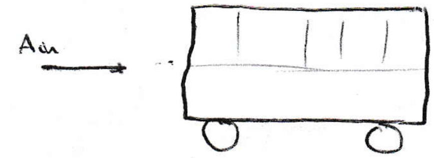
Solution Assignment #4
Summer 2016

9.112 / # 9.125 / # 9.165 / x foil

9.112



A bus travel at 80 km/hr in Standard air.

$A_{frontal} = 7{,}5\ m^2 \qquad C_D = 0{,}92$

a) Power to overcome drag ?

b) Maximum speed of bus if engine is rated at 465 hp

Add fairings on Front and rear of bus to reduce drag ⇒ C_D reduces to 0,86 without changing A ~~frontal~~ frontal

c) Required Power at 80 km/hr and at new top speed

d) if fuel cost is \$300/day ⇒ \$4800 to install. How long it would take for the modifications to pay for itself

a) We know $V = 80\ km/hr = 22{,}2\ m/s$

Let's estimate the drag force based on C_D ⇒ $C_D = \dfrac{F_D}{\frac{1}{2}\rho V^2 A}$

$\Rightarrow F_D = \frac{1}{2} C_D \rho V^2 A$

at standard air conditions : $\rho = 1{,}23\ kg/m^3$

$\Rightarrow F_D = 0{,}5 \times 0{,}92 \times 1{,}23 \times (22{,}2)^2 \times 7{,}5 = 2091{,}36\ N$

⇒ Power to overcome drag ⇒ $P = F_D \times V = 2091{,}36 \times 22{,}2 \Rightarrow \boxed{P = 46{,}42\ kW}$

b) $P_{max} = 465\ Hp \times 745{,}7\ \frac{W}{Hp} = 346{,}75\ kW$

$\Rightarrow P_{max} = F_{D_{max}} \times V_{max} = \frac{1}{2} C_D \rho V_{max}^2 A \times V_{max} = \frac{1}{2} C_D \rho A V_{max}^3$

$\Rightarrow V_{max} = \sqrt[3]{\dfrac{P_{max}}{\frac{1}{2} C_D \rho A}} \Rightarrow V_{max} = \sqrt[3]{\dfrac{346{,}75\ 10^3}{0{,}5 \times 0{,}92 \times 1{,}23 \times 7{,}5}} \Rightarrow \boxed{V_{max} = 43{,}39\ m/s}$

c) Now $C_{D_{new}} = 0,86$ without changing $A_{frontal}$

$$F_{D_{new}} = \frac{1}{2} C_{D_{new}} \times \rho V^2 A = 0,5 \times 0,86 \times 1,23 \times (22,2)^2 \times 7,5$$

$$\Rightarrow F_{D_{new}} = 1954,97 \text{ N}$$

$$\Rightarrow Power_{new} = F_{D_{new}} \times V = 1954,97 \times 22,2. \Rightarrow \boxed{Power_{new} = 43,4 \text{ kW.}}$$

$$Power_{Max} = F_{D_{new\,Max}} \times V_{Max_{new}} = 0,5 \times C_{D_{new}} \times \rho \times V_{max_{new}}^2 A \times V_{max_{new}}$$

$$= 0,5 \times C_{D_{new}} \times \rho \times V_{max_{new}}^3 \times A$$

$$\Rightarrow V_{max_{new}} = \sqrt[3]{\frac{Power_{Max}}{0,5 \times C_{D_{new}} \times \rho \times A}} = \sqrt[3]{\frac{346,1 \cdot 10^3}{0,5 \times 0,86 \times 1,23 \times 7,5}} = 44,38 \text{ m/s}$$

$$\boxed{V_{max_{new}} = 44,38 \text{ m/s}}$$

d) The fairings cost \$4800 to install and Fuel cost $= \frac{\$300}{day}$.

