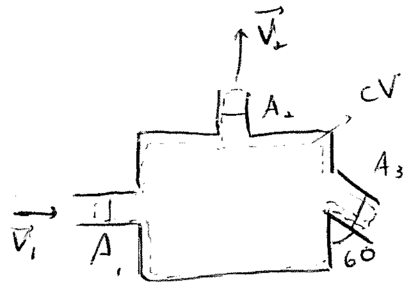
Solution Assignment #1

Summer 2016

4.24 / 4.102 / 4.109 / 6.71 / 7.77 / 5.46

4.24



$\rho = 65$ lbm/ft^3 $\quad \vec{V}_1 = 10\vec{i}$ ft/s

$A_1 = 0.5$ ft^2 $\quad \vec{V}_2 = 20\vec{j}$ ft/s

$A_2 = 0.1$ ft^2

$A_3 = 0.6$ ft^2 $\quad \vec{V}_3 = ?$

Find $\vec{V}_3$?

Assumptions

* steady state flow
* incompressible flow
* Uniform flow at section (1), (2) and (3)

Mass conservation

$$\frac{dm}{dt} = 0 = \cancel{\frac{\partial}{\partial t}\int_{vc} \rho \, d\forall}^{\,s.s} + \int_{sc} \rho \vec{V} \cdot \vec{dA}$$

we will assume the flow is going out of the control volume at section (3)

$$\Rightarrow -\rho V_1 A_1 + \rho V_2 A_2 + \rho V_3 A_3 = 0$$

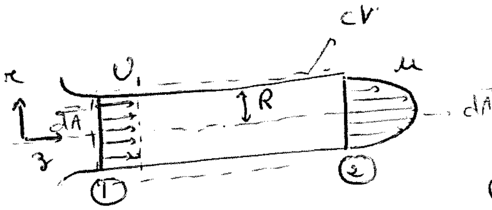
$$\Rightarrow -V_1 A_1 + V_2 A_2 + V_3 A_3 = 0 \Rightarrow V_3 = \frac{-V_2 A_2 + V_1 A_1}{A_3} = \frac{-20 \times 0.1 + 10 \times 0.5}{0.6} = 5 \text{ ft/s}$$

$$\Rightarrow \boxed{\vec{V}_3 = 5 \text{ ft/s} \; \searrow 30}$$

$V_{3x} = 5 \cos 30 = 4.33$ ft/s

$V_{3y} = -5 \sin 30 = -2.5$ ft/s

#4.102



$\rho = 850\ kg/m^3$

$R = 75\ mm$

$Q = 0{,}1\ m^3/s$

a) Find U_1 ?

b) $\dfrac{u}{u_{max}} = 1 - \left(\dfrac{r}{R}\right)^2$

Find the Maximum velocity at (2) ?

c) ΔP if viscous forces at the walls are neglected ?

Assumptions:

* incompressible
* steady state
* Uniform flow at (1)
* neglect viscous forces

Mass conservation

$$\frac{dm}{dt} = 0 = \cancel{\frac{\partial}{\partial t}}\int_{vc} \rho\, d\forall + \int_{sc} \rho \vec{V} \cdot d\vec{A} \quad (\text{s.s})$$

$$-\rho \underbrace{U_1 A_1}_{Q} + \int \rho u_2 dA_2 = 0 \qquad (I)$$

a) $Q = U_1 A_1 \Rightarrow U_1 = \dfrac{Q}{A_1} = \dfrac{0{,}1}{\frac{\pi \times D_1^2}{4}} = \dfrac{0{,}1}{\pi \times R_1^2} = \dfrac{0{,}1}{\pi \times (75 \cdot 10^{-3})^2} = \boxed{5{,}65\ m/s = U_1}$

b) $(I) \rightarrow -\cancel{\rho} U_1 A_1 + \cancel{\rho} \displaystyle\int_0^R \int_0^{2\pi} u_{max}\left(1 - \left(\frac{r}{R}\right)^2\right) r\, d\theta\, dr = 0$

$$\Rightarrow \int_0^R 2\pi u_{max}\left(r - \frac{r^3}{R^2}\right) dr = U_1 A_1$$

$$\Rightarrow 2\pi u_{max}\left[\frac{r^2}{2} - \frac{1}{4}\frac{r^4}{R^2}\right]_0^R = U_1 A_1$$

