12.27 /
12.28 / 12.30 / 12.31 / 12.68 / 12.70 / 13.13

* 9.10

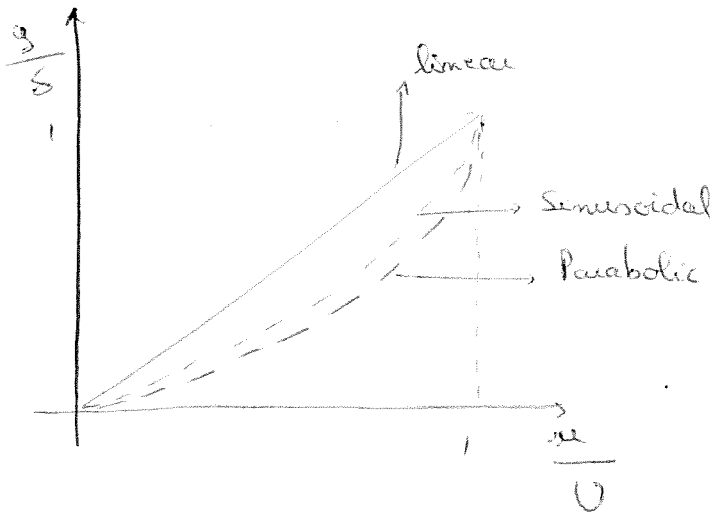
Velocity profiles in laminar BL =

$$\text{Linear} : \frac{u}{U} = \frac{y}{\delta}$$

$$\text{Semi-sinoidal} : \frac{u}{U} = \sin\left(\frac{\pi}{2} \frac{y}{\delta}\right)$$

$$\text{Parabolic} : \frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

Plot $\frac{y}{\delta}$ vs $\frac{u}{U}$ and compare.



9.11

Velocity profile in laminar BL is :

$$\frac{u}{U} = \frac{3}{2} \frac{y}{\delta} - \frac{1}{2} \left(\frac{y}{\delta}\right)^3$$

a) Satisfy BC?

b) Evaluate $\frac{\delta^*}{\delta}$ and $\frac{\theta}{\delta}$

a) BC für laminaire BL:

$$u(0) = 0 \text{ and } \frac{du}{dy} \Big|_{y=\delta} = 0$$

(2)

wir know that $u(y) = U \left(\frac{3}{2} \frac{y}{\delta} - \frac{1}{2} \left(\frac{y}{\delta} \right)^3 \right)$

$$u(0) = U(0 - 0) = 0 \quad \checkmark$$

$$\frac{du}{dy} = U \left(\frac{3}{2} \times \frac{1}{\delta} - \frac{3}{2} \frac{y^2}{\delta^3} \right)$$

$$\frac{du}{dy} \Big|_{y=\delta} = U \left(\frac{3}{2} \times \frac{1}{\delta} - \frac{3}{2} \frac{\delta^2}{\delta^3} \right) = U \left(\frac{3}{2\delta} - \frac{3}{2\delta} \right) = 0 \quad \checkmark$$

Satisfy BC \checkmark

$$b) \delta^* = \int_0^{\infty} \left(1 - \frac{u}{U} \right) dy = \int_0^{\delta} \left(1 - \frac{u}{U} \right) dy$$

$$\Rightarrow \delta^* = \int_0^{\delta} \left(1 - \frac{3}{2} \times \frac{y}{\delta} + \frac{1}{2} \left(\frac{y}{\delta} \right)^3 \right) dy$$

$$\Rightarrow \delta^* = \left[y - \frac{3}{4} \frac{y^2}{\delta} + \frac{1}{2 \times 4} \frac{y^4}{\delta^3} \right]_0^{\delta}$$

$$\Rightarrow \delta^* = \delta - \frac{3}{4} \frac{\delta^2}{\delta} + \frac{1}{8} \frac{\delta^4}{\delta^3} = \delta - \frac{3}{4} \delta + \frac{1}{8} \delta = \frac{3}{8} \delta$$