Let's see how much saving we have because of the fairings

$$Power\ gain = \frac{Power\ new}{P} = \frac{43,4 \cdot 10^3}{46,42 \cdot 10^3} = 93,49 \simeq 93,5\%$$

$\Rightarrow$ The new cost per day $\Rightarrow$ $New\ cost/day = Power\ gain \times Fuel\ cost$

$$= 0,935 \times 300$$

$$New\ cost/day = \frac{\$280,5}{day}$$

$$\Rightarrow Savings = Fuel\ cost - \frac{New\ cost}{day} = 300 - 280,5 = \$19,5/day$$

$$time = \frac{Initial\ cost}{savings} = \frac{\$4800}{\$19,5/day} = \boxed{246,153 \text{ days.} = time} \simeq 8,2 \text{ months}$$

9.125

Runner Maintains a speed of 7,5 mph during a 4 mile run

2 miles straight then turn around and return home for 2 miles.

$C_D A = 9\,ft^2$ for the runner

a) on a Windless day, how many calories in kcal will the runner burn overcoming drag

b) if wind is blowing 5 mph directly along the runner's route, how many calories burn to overcome drag.

a) $V_{wind} = 0\,mph$

$V_{runner} = 7,5\,mph$, Runner in air $\Rightarrow \rho_{air} = 0{,}002377\,slug/ft^3$

Calories burned = Energy $\Rightarrow$ Energy = Power × time

$V = \frac{distance}{time} \Rightarrow time = \frac{distance}{V} = \frac{4\,miles}{7{,}5\,mph} = 0{,}533\,h = 0{,}533\,h \times \frac{3600\,s}{h} = 1920\,s$

Power = $F_D \times V$

$C_D = \frac{F_D}{\frac{1}{2} \times V^2 \times A \times \rho} \Rightarrow F_D = \frac{1}{2} C_D V^2 \times A \times \rho = 0{,}5 \times (\overbrace{7{,}5 \times 1{,}4667}^{ft/s})^2 \times 0{,}002377 \times 9$ (given)

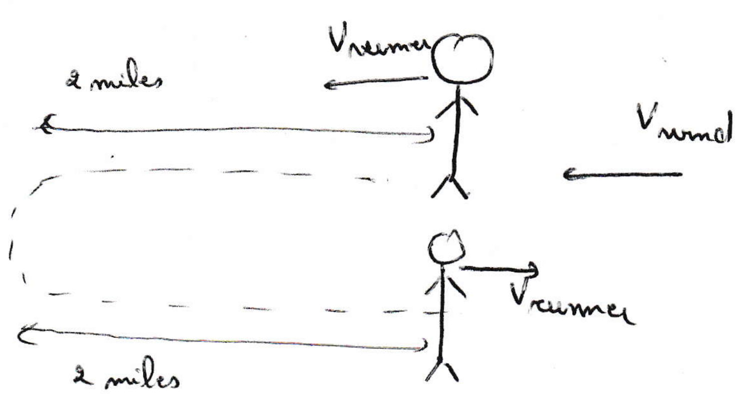
$\Rightarrow F_D = 1{,}294\,lbf$

$\Rightarrow$ Power $= 1{,}294 \times 11 = 14{,}237\,lbf\,\frac{ft}{s}$

$\Rightarrow$ Energy (in kcal) = Power × time $= 14{,}237 \times 1920\ \frac{\cancel{lbf} \times \cancel{ft}}{\cancel{s}} \times \cancel{s} \times \frac{0{,}00032\,38\,kcal}{\cancel{ft} \times \cancel{lbf}}$

$\Rightarrow \boxed{Energy\,(kcal) = 8{,}86\,kcal}$

b) $V_{wind} = 5\,mph$



1. For the first 2 miles ⇒ the relative velocity $V_{rel} = V_{runner} + V_{wind}$ ⇒

⇒ $V_{rel} = 7,5 + 5 = 12,5\ mph = 18,33\ ft/s$

2. For the next 2 miles ⇒ $V_{rel} = 7,5 - 5 = 2,5\ mph = 3,667\ ft/s$

1. $time = \frac{2\ miles}{7,5\ mph} = 960\ s$

⇒ $Energy_{①} = Power \times time$

$= F_D \times V \times time$

$= \frac{1}{2} C_D \times V^2 \times A \rho \times V \times time = 0,5 \times 9 \times (18,33)^2 \times 18,33 \times \overset{kcal/ft.lbf}{\overline{0,0003238}} \times 960 \times 0,00238$