$$\Rightarrow 2\pi u_{max}\left(\frac{R^2}{2} - \frac{R^{\cancel{4}\,2}}{4\cancel{R^2}}\right) = U_1 A_1 \quad \Rightarrow \cancel{2}\pi u_{max} \frac{R^2}{\cancel{4}\,2} = U_1 A_1$$

$$\Rightarrow u_{max} = \frac{2 U_1 A_1}{\pi R^2} = \frac{U_1 \times 2\cancel{\pi R^2}}{\cancel{\pi R^2}} = 2 U_1 \Rightarrow u_{max} = 2 \times 5{,}65$$

$$\boxed{u_{max} = 11{,}31\ m/s}$$

c) conservation of Momentum

$$\sum \vec{F} = \cancel{\frac{\partial}{\partial t}\int_{vc} \rho \vec{V}\, dV}^{\,S.S} + \int_{sc} \rho \vec{V} \, \vec{V} \cdot d\vec{A}$$

$$\text{in } x : \underbrace{\sum F_x}_{\text{only pressure forces since we are neglecting friction at the wall}} = \rho(-U_1 A_1) \times \vec{V_1}^{\,U_1} + \int \rho u_2^2 \, dA_2$$

$$P_1 A_1 - P_2 A_2 = -\rho U_1^2 A_1 + \rho \int_0^R \int_0^{2\pi} u_{max}^2 \left(1 - \left(\frac{r}{R}\right)^2\right)^2 r \, d\theta \, dr$$

$A_1 = A_2$

$$(P_1 - P_2) A_1 = -\rho U_1^2 A_1 + \rho \times 2\pi \times u_{max}^2 \int_0^R r\left(1 - \frac{2r^2}{R^2} + \frac{r^4}{R^4}\right) dr$$

$$\Rightarrow (P_1 - P_2) A_1 = -\rho U_1^2 A_1 + \rho \times 2\pi \times u_{max}^2 \left[\frac{1}{2} r^2 - \frac{2}{4} \frac{r^4}{R^2} + \frac{r^6}{6R^4}\right]_0^R$$

$$\Rightarrow \Delta P A_1 = -\rho U_1^2 A_1 + \rho \times 2\pi \times u_{max}^2 \left(\frac{R^2}{2} - \frac{R^2}{2} + \frac{R^2}{6}\right)$$

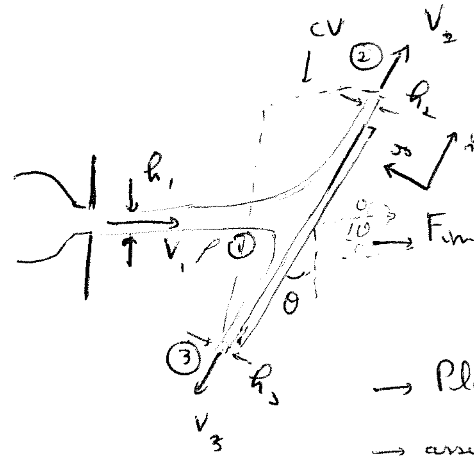
$$\Rightarrow \Delta P = -\rho U_1^2 + \rho \times \frac{2\pi}{A_1} u_{max}^2 \left(\frac{R^2}{6}\right) = -\rho U_1^2 + \rho \frac{u_{max}^2}{3} \times \frac{\pi R^2}{A_1}$$

$$\Delta P = -\rho U_1^2 + \frac{4\rho U_1^2}{3} = \frac{\rho U_1^2}{3}$$

$$\Rightarrow \Delta P = \frac{850 \times 5{,}65^2}{3} = \boxed{9.044 \text{ kPa} = \Delta P}$$

4.109

4



plane liquid jet on an inclined plate

two stream of equal speed

→ Find an expression for $\frac{h_2}{h_1}$ as a function of the plate angle θ ?

→ Plot the results and comment for $\theta = 0^\circ$ and $\theta = 90^\circ$

→ assume dept of w

$\frac{h_2}{h}$?