$$\Rightarrow \boxed{\frac{\delta^*}{\delta} = \frac{3}{8}}$$

$$\Theta = \int_0^{\infty} \frac{u}{U} \left(1 - \frac{u}{U} \right) dy = \int_0^{\delta} \frac{u}{U} \left(1 - \frac{u}{U} \right) dy$$

$$\Rightarrow \Theta = \int_0^{\delta} \left(\frac{3}{2} \frac{y}{\delta} - \frac{1}{2} \left(\frac{y}{\delta} \right)^3 \right) \left(1 - \frac{3}{2} \frac{y}{\delta} + \frac{1}{2} \left(\frac{y}{\delta} \right)^3 \right) dy$$

$$= \int_0^{\delta} \frac{3}{2} \frac{y}{\delta} - \frac{9}{4} \frac{y^2}{\delta^2} + \frac{3}{4} \frac{y^4}{\delta^4} - \frac{1}{2} \left(\frac{y}{\delta} \right)^3 + \frac{3}{4} \frac{y^4}{\delta^4} - \frac{1}{4} \frac{y^6}{\delta^6} dy$$

$$= \int_0^{\delta} \frac{3}{2} \frac{y}{\delta} - \frac{9}{4} \frac{y^2}{\delta^2} + \frac{3}{2} \frac{y^4}{\delta^4} - \frac{1}{2} \left(\frac{y}{\delta} \right)^3 - \frac{1}{4} \frac{y^6}{\delta^6} dy$$

$$= \left[\frac{3}{4} \frac{y^2}{\delta} - \frac{9}{4 \times 3} \frac{y^3}{\delta^2} + \frac{3}{2 \times 5} \frac{y^5}{\delta^4} - \frac{1}{2 \times 4} \frac{y^4}{\delta^3} - \frac{1}{4 \times 7} \frac{y^7}{\delta^6} \right]_0^{\delta}$$

$$= \frac{3}{4} \delta - \frac{3}{4} \delta + \frac{3}{10} \delta - \frac{1}{8} \delta - \frac{1}{28} \delta \Rightarrow \boxed{\frac{\Theta}{\delta} = 0,139}$$

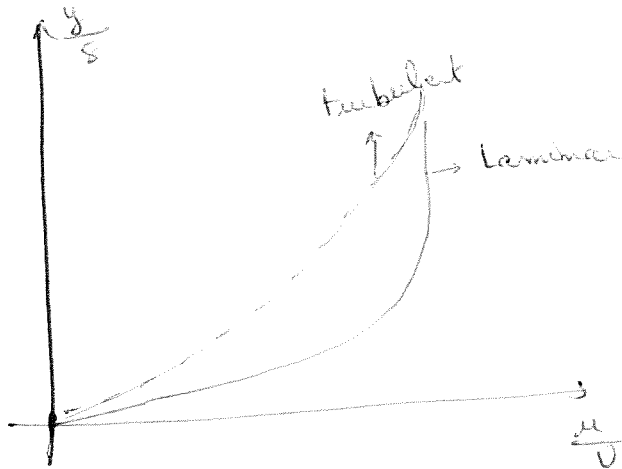
9.14

3

Velocity profile turbulent BL:

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$$

Compare shape of profile with laminar BL: $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$
 by plotting y/δ vs. u/U



$$\Rightarrow \frac{du}{dy} \rightarrow \infty \text{ when } y \rightarrow 0$$

9.17

 $\frac{\delta^4}{\delta}$ and $\frac{\theta}{\delta}$ for turbulent $1/7$ power law profile

compare with laminar values.