⇒ $Energy_{①} = 12,3\ kcal$

2. $time = 960\ s$

⇒ $Energy_{①} = \frac{1}{2} C_D V^2 \times A \times \rho \times V \times time = 0,5 \times 9 \times (3,667)^3 \times 0,0003238 \times 960 \times 0,00238$

⇒ $Energy_{②} = 0,49\ kcal$

⇒ Total Energy $= Energy_{①} + Energy_{②} = 12,3 + 0,49 = \boxed{12,79\ kcal = Energy\ total}$

9,165

Design of airplane

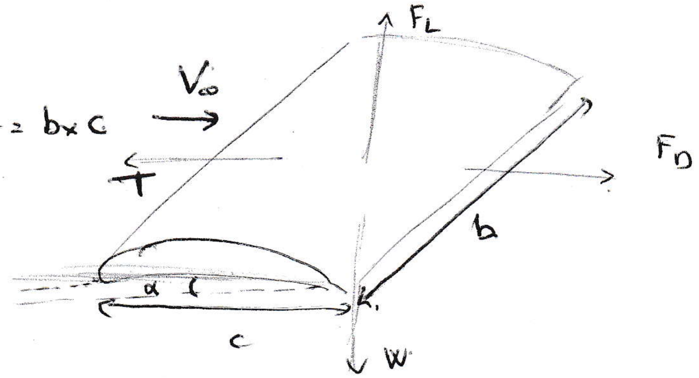
Evaluate the maximum total payload and required power to maintain flight, if sheet of plastic is replaced by NACA 23015 with same AR and angle of attack as in (9.163)

To maintain the flight ⇒ Thrust = F_D
Weight = Lift

NACA 23015

$AR = \frac{b^2}{A}$ where $A = b \times c$

$\Rightarrow AR = \frac{b^2}{bc} = \frac{b}{c}$



From 9.163: $b = W_{9,163} = 7\,ft$ and $c = L_{9,163} = 5\,ft$

Angle of attack $\alpha = 10°$, $V = 40\,ft/s$

* Payload ⇒ M ? since $W = F_L \Rightarrow M \times g = F_L$

we know: $C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A} \Rightarrow F_L = \frac{1}{2} C_L \rho V^2 A$?

Figure 9.17 in the book ⇒ for $\alpha = 10°$ ⇒ $C_L = 1,2$ and $C_{Di} = 0,01$

Assume standard conditions for air ⇒ $\rho_{air} = 0,00234\ \frac{slug}{ft^3}$

$$\Rightarrow M = \frac{F_L}{g} = \frac{0,5 \times 1,2 \times 0,00234 \times (40)^2 \times \underbrace{7 \times 5}_{A}}{32,2} = \frac{78,624}{32,2} = 2,44\ slug = \boxed{78,624\,lb = M}$$

* Power required ⇒ $P = \overset{T}{Thrust} \times V$, $C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$

we know $T = F_D \Rightarrow T = \frac{1}{2}\rho V^2 A\, C_D \rightarrow$?

For a wing, the drag coefficient can be found using $C_D = C_{Di} + \frac{C_L^2}{\pi\, AR}$

$$\Rightarrow C_D = 0,01 + \frac{1,2^2}{\pi \times \frac{7}{5}} = 0,337$$

$T = 0{,}5 \times 0{,}00234 \times (40)^2 \times 7 \times 5 \times 0{,}337 = 22{,}08\ lbf$

$Power = T \times V = 22{,}08 \times 40 = \boxed{883{,}2 \cdot lbf\ \frac{ft}{s} = Power} \Rightarrow Power = 1{,}605\ hp$

