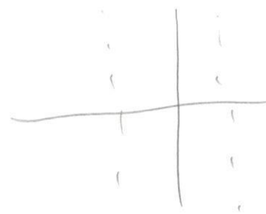
4

1. [1 point] What is the domain of the function $f(x) = \sqrt{4 - x^2}$?

(A) $x \geq 2$ (B) $-2 \leq x \leq 2$ (C) $x \leq -2$

(D) $x < 2$ (E) all x (F) $x \leq -2$ or $x \geq 2$

$X \neq \pm 2$



2. [1 point] Find a formula for the inverse of $f(x) = \ln(x + 6)$.

(A) e^{x+6} (B) $e^x + 6$ (C) $e^x - 6$

(D) $x + 6$ (E) e^{x-6} (F) $6 - e^x$

$y = \ln(x+6)$

$x = \ln(y+6)$

$e^x - 6 = y$

3. [1 point] What is the equation of the tangent line to the curve $y = f(x) = 2x+3x^2$ at the point $(1, 5)$?

(A) $y = 32x - 27$ (B) $y = 6x - 1$ (C) $y = 8x - 3$

(D) $y = 3x + 2$ (E) $y = 2x + 3$ (F) $y = 2x + 3x^2$

$f(x) = 2x + 3x^2$

$f'(x) = 2 + 6x$

$= 2 + 6$

$= 8$

$y = 8x + b$

$5 = 8(1) + b$

$5 - 8 = b$

$-3 = b$

$y = 8x - 3$

4. [1 point] If a function $f(x)$ is continuous at $x = a$, then it must be differentiable there.

(A) TRUE (B) FALSE

5. [1 point] What is $\lim_{x\to\infty} \dfrac{5x^3 + 7x^2 - 8x + 9}{11x^2 + 7x - 6}$?

(A) 3/2 (B) 0 (C) ∞ (D) $-\infty$ (E) 5/11 (F) 11/5

$$\frac{5x^3 + 7x^2 - 8x + 9}{11x^2 + 7x - 6}$$

$$\frac{5x + 7 - \cancel{\frac{8}{x}} + \cancel{\frac{9}{x^2}}}{11 + \cancel{\frac{7}{x}} - \cancel{\frac{6}{x^2}}}$$

$$\frac{5x + 7}{11}$$

6. [2 points] Find the limit $\lim_{x\to 2} \frac{\sqrt{x^2+12}-4}{x-2}$. You must do this properly, showing all steps.

2

$$\lim_{x\to 2} \frac{\sqrt{x^2+12}-4}{x-2} \times \left(\frac{\sqrt{x^2+12}+4}{\sqrt{x^2+12}+4}\right)$$

$$\lim_{x\to 2} \frac{x^2+12-16}{(\sqrt{x^2+12}+4)(x-2)}$$

$$\lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{(\sqrt{x^2+12}+4)\cancel{(x-2)}}$$

$$\lim_{x\to 2} \frac{x+2}{\sqrt{x^2+12}+4}$$

$$= \frac{(2)+2}{\sqrt{2^2+12}+4} = \frac{4}{8} = \boxed{\frac{1}{2}}$$

$\therefore \lim = \frac{1}{2}$ at $x \to 2$

7. [3 points] Use the definition of the derivative to find $f'(x)$ if $f(x) = \frac{2x^2}{x+3}$. Then verify your answer with the Quotient Rule.

3

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\lim_{h\to 0} \frac{\frac{2(x+h)^2}{(x+h)+3} - \frac{2x^2}{x+3}}{h}$$

$$\lim_{h\to 0} \left(\frac{2x^2+4xh+2h^2}{x+h+3} - \frac{2x^2}{x+3}\right) \times \left(\frac{1}{h}\right)$$

$$\lim_{h\to 0} \frac{(2x^2+4xh+2h^2)(x+3) - 2x^2(x+h+3)}{(x+h+3)(x+3)} \times \left(\frac{1}{h}\right)$$

$$\frac{\cancel{2x^3}+\cancel{6x^2}+4x^2h+12xh+2xh^2+6h^2-\cancel{2x^3}-2x^2h-\cancel{6x^2}}{(x+h+3)(x+3)} \times \left(\frac{1}{h}\right)$$

$(x+h)(x+h)$

$x^2+2xh+h^2$

$$\lim_{h\to 0} \frac{\cancel{h}(4x^2+12x+2xh+6h-2x^2)}{(x+h+3)(x+3)} \times \frac{1}{\cancel{h}}$$

$$\lim_{h\to 0} \frac{2x^2+12x+2xh^0+6h^0}{(x+h+3)(x+3)}$$

$$\boxed{= \frac{2x^2+12x}{(x+3)^2}}$$

Checked w/ quotient rule

$$\frac{4x(x+3)-2x^2}{(x+3)^2} = \frac{4x^2+12x-2x^2}{x+3^2} = \frac{2x^2+12x}{(x+3)^2}$$

8. [5 points] Find the first derivatives of the following functions.

5

(a) $f(x) = e^{-3x}\sin(4x^2)$

$$f'(x) = e^{-3x}(-3)(\sin(4x^2)) + e^{-3x}(\cos(4x^2))(8x)$$

$$= e^{-3x}(-3\sin(4x^2) + 8x\cos(4x^2))$$

$$= e^{-3x}(-3\sin(4x^2) + 8x\cos(4x^2))$$ 1

(b) $g(t) = \sqrt{3t^2 + 7t - 2}$

$$g'(t) = \frac{1}{2}(3t^2+7t-2)^{-\frac{1}{2}}(6t+7)$$

$$g'(t) = \frac{6t+7}{2(\sqrt{3t^2+7t-2})}$$

Final Answer $= \dfrac{6t+7}{2(\sqrt{3t^2+7t-2})}$ 1

(c) $\varphi(\theta) = \tan^2(3e^\theta)$

$$\varnothing(\theta) = \tan^2(3e^\theta)$$

$3e^x$

$$\varphi'(\theta) = 2(\tan(3e^\theta))(\sec^2(3e^\theta))(3e^\theta)$$

$$\varphi''(\theta) = 6e^\theta(\tan(3e^\theta))(\sec^2(3e^\theta))$$ 1

(d) $p(t) = 5^{\sqrt{t}}$

$$p(t) = 5^{\sqrt{t}}$$

$$= 5^{\sqrt{t}}(\ln 5)(\tfrac{1}{2}t^{-\frac{1}{2}})$$

$$= 5^{\sqrt{t}}(\ln 5)(\tfrac{1}{2t})$$

$$= 5^{\sqrt{t}}(\ln 5)(\tfrac{1}{2\sqrt{t}})$$ 1

~~$\varnothing(x) = \tan^2(3e^x)$~~

$= 2(\tan(3e^x))(\sec^2(3e^x))$

$p(t) = 5^{\sqrt{x}}$

$= (\ln 5)(\frac{1}{2}x^{-\frac{1}{2}})(5^{\sqrt{x}}$

(e) $y = e^{x\cos(2x)}$

$$y = e^{x\cos(2x)}$$

$$y' = e^{x\cos(2x)}(\cos(2x) + x(-\sin(2x))(2))$$

$$= e^{x\cos(2x)}(\cos(2x) - 2x\sin(2x))$$

Final Answer $= e^{x\cos(2x)}(\cos(2x) - 2x\sin(2x))$ 1