- turbulent

$$\delta^4 = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy = \int_0^{\delta} \left(1 - \frac{y^{1/7}}{\delta^{1/7}}\right) dy = \left[y - \frac{1}{\frac{1}{7}+1} \frac{y^{1/7+1}}{\delta^{1/7}} \right]_0^{\delta}$$

$$= \delta - \frac{7}{8} \frac{\delta^{8/7}}{\delta^{1/7}} = \delta - \frac{7}{8} \delta = \frac{\delta^8}{8} \Rightarrow \frac{\delta^4}{\delta} = 1 - \frac{7}{8} = \frac{1}{8} = \boxed{\frac{0.125 = \delta^4}{\delta}}$$

$$\theta = \int_0^{\delta} \frac{u}{U} \left(1 - \frac{u}{U}\right) dy = \int_0^{\delta} \frac{y^{1/7}}{\delta^{1/7}} \left(1 - \frac{y^{1/7}}{\delta^{1/7}}\right) dy = \int_0^{\delta} \frac{y^{1/7}}{\delta^{1/7}} - \frac{y^{2/7}}{\delta^{2/7}} dy$$

$$\theta = \left[\frac{7}{8} \frac{y^{8/7}}{\delta^{1/7}} - \frac{1}{\frac{2}{7}+1} \frac{y^{2/7+1}}{\delta^{2/7}} \right]_0^{\delta} = \frac{7}{8} \delta - \frac{7}{9} \delta = \frac{\theta}{\delta} = \frac{7}{8} - \frac{7}{9} = \boxed{\frac{0.0972 = \theta}{\delta}}$$

Laminar: (same analysis with $\frac{\mu}{U} = \frac{2\gamma}{\delta} - \left(\frac{\gamma}{\delta}\right)^2$)

(4)

$$\frac{\delta^*}{\delta} = 0,33$$

$$\frac{\theta}{\delta} = 0,133$$

turbulent

Laminar

$$\frac{\delta^*}{\delta}$$

$$0,125$$

$$0,33$$

$$\frac{\theta}{\delta}$$

$$0,0972$$

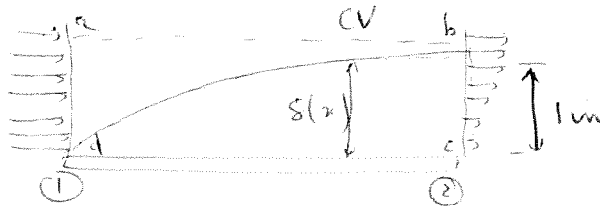
$$0,133$$

9.18

Fluid $\rho = 1,7 \text{ slug/ft}^3$ $U = 10 \text{ ft/s}$ $L = 10 \text{ ft}$, $b = 3 \text{ ft}$ $S_{bc} = L = 1 \text{ in}$
 Velocity profile linear

a) \dot{m} in control system ?

b) Drag force on upper surface ? How this drag can be computed from given data



- a) Assumptions:
- * steady flow
 - * No pressure force
 - * uniform flow at ①

Mass conservation: $0 = \frac{d}{dt} \int_{CV} \rho dV + \int_{sc} \rho \vec{V} \cdot d\vec{A}$

$$\Rightarrow -\rho U b S + \int_0^S \rho u b dy + \dot{m}_{ab} = 0$$

Velocity profile linear $= \frac{u}{U} = \frac{y}{S}$

$$\Rightarrow -\rho U b S + \left[\frac{\rho U y^2}{2 S} b \right]_0^S + \dot{m}_{ab} = 0$$

$$\Rightarrow \dot{m}_{ab} = \rho U b S - \frac{\rho}{2} S b U = \frac{\rho S b U}{2} = 1,7 \times \frac{1}{12} \times \frac{3 \times 10}{2} = \boxed{1,875 \frac{\text{slug}}{\text{s}} = \frac{\dot{m}}{s}}$$

b) Drag?