Assumptions :

* steady state (1)
* incompressible (2)
* Uniform flow (3)
* $V_1 = V_2 = V_3$ (4)
* no surface and body forces (5) → friction neglected; atmosphere

Mass conservation : (1)

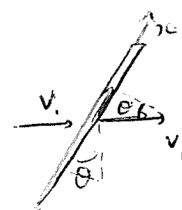
$$\frac{dm}{dt} = 0 = \cancel{\frac{\partial}{\partial t}\int_{VC} \rho \, dV} + \int_{SC} \rho \vec{V} \cdot d\vec{A}$$

$$\Rightarrow -\rho V_1 A_1 + \rho V_2 A_2 + \rho V_3 A_3 = 0 \Rightarrow A_1 = A_2 + A_3 \Rightarrow h_1 w = h_2 w + h_3 w$$

$$\Rightarrow h_1 = h_2 + h_3 \quad \text{(I)} \Rightarrow h_3 = h_1 - h_2$$

Conservation of Momentum

$$\cancel{\Sigma \vec{F}} = \cancel{\frac{\partial}{\partial t}\int_{VC} \rho \vec{V} \, dV} + \int_{SC} \rho \vec{V} \vec{V} \cdot d\vec{A}$$



in x :

$$\Rightarrow \rho(-V_1 A_1)\vec{V_1} + (V_2 A_2)\rho \vec{V_2} + (V_3 A_3)\rho \vec{V_3}$$

$$\vec{V_1} = V_1 \sin\theta \quad \vec{V_2} = V_2 \quad \vec{V_3} = -V_3$$

$$\Rightarrow -\rho V_1^2 \sin\theta A_1 + \rho V_2^2 A_2 - V_3^2 \rho A_3 = 0$$

$$\Rightarrow -\rho V_1^2 \sin\theta \, w h_1 + \rho V_2^2 w h_2 - \rho V_3^2 h_3 w = 0 \quad \text{(II)} \quad \text{(I)} \text{(III)}$$

$$\Rightarrow -h_1 \sin\theta + h_2 - h_3 = 0 \Rightarrow h_1 \sin\theta + h_2 - h_1 + h_2 = 0$$

$$\Rightarrow -h_1(\sin\theta + 1) + 2h_2 = 0 \Rightarrow \boxed{\frac{h_2}{h_1} = \frac{1 + \sin\theta}{2}}$$

Fig. 1

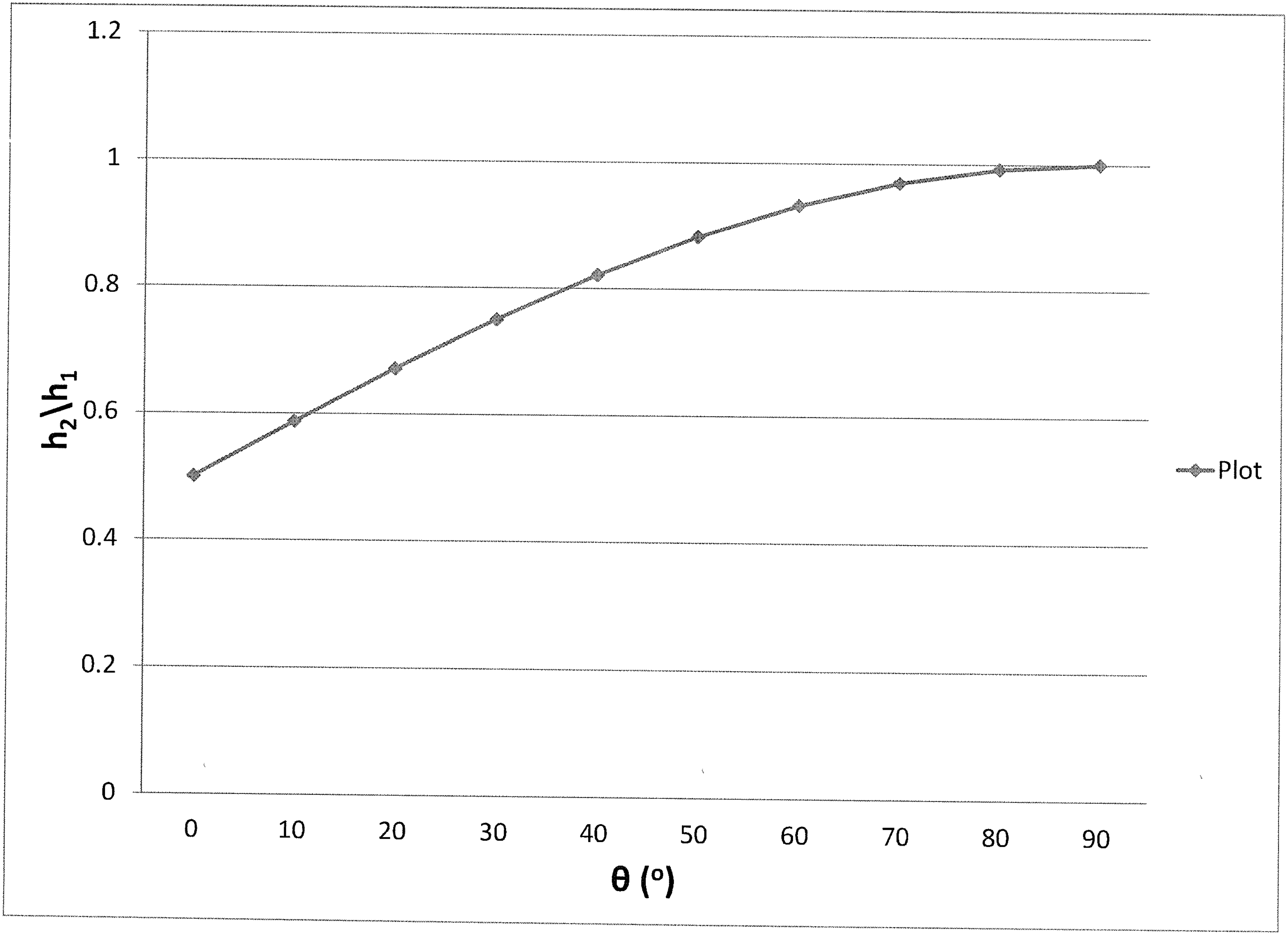
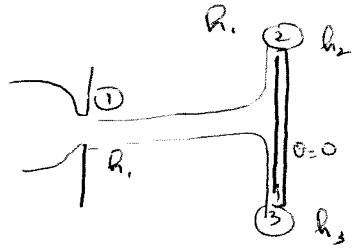