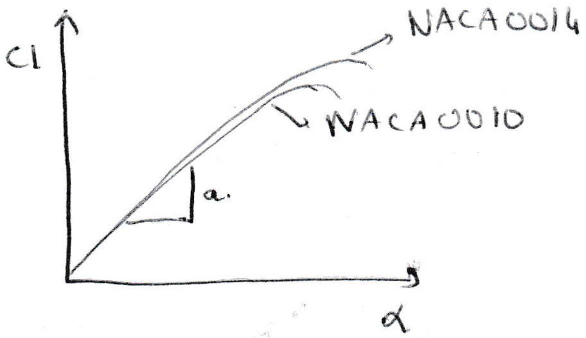
X-foil

C_L was calculated for 3 Different NACA airfoils ⇒ NACA 0006 ⇒ 6% thickness
⇒ NACA 0010 ⇒ 10% "
⇒ NACA 0014 ⇒ 14% thickness

These airfoils are not cambered, they are symmetrical.

angle of attack
* Fig 1. ⇒ CL vs. α (see page 9)

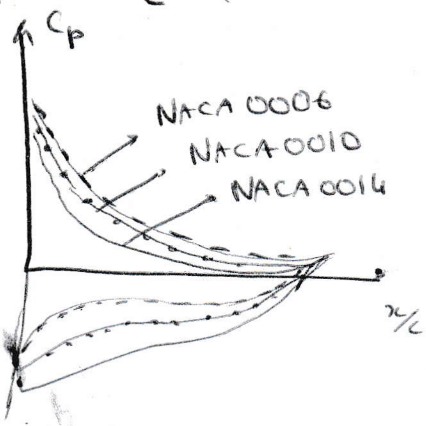
- The results were generated for $Re = 10^6$



⇒ $a \simeq 2\pi$ for all airfoils until reaching a critical angle of attack where C_L decreases.

- we noticed that for the same angle of attack the airfoil with higher thicknesses have higher lift coefficients.
- The same analysis can be performed with cambered airfoils with comparable thicknesses (NACA 2406, NACA 2410 and NACA 2414) ⇒ Max camber 2% at 40% of chord.
- it was found that cambered airfoils have higher lift coefficients for the same angles of attack ⇒ this shows the importance of the airfoil camber.

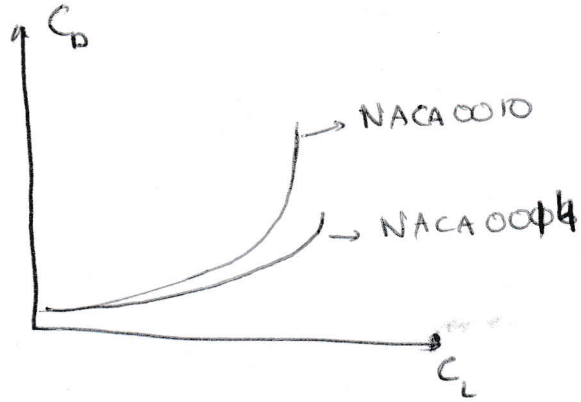
* Fig 2. ⇒ C_p vs. $\frac{x}{c}$ (using the same Re) for angle of attack at $2° = \alpha$