5

$$\vec{F} = \frac{d}{dt} \int_{V_c} \rho \vec{V} dV + \int_{S_c} \rho \vec{V} \vec{V} \cdot d\vec{A}$$

$$u = U \frac{y}{\delta}$$

$$R_x = (-\rho U b \delta) U + \dot{m}_{ab} U + \int_0^{\delta} (\rho u b) u dy$$

$$= -\rho U^2 b \delta + \dot{m}_{ab} U + \int_0^{\delta} \rho U \frac{y^2}{\delta^2} b dy$$

$$= -\rho U^2 b \delta + \dot{m}_{ab} U + \left[\frac{\rho U^2 b}{\delta^2} \times \frac{1}{3} y^3 \right]_0^{\delta}$$

$$= -\rho U^2 b \delta + \dot{m}_{ab} U + \frac{\rho U^2 b}{3} \delta$$

$$\boxed{R_x = -6,75 \text{ lbf}}$$

9.19

turn over plate in 9.18 so $L = 3 \text{ ft}$ $b = 10 \text{ ft}$

a) Drag increases or decreases, why?

b) $\delta|_{x=L} = 0,6 \text{ in}$, repeat analysis of 9.18 as \dot{m} and Drag.

Same assumptions as in 9.18

b) $\dot{m}_{ab} = \frac{1}{2} \rho U b \delta = 3,75 \text{ slug/s}$ (from mass conservation as previously)

$$R_x = -\rho U^2 b \delta + \dot{m}_{ab} U + \frac{\rho U^2 b \delta}{3} = -12,5 \text{ lbf}$$

a) Drag is larger than for problem 9.18 as expected since the viscous friction is near the leading edge so (3 ft in 9.18 and 10 ft in 9.19)

The BL is small so $\tau = \mu \frac{\partial u}{\partial y} = \mu \frac{4U}{\delta}$ is large.

9.40

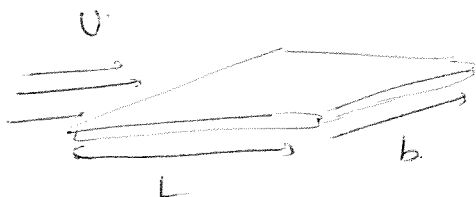
6

Thin flat plate installed in water tunnel as a splitter

$$U = 1,6 \text{ m/s} \quad L = 0,3 \text{ m} \quad b = 1 \text{ m}$$

Parabolic laminar BL

Determine viscous drag on plate (no pressure drag)?



$$\rho = 1000 \text{ kg/m}^3$$

water

$$\mu = 10^{-3} \text{ Pa}\cdot\text{s}$$

The flow is laminar but calculate Re

$$Re|_{x=L} = \frac{\rho U L}{\mu} = \frac{1000 \times 1,6 \times 0,3}{10^{-3}} = 4,8 \cdot 10^5 \text{ so laminar } \checkmark$$

viscous drag $\Rightarrow F_D = \int_0^L \tau_w b dx$ for one side

$$\Rightarrow F_{D \text{ total}} = 2 \int_0^L \tau_w b dx$$

$$c_f = \frac{\tau_w}{\frac{1}{2} \rho U^2}$$

$$\text{and } \tau_w = \mu \frac{du}{dy} = \frac{24U}{S} \quad \text{where } S = \frac{5,48 x}{Re^{1/2}}$$

$$\Rightarrow F_{D \text{ total}} = 2b \int_0^L \frac{24U}{4^{1/2}} \times \frac{\rho U^{1/2} x^{1/2}}{5,48 x} dx = \frac{4bU^{3/2} \rho^{1/2}}{4^{1/2} 5,48} \left[\frac{2}{3} x^{3/2} \right]_0^L$$

$$F_{D \text{ total}} = \frac{4bU^{3/2} \rho^{1/2}}{5,48 \cdot 4^{1/2}} \times 2L^{3/2} \Rightarrow \boxed{F_{D \text{ total}} = 1,617 \text{ N}}$$

9.78

7

Flat bottomed barge $L=80\text{ft}$, $b=35\text{ft}$, $d=5\text{ft}$ $T=60^\circ\text{F} \Rightarrow \text{water}$ Power required to overcome drag for $V_{\text{max}}=15\text{mph}$?