Fig. 1

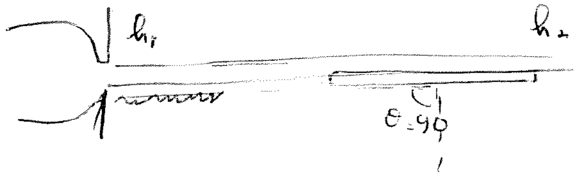
when $\theta = 0 \Rightarrow \frac{h_2}{h_1} = \frac{1+\sin(0)}{2} = \frac{1}{2}$

$\Rightarrow$



this means that when $\theta = 0$
$h_2 = \frac{1}{2} h_1 \Rightarrow$ the stream splits into equal stream when hitting the perpendicular plate
$h_3 = h_1 - h_2 = \frac{1}{2} h_1$

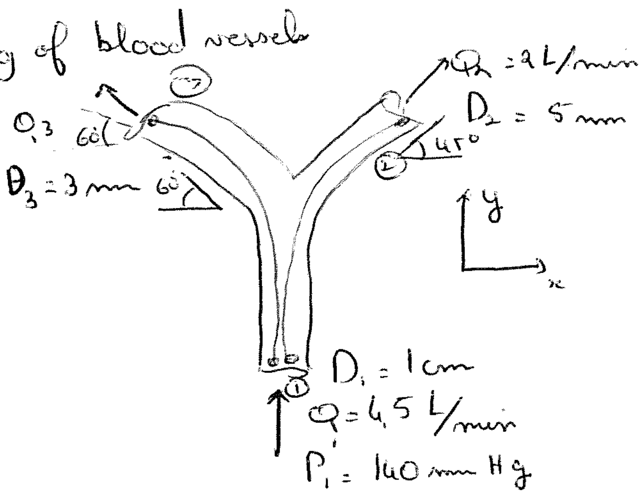
when $\theta = 90° \Rightarrow \frac{h_2}{h_1} = \frac{1+\sin(90°)}{2} = \frac{1+1}{2} = 1 \Rightarrow h_2 = h_1$ and $h_3 = h_1 - h_2 = 0$



this means that when $\theta = 90°$ the plate is horizontal and the stream is not affected by the plate.

6.71

Branching of blood vessels

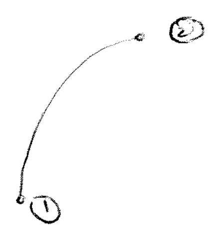


Assumptions:
* incompressible (blood) (1)
* Rigid tubes (2)
* inviscid (3)
* vessel in horizontal plane (4) $\Rightarrow \Delta z = 0$
* steady state (6)
* $\rho = 1060\ kg/m^3$ (7)

a) Estimate blood pressure in each branch P_2 and P_3 ?

b) Force generated by blood ?

a) We can use Bernoulli between (1) $\to$ (2) on a streamline
* incompressible
* steady state
* inviscid



$P_1 + \frac{1}{2}\rho V_1^2 + \cancel{\rho g z_1} = P_2 + \frac{1}{2}\rho V_2^2 + \cancel{\rho g z_2}$ (4) (4)

$P_1 = 140\ mmHg = 18665.13\ Pa$

V_1 ? , V_2 ? P_2 ?