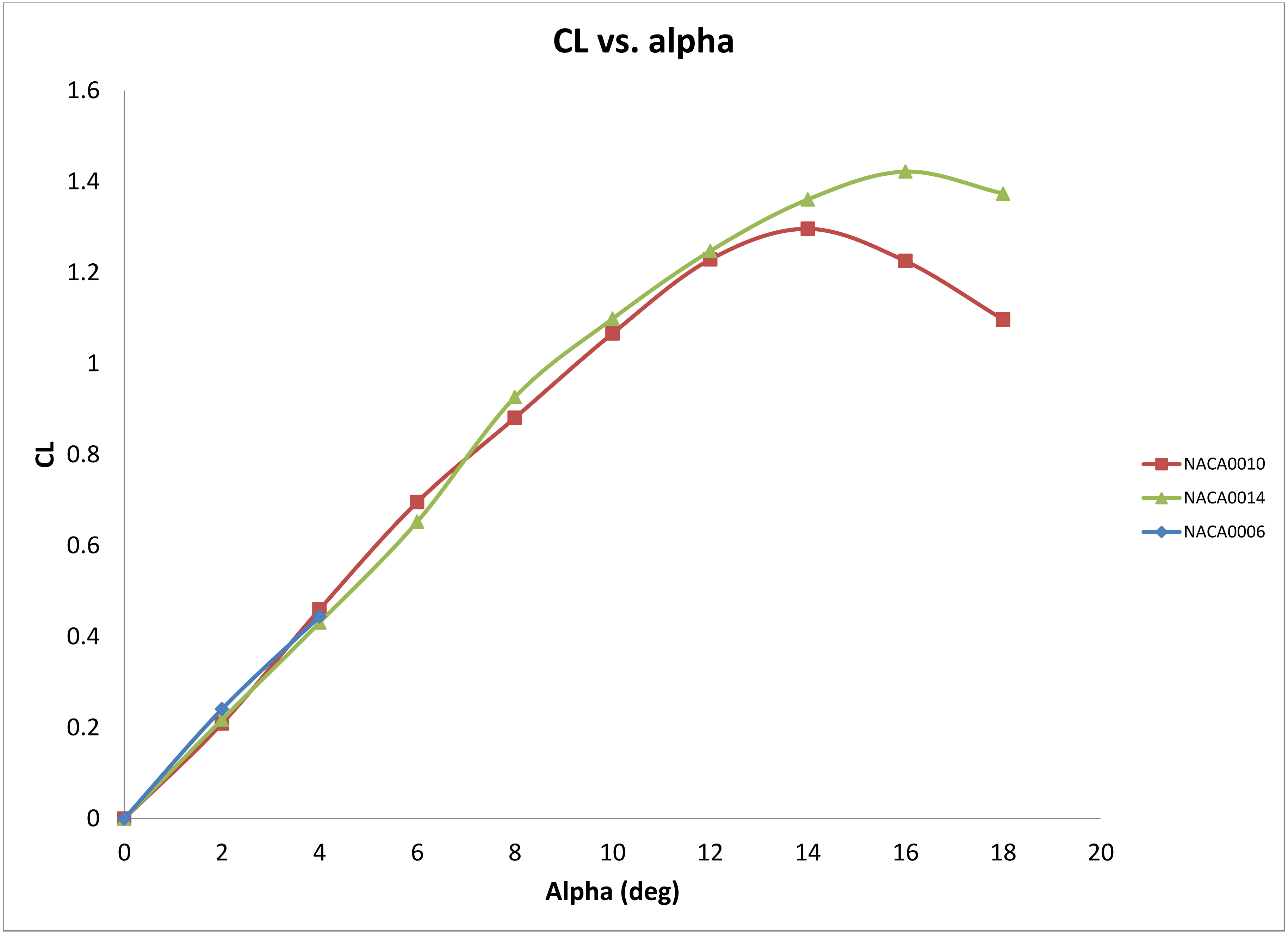
$$C_p = \frac{P - P_\infty}{\frac{1}{2}\rho_\infty V_\infty^2}$$