P?

$$F_D? \Rightarrow F_D = C_D \times \frac{1}{2} \rho V^2 A_{\text{wetted}}?$$

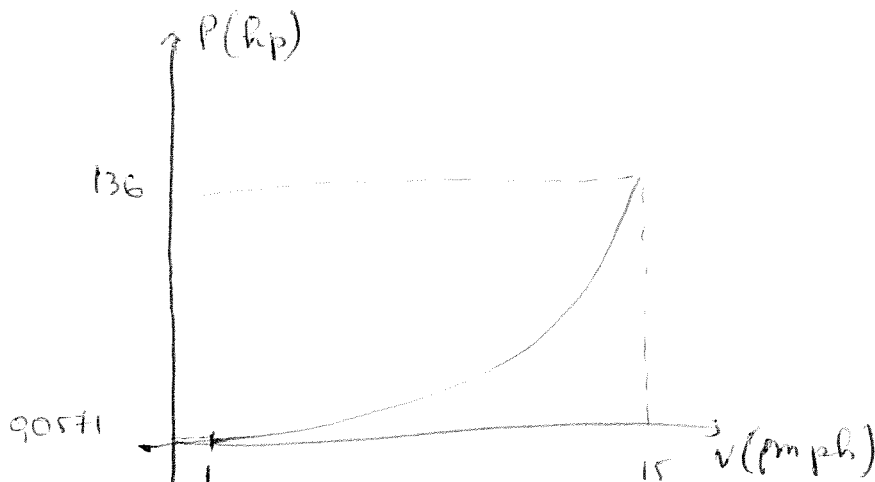
$$A_{\text{wetted}} = L(b + 2d) = 3600\text{ft}^2$$

(Table A.7) $T=60^\circ\text{F} \Rightarrow \rho = 1.94\text{ slug/ft}^3$, $\nu = 1.321 \times 10^{-5}\text{ ft}^2/\text{s}$

$$\Rightarrow C_D? \Rightarrow \text{check } Re \Rightarrow Re = \frac{\rho V L}{\mu} = \frac{\rho V L}{\rho \nu} = \frac{V L}{\nu} = 1.45 \times 10^3$$

$$\Rightarrow C_D = \frac{0.477}{(\log Re)^{2.58}} - \frac{1610}{Re}$$

Power = $F_D \times V = C_D \times \frac{1}{2} \rho V^3 \times L(b + 2d)$ where $C_D = \frac{0.477}{(\log \frac{V L}{\nu})^{2.58}} - \frac{1610}{\frac{V L}{\nu}}$
 for V varying from 1mph to $15\text{mph} \Rightarrow$ Powers from 0.0571 hp to 136 hp



9,81

8

Jet transport aircraft $H = 12 \text{ km}$, $V = 800 \text{ km/h}$ circular cylinder $D = 4 \text{ m}$, $L = 38 \text{ m}$

Skin friction drag and Power to overcome drag

$$C_D = \frac{F_D}{\frac{1}{2} \rho V^2 A}$$

Assume flat ~~plate~~ plate with Area $A = L \pi D$

$$\text{Pa at } H = 12 \text{ km} \Rightarrow \frac{P}{P_{SL}} = 0,2766 \text{ where } P_{SL} = 1,225 \text{ kg/m}^3$$

$$\Rightarrow \rho = 0,3119 \text{ kg/m}^3 \quad T = 216,7 \text{ K}$$

$$\mu? \text{ use equation } \mu(T) \Rightarrow \mu = \frac{b T^{1/2}}{1 + \frac{S}{T}} \quad b = 1,458 \cdot 10^{-6} \text{ kg/m} \cdot \text{K}^{1/2} \quad S = 110,4 \text{ K}$$

$$\Rightarrow \mu = 1,42 \cdot 10^{-5} \text{ Pa} \cdot \text{s}$$

$$Re = \frac{\rho V L}{\mu} = 1,87 \cdot 10^8 \Rightarrow \text{turbulent} \Rightarrow C_D = \frac{0,455}{(\log Re_L)^{1,58}} = 0,00196$$

$$\Rightarrow \boxed{F_D = 7189 \text{ W}}$$

$$\Rightarrow \text{Power} = F_D \times V = \boxed{1,598 \cdot 10^6 \text{ W} = \text{Power}}$$