Mass conservation $\Rightarrow \frac{dm}{dt} = 0 = \cancel{\frac{\partial}{\partial t}\int_{vc} \rho\, dV}^{(6)} + \int_{sc} \rho\, \vec{V}\cdot d\vec{A} \Rightarrow -\rho V_1 A_1 + \rho V_2 A_2 + \rho V_3 A_3 =$

$\Rightarrow -Q_1 + Q_2 + Q_3 = 0$

$\Rightarrow Q_1 = V_1 A_1 \quad Q_1 = 4.5\ L/min = 7.5\ 10^{-5}\ m^3/s$

$\Rightarrow V_1 = \frac{Q_1}{A_1} = \frac{Q_1}{\pi D_1^2} = \frac{7.5\ 10^{-5}}{\pi \times (1\ 10^{-2})^2} = 0.955\ m/s$

$Q_2 = 2L/min = 3{,}33\,10^{-5}\ m^3/s$

$\Rightarrow V_2 = \frac{Q_2}{A_2} = \frac{Q_2}{\frac{\pi \times D_2^2}{4}} = \frac{3{,}33\,10^{-5}}{\frac{\pi \times (5\,10^{-3})^2}{4}} = 1{,}7\ m/s$

$Q_3 = Q_1 - Q_2 = 7{,}5\,10^{-5} - 3{,}33\,10^{-5} = 4{,}17\,10^{-5}$

$\Rightarrow V_3 = \frac{Q_3}{A_3} = \frac{Q_3}{\frac{\pi \times D_3^2}{4}} = \frac{4{,}17\,10^{-5}}{\frac{\pi \times (3\,10^{-3})^2}{4}} = 5{,}9\ m/s$

$\Rightarrow P_2 = P_1 + \frac{1}{2}\rho V_1^2 - \frac{1}{2}\rho V_2^2 = P_1 + \frac{1}{2}\rho(V_1^2 - V_2^2)$

$\Rightarrow P_2 = 18665{,}13 + 0{,}5 \times 1060(0{,}955^2 - 1{,}7^2) \approx$

$\boxed{P_2 = 1{,}76\,10^4\ Pa}$

P_3? Bernoulli from ① → ③

$\Rightarrow P_1 + \frac{1}{2}\rho V_1^2 + \cancel{\rho g z_1} = P_3 + \frac{1}{2}\rho V_3^2 + \cancel{\rho g z_3}$

$\Rightarrow P_3 = P_1 + \frac{1}{2}\rho(V_1^2 - V_3^2)$

$= 18665{,}13 + 0{,}5 \times 1060(0{,}955^2 - 5{,}9^2)$

$\boxed{P_3 = 699{,}2\ Pa}$

b) Force generated by blood?

Conservation of Momentum $\quad \sum\vec{F} = \cancel{\frac{\partial}{\partial t}\int_{vc}\rho\vec{V}\,d\forall} + \int_{sc}\rho\vec{V}\,\vec{V}\cdot d\vec{A}$

in x:

$F_x - P_2 A_2 \cos 45^\circ + P_3 A_3 \cos 60^\circ = +\rho |V_2 A_2| \vec{V}_{2x} + \rho |V_3 A_3| \vec{V}_{3x}$

$\vec{V}_{2x} = V_2 \cos 45 \quad , \quad \vec{V}_{3x} = -V_3 \cos 60$

$\Rightarrow F_x - P_2 A_2 \cos 45^\circ + P_3 A_3 \cos 60^\circ = \rho V_2^2 A_2 \cos 45 - \rho V_3^2 A_3 \cos 60^\circ$

$\Rightarrow F_x = \rho V_2^2 A_2 \cos 45 - \rho V_3^2 A_3 \cos 60 + P_2 A_2 \cos 45^\circ - P_3 A_3 \cos 60^\circ$