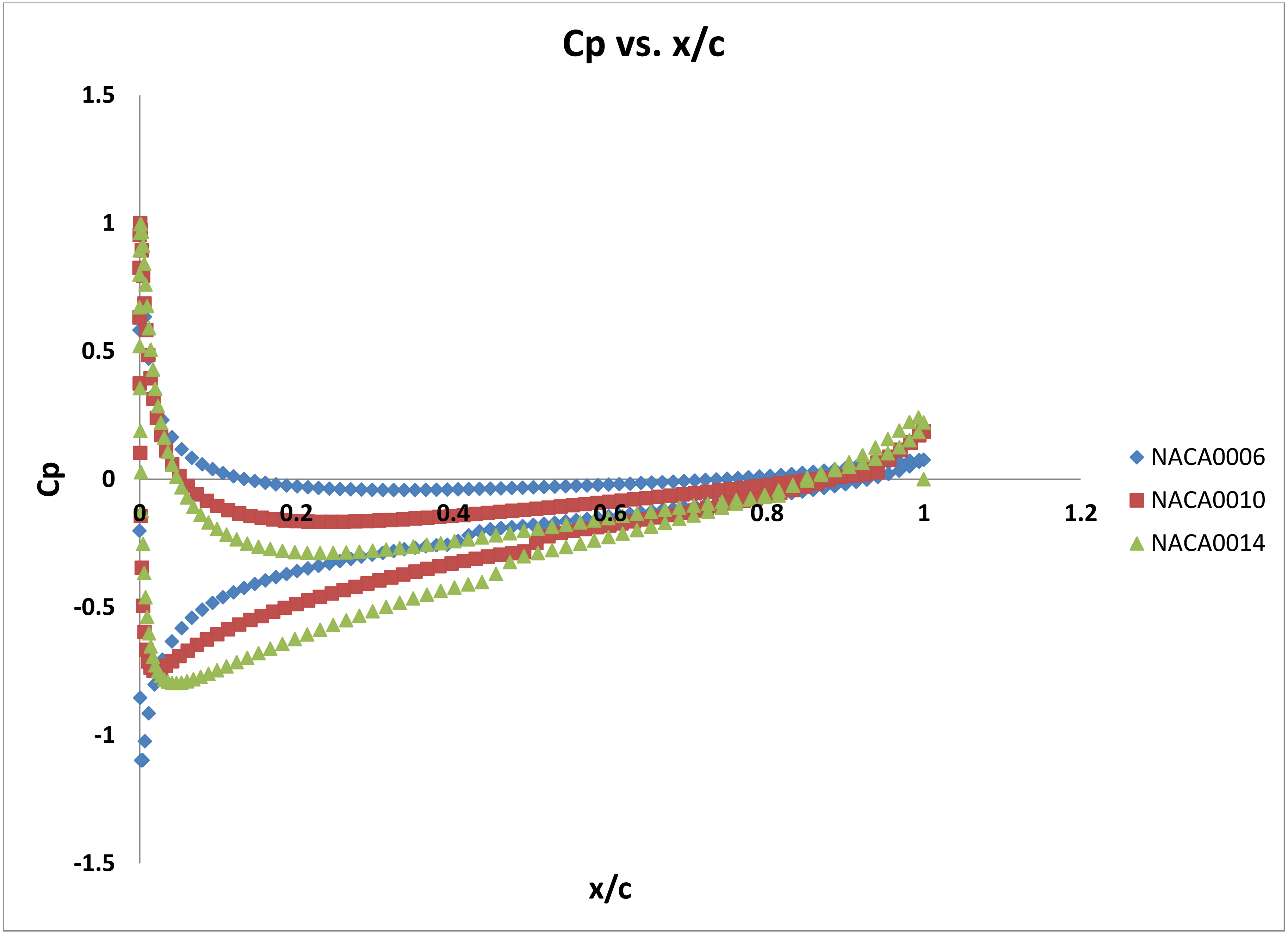
- We noticed that the pressure coefficient tend to decrease on the top of the airfoil for larger thicknesses, which explains the higher C_L

* Fig 3. C_L vs. C_D (for the same Re)