9.90

American flag \Rightarrow $H = 194 \text{ ft}$ high, $W = 367 \text{ ft}$ wide \Rightarrow no holes

$$V = 10 \text{ mph} = 14,67 \text{ ft/s}$$

Estimate the wind force on flag \Rightarrow should they have been surprised?

$$\rho_{air} = 0,00234 \frac{\text{slug}}{\text{ft}^3} \quad \nu = 1,62 \cdot 10^{-4} \frac{\text{ft}^2}{\text{s}}$$

$$Re? \Rightarrow \frac{\rho V L}{\mu} = \frac{V L}{\nu} = \frac{14,67 \times 367}{1,62 \cdot 10^{-4}} = 3,32 \cdot 10^7$$

$$F_D = C_D \times \frac{1}{2} \rho V^2 A \Rightarrow A = H \times W$$

$$C_D \Rightarrow \text{fig 9.10} \Rightarrow C_D = 1,15$$

$$\Rightarrow \boxed{F_D = 2,06 \cdot 10^4 \text{ lbf}} \Rightarrow \text{very large} = \text{building should have been expected!}$$

12.23

9

speed of sound at 20° for:

- a) hydrogen,
- b) helium
- c) methane
- d) nitrogen
- e) carbon dioxide

a) $c = \sqrt{kRT}$ $T = 20 + 273 = 293 \text{ K}$

$k_{\text{hydrogen}} = 1,41$
 $R = 4124 \frac{\text{J}}{\text{kg K}}$ $\Rightarrow c = \sqrt{kRT} = 1305 \text{ m/s}$

b) $k_{\text{helium}} = 1,66$
 $R_{\text{helium}} = 2077 \frac{\text{J}}{\text{kg K}}$ $\Rightarrow c = \sqrt{kRT} = 1005 \text{ m/s}$

c) $k_{\text{methane}} = 1,31$
 $R_{\text{methane}} = 518,3 \frac{\text{J}}{\text{kg K}}$ $\Rightarrow c = \sqrt{kRT} = 446 \text{ m/s}$

d) $k_{\text{nitrogen}} = 1,4$
 $R_{\text{nitrogen}} = 296,8 \frac{\text{J}}{\text{kg K}}$ $\Rightarrow c = \sqrt{kRT} = 349 \text{ m/s}$

e) $k_{\text{carbon dioxide}} = 1,29$
 $R_{\text{carbon dioxide}} = 188,9 \frac{\text{J}}{\text{kg K}}$ $\Rightarrow c = \sqrt{kRT} = 267 \text{ m/s}$

12.27

Submarine sends a sonar signal to detect enemy.

\Rightarrow Reflected wave returns after 3,27 s

Separation between submarines? for $T = 20^\circ$ in seawater

\Rightarrow we know that $t = 3,27 \text{ s} \Rightarrow d?$ $V = \frac{d}{t}$

$\Rightarrow c = V$ for water we know that $c = \sqrt{\frac{E_r}{\rho}}$

At $T = 20^\circ$ in seawater: $\rho = 1,025$, $E_r = 2,42 \frac{\text{GN}}{\text{m}^2}$ $\rho_{\text{water}} = 1000 \frac{\text{kg}}{\text{m}^3}$

$$c = \sqrt{\frac{E_r}{\rho \text{ water}}} = \sqrt{\frac{2,4210^9}{1000 \times 1025}} = 1537 \text{ m/s}$$

(10)

$$d = ct \approx 5000 \text{ m}$$

Since the wave is reflected \Rightarrow the distance between both submarines is $d = 2500 \text{ m}$

12.28

Airplane $\Rightarrow V = 550 \text{ km/hr}$ at $H = 1500 \text{ m}$ on standard day (1)

Now $\Rightarrow V = 1200 \text{ km/hr}$ at $H = 15000 \text{ m}$ (2)

Mach number for (1) and (2)?