$= 1060 \times 1{,}7^2 \times \frac{\pi \times (5\,10^{-3})^2}{4} \times \cos 45 - 1060 \times 5{,}9^2 \times \frac{\pi \times (3\,10^{-3})^2}{4} \cos 60 + 1{,}76\,10^4 \times \frac{\pi \times (5\,10^{-3})^2}{4} \cos 45$

$- 699{,}2 \times \frac{\pi \times (3\,10^{-3})^2}{4} \cos 60$

$\Rightarrow \boxed{F_x = 0{,}154\ N}$

In y:

$$F_y + P_1 A_1 - P_2 A_2 \sin 45 - P_3 A_3 \sin 60 = \rho (V_2 A_2)\vec{V}_{2y} + \rho (V_3 A_3)\vec{V}_{3y} + \rho(-V_1 A_1)\vec{V}_{1y}$$

$$\vec{V}_{2y} = V_2 \sin 45 \qquad \vec{V}_{3y} = V_3 \sin 60 \qquad \vec{V}_1 = V_1$$

$$\Rightarrow F_y = \rho V_2^2 A_2 \sin 45° + \rho V_3^2 A_3 \sin 60° - \rho V_1^2 A_1 + P_3 A_3 \sin 60° + P_2 A_2 \sin 45 - P_1 A_1$$

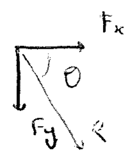
$$F_y = 1060 \times 1{,}7^2 \times \frac{\pi \times (5\,10^{-3})^2}{4} \sin 45° + 1060 \times 5{,}9^2 \times \frac{\pi \times (3\,10^{-3})^2}{4} \sin 60° - 1060 \times 0{,}955^2 \times \frac{\pi \times (10^{-2})^2}{4}$$

$$+ 699{,}2 \times \frac{\pi (3\,10^{-3})^2}{4} \sin 60 + 1{,}76\,10^4 \times \frac{\pi (5\,10^{-3})^2}{4} \sin 45 - 18665{,}13 \times \frac{\pi (10}{4}$$

$$\boxed{F_y = -1{,}02\ N}$$

$$F = \sqrt{F_x^2 + F_y^2} = 1{,}03\ N$$

$$\theta = tg^{-1}\left(\frac{F_y}{F_x}\right) = 81°$$



7.77

Automobile → standard air at 60mph

Pressure distribution: 1/5 scale model in water

a) what factors must be considered to ensure kinematic similarity?

b) Determine the water speed

c) Ratio of Drag force prototype / model

d) $C_p = -1{,}4$ at minimum static pressure on the surface
Estimate Minimum tunnel pressure to avoid cavitation if cavitation number = 0,5?

a) $P = f(F_D, V, \rho, \mu, L)$

$$\left[\frac{M}{LT^2}\right] \quad \left[\frac{ML}{T^2}\right] \left[\frac{L}{T}\right] \left[\frac{M}{L^3}\right] \left[\frac{M}{LT}\right] [L]$$

M: Mass
L: Length
T: time

n-d group = # parameter − # units ⇒ # N-D groups = 6 − 3 = 3

$\#$ ① $\Rightarrow$ $Re = \frac{\rho V \cancel{L}}{\mu} = [1]$

$\#$ ② $\Rightarrow$ $\frac{F_D}{V^2} = \left[\frac{M\cancel{L}}{\cancel{T^2}} \times \frac{\cancel{T^2}}{\cancel{L^2}}\right] = \left[\frac{M}{L}\right]$

$\Rightarrow \frac{F_D}{V^2 \rho} = \left[\frac{\cancel{M}}{\cancel{L}} \times \frac{L^3}{\cancel{M}}\right] = [L^2] \Rightarrow \frac{F_D}{V^2 \rho L^2} = [1]$

$\#$ ③ $\Rightarrow$ $\frac{P}{V^2} = \left(\frac{M}{L\cancel{T^2}} \times \frac{\cancel{T^2}}{L^2}\right) = \left(\frac{M}{L^3}\right) \Rightarrow \frac{P}{V^2 \rho} = [1]$

~~For dynamic similarity~~ ;

For kinematic similarity : Re prototype = Re model

Prototype : real (air)

model : (water)

b)

$$\Rightarrow \frac{\rho_{real} \times V_{real} \times L_{real}}{\mu_{real}} = \frac{\rho_{model} \times V_{model} \times L_{model}}{\mu_{model}}$$

$$V_{model} = \frac{\rho_{real}}{\rho_{model}} \times \frac{\mu_{model}}{\mu_{real}} \times \frac{L_{real}}{L_{model}} \, V_{real}$$