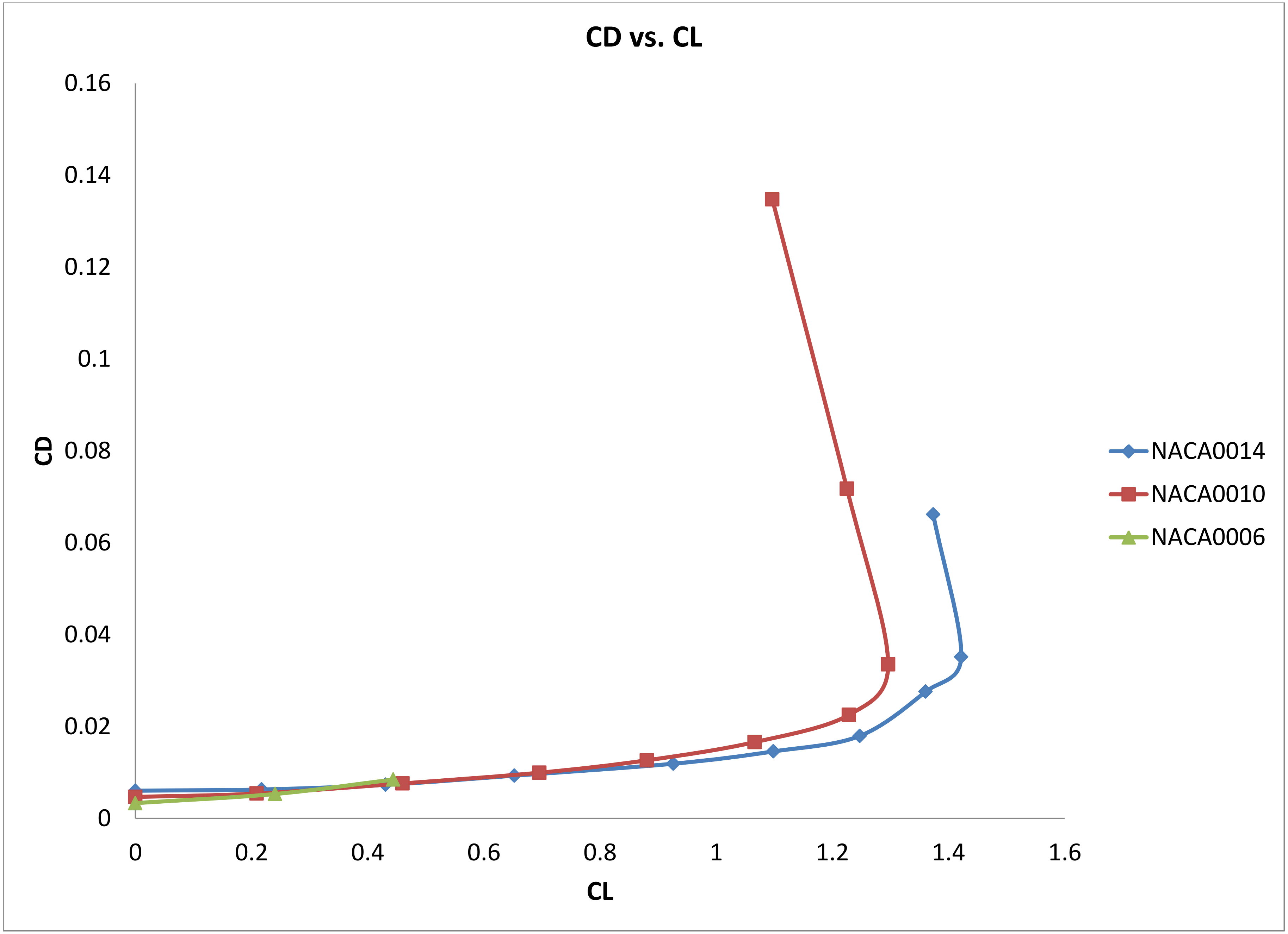


- This figure shows that the larger lift coefficients are obtained for smaller drag coefficients with the airfoil NACA 0014 (larger thickness) compared to NACA 0010, which is preferred.


CL vs. alpha
CL
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10
12
14
16
18
20
Alpha (deg)
NACA0010
NACA0014
NACA0006



Cp vs. x/c
Cp
x/c
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.2
0.4
0.6
0.8
1
1.2
NACA0006
NACA0010
NACA0014


CD vs. CL
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
CD
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
CL
NACA0014
NACA0010
NACA0006