(1) $\Rightarrow H = 1500 \text{ m} \Rightarrow T = 278,4 \text{ K}$, $V = 550 \text{ km/hr} = 152,778 \text{ m/s}$

$$\Rightarrow M = \frac{V}{c} = \frac{V}{\sqrt{kRT}} \quad \text{Air} \Rightarrow k = 1,4 \text{ and } R = 287 \frac{\text{J}}{\text{kgK}}$$

$$\Rightarrow M_1 = 0,477$$

(2) $\Rightarrow H = 15000 \text{ m} \Rightarrow T = 216,7 \text{ K}$, $V = 1200 \text{ km/hr} = 333,33 \text{ m/s}$

$$\Rightarrow M = \frac{V}{\sqrt{kRT}} = 1,13 = M_2$$

12.30

Aircraft performance never released but it was thought to cruise at $M = 3,3$ at $H = 85000 \text{ ft}$

Speed of sound? flight speed? compare to muzzle speed of 30-06 rifle bullet (700 m/s)?

(a) $H = 85000 \text{ ft} \Rightarrow T = 222 \text{ K}$ Air $\Rightarrow k = 1,4$ $R = 287 \frac{\text{J}}{\text{kgK}}$

$$\Rightarrow c = \sqrt{kRT} = \sqrt{1,4 \times 287 \times 222} = 299 \text{ m/s}$$

$$\Rightarrow M = \frac{V}{c} \Rightarrow V = M \times c = 3,3 \times 299 = 987 \text{ m/s}$$

$$\Rightarrow \frac{V_{\text{aircraft}}}{V_{\text{bullet}}} = \frac{987}{700} = 1,41 \Rightarrow \text{larger than } V_{\text{bullet}}$$

12.31

11

Boeing 727 aircraft of Ex. 9.8 $\Rightarrow V = 520 \text{ mph}$ at 33000 ft on standard day

a) M ?

b) If Mass Mach number is 0.9, what is V ?

$$a) M = \frac{V}{c} = \frac{V}{\sqrt{kRT}}$$

$$\textcircled{a} H = 33000 \text{ ft} \Rightarrow T = 223 \text{ K} \quad \text{air } k = 1.4, R = 287 \frac{\text{J}}{\text{kg K}}$$

$$\Rightarrow c = \sqrt{kRT} = 299 \text{ m/s}$$

$$\Rightarrow V = 520 \text{ mph} = 231 \text{ m/s}$$

$$\Rightarrow M = \frac{231}{299} = \boxed{0.776 = M}$$

$$b) \text{ for } M = 0.9 = \frac{V}{c} \Rightarrow V = c \times M = 299 \times 0.9 = \boxed{269 \text{ m/s} = V}$$

12.68

Anglo-French conic superersonic transport $\Rightarrow M = 2.2$ at 20 km altitude.

a) c ? V ? and α ?

b) Maximum air temperature at stagnation point on aircraft?

$$a) c = \sqrt{kRT}$$

$$\textcircled{a} H = 20 \text{ km} \Rightarrow T = 216.7 \text{ K} \quad \text{Air } \Rightarrow k = 1.4, R = 287 \frac{\text{J}}{\text{kg K}}$$

$$\Rightarrow c = \sqrt{1.4 \times 286.9 \times 216.7} = \boxed{295 \text{ m/s} = c}$$

$$\Rightarrow M = \frac{V}{c}$$

$$\Rightarrow V = M \cdot c = 2.2 \times 295 = \boxed{649 \text{ m/s} = V}$$

$$\text{we know for supersonic Mach cone } \Rightarrow \alpha = \sin^{-1}\left(\frac{1}{M}\right) = \sin^{-1}\left(\frac{1}{2.2}\right) = \boxed{27^\circ = \alpha}$$

b) we know that T_0 stagnation temperature:

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2 \Rightarrow T_0 = T \left(1 + \frac{k-1}{2} M^2\right) = 216.7 \left(1 + \frac{1.4-1}{2} 2.2^2\right) = \boxed{426 \text{ K} = T_0}$$

Supersonic tunnel test section designed to flow $M = 2.7$ @ $T = 11^\circ\text{C}$ and $P = 37\text{ kPa (abs)}$

⇒ Air

a) T_0 ? P_0 ?

b) Mass flow rate for test section area of $0.175\text{ m}^2 = A$

$$\text{a) we know } \frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$$

$$\frac{P_0}{P} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{k}{k-1}}$$

$$T_0 = T \left(1 + \frac{k-1}{2} M^2\right) \quad T = (11 + 273)\text{ K} = 288\text{ K}$$

$$\Rightarrow T_0 = 288 \left(1 + \frac{1.4-1}{2} 2.7^2\right) = \boxed{648\text{ K} = T_0}$$

$$P_0 = P \left(1 + \frac{k-1}{2} M^2\right)^{\frac{k}{k-1}} = 3710^3 \left(1 + \frac{1.4-1}{2} 2.7^2\right)^{\frac{1.4}{1.4-1}}$$

$$\Rightarrow \boxed{P_0 = 598\text{ kPa}}$$

$$\text{b) } \dot{m} = \rho VA \quad V? \quad \rho? \quad k = 1.4 \quad R = 287 \frac{\text{J}}{\text{kg K}} (\text{air})$$

$$M = \frac{V}{c} \Rightarrow V = M \times c \quad c = \sqrt{\gamma R T}$$

$$\Rightarrow V = 2.7 \times \sqrt{1.4 \times 287 \times 288} = 850\text{ m/s}$$

$$P = \rho R T \Rightarrow \rho = \frac{P}{R T} = \frac{3710^3}{287 \times 288} = 0.424\text{ kg/m}^3$$

$$\Rightarrow \dot{m} = 0.424 \times 850 \times 0.175 = \boxed{63 \frac{\text{kg}}{\text{s}} = \dot{m}}$$

Air at (1)

$k = 1.4$

$P_1 = 60 \text{ kPa (abs)}$

$R = 287 \text{ J/kg K}$

$T_1 = 27^\circ\text{C}$

$V_1 = 486 \text{ m/s}$

$A_1 = 0.02 \text{ m}^2$

(2)

$P = 78.8 \text{ kPa (abs)}$

$M_2 ?$

Assuming isentropic flow

Sketch passage?

since isentropic $\Rightarrow P_{01} = P_{02}$

we know $\Rightarrow \frac{P_0}{P} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{k}{k-1}}$ $M_1 ? \Rightarrow M_1 = \frac{V_1}{C_1} = \frac{V_1}{\sqrt{kRT}} = \frac{486}{\sqrt{1.4 \times 287 \times 27 + 273}} = \frac{486}{347 \text{ m/s}} = 1.4$

$\frac{P_{01}}{P_1} = \left(1 + \frac{k-1}{2} M_1^2\right)^{\frac{k}{k-1}} \Rightarrow P_{01} = P_1 \left(1 + \frac{k-1}{2} M_1^2\right)^{\frac{k}{k-1}} \Rightarrow P_{01} = 6010^3 \left(1 + \frac{1.4-1}{2} 1.4^2\right)^{\frac{1.4}{1.4-1}}$

$\Rightarrow P_{01} = 19110^3 \text{ Pa}$

$P_{01} = P_{02} = P_2 \left(1 + \frac{k-1}{2} M_2^2\right)^{\frac{k}{k-1}} \Rightarrow M_2 = \sqrt{\frac{2}{k-1} \left(\left(\frac{P_{01}}{P_2} \right)^{\frac{k-1}{k}} - 1 \right)} = \boxed{1.2 = M_2}$

$M_1 = 1.4$
 $M_2 = 1.2$ \Rightarrow fluid being slowed down and $M > 1 \Rightarrow$ supersonic diffuser