$\frac{1}{5}$ scale $\Rightarrow \frac{L_{model}}{L_{real}} = \frac{1}{5}$

For air at 68°F : $\rho_{real} = 0{,}00234 \frac{slug}{ft^3}$ $\mu_{real} = 3{,}79 \, 10^{-7} \frac{lbf \cdot s}{ft^2}$

For water at 68°F $\Rightarrow$ $\rho_{model} = 1{,}94 \frac{slug}{ft^3}$ $\mu_{model} = 2{,}1 \, 10^{-5} \frac{lbf \cdot s}{ft^2}$

$V_{real} = 60 mph = 88 \, ft/s$

$$\Rightarrow V_{model} = \frac{0{,}00234}{1{,}94} \times \frac{2{,}1 \, 10^{-5}}{3{,}79 \, 10^{-7}} \times 5 \times 88 = \boxed{29{,}4 \, ft/s = V_{model}}$$

c) $\frac{F_{D\,prototype}}{F_{D\,model}}$?

$$\frac{F_{D\,real}}{\rho_{real} V_{real}^2 L_{real}^2} = \frac{F_{D\,model}}{\rho_{model} \times V_{model}^2 \times L_{model}^2}$$

$$\Rightarrow \frac{F_{D\,real}}{F_{D\,model}} = \frac{\rho_{real}}{\rho_{model}} \times \frac{V_{real}^2}{V_{model}^2} \times \frac{L_{real}^2}{L_{model}^2} = \frac{0{,}00234}{1{,}94} \times \left(\frac{88}{29{,}4}\right)^2 \times (5)^2 = \boxed{0{,}270 = \frac{F_{D\,real}}{F_{D\,model}}}$$

d)

$$Ca = \frac{P_{min} - P_{vapor}}{\frac{1}{2}\rho V^2} = 0{,}5 \Rightarrow P_{min} = \frac{1}{4}\rho V^2 + P_{vapor}$$

for water at 68°F $\Rightarrow P_{vapor} = 0{,}339$ psi

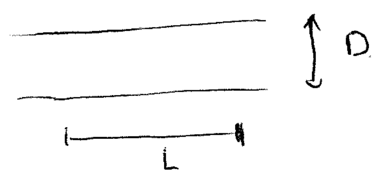
$$\Rightarrow P_{min} = \frac{1}{4} \times 1{,}94 \times \frac{29{,}4^2}{12^2} + 0{,}339 \Rightarrow P_{min} = 3{,}25 \text{ psi}$$

$$\Rightarrow C_p = \frac{P_{min} - P_{tunnel}}{\frac{1}{2}\rho V^2} = -1{,}4 \Rightarrow P_{tunnel} = (+1{,}4) \times \frac{1}{2}\rho V^2 + P_{min}$$

$$\Rightarrow P_{tunnel} = 1{,}4 \times 0{,}5 \times 1{,}94 \times \frac{29{,}4^2}{12^2} + 3{,}25$$

$$\Rightarrow \boxed{P_{tunnel} = 11{,}4 \text{ psi}}$$

5.46



$u(x) = U\left(1 - \frac{x}{2L}\right)$ $U = 5$ m/s $L = 0{,}3$ m

Develop an expression for acceleration of fluid ?

$$\vec{a}_p = \frac{D\vec{V}}{Dt} = u\frac{\partial \vec{V}}{\partial x} + \cancel{v\frac{\partial \vec{V}}{\partial y}}\ (3) + \cancel{w\frac{\partial \vec{V}}{\partial z}}\ (3) + \cancel{\frac{\partial \vec{V}}{\partial t}}\ (2)$$

Assumptions:
* incompressible (1)
* steady state (2)
* 1D $\Rightarrow u = u(x)$ (3)

$$\Rightarrow \vec{a}_p = u\frac{\partial(u)}{\partial x} = U\left(1 - \frac{x}{2L}\right) \times \left(-\frac{U}{2L}\right)$$

$$\Rightarrow a_{px} = -\frac{U^2}{2L}\left(1 - \frac{x}{2L}\right)$$

$$\Rightarrow a_{px} = -\frac{5^2}{2 \times 0{,}3}\left(1 - \frac{x}{2 \times 0{,}3}\right) = \boxed{-41{,}66\left(1 - \frac{x}{0{,}6}\right) = a_{px}}$$