MATH2004B — Test 5 — 16:35–17:25, Nov 22

Surname ______________ **First Name** ______________**Student #** __________

Total: 15 points. No partial marks for Questions 1-3.
Closed book! Non-programmer calculators are allowed!

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1. (1 point) Evaluate $\int_C x^2 ds$, where C is the curve $\vec{r}(t) = (1-3t, 2-4t)$, $0 \le t \le 1$.

   (a) 3 (b) 5 (c) 8 (d) 10 (e) None of these.

   Solution: (b).

$$\int_C x^2 ds = \int_0^1 (1-3t)^2 \sqrt{(-3)^2 + (-4)^2} dt = \int_0^1 5(1-6t+9t^2) dt = 5.$$

2. (1 point) Let $\vec{F}(x,y,z) = (y^2, 2xy + e^{3z}, 3ye^{3z})$. A potential function is $f(x,y,z) = xy^2 + ye^{3z} + 1$. Find $\int_C \vec{F} \cdot d\vec{r}$, where C is the curve $\vec{r}(t) = (\cos(\pi t), t+1, \sin(\pi t)), 0 \le t \le 1$.

   (a) 0 (b) -3 (c) -4 (d) -5 (e) None of these.

   Solution: (c). Note that $\vec{r}(0) = (1,1,0)$, $\vec{r}(1) = (-1,2,0)$. Hence

$$\int_C \vec{F} \cdot d\vec{r} = f(\vec{r}(1)) - f(\vec{r}(0)) = f(-1,2,0) - f(1,1,0) = -1 - 3 = -4.$$

3. (1 point) Find $\displaystyle\iint_R (2x+y) dA$, where $R = \{0 \le x \le 2,\ 0 \le y \le 1\}$.

   (a) 3 (b) 5 (c) 8 (d) 10 (e) 13

   Solution: (b).

$$\iint_R (2x+y) dA = \int_0^1 \left(\int_0^2 (2x+y) dx \right) dy = \int_0^1 \left(x^2 + xy\right)|_{x=0}^2 dy = \int_0^1 (4+2y)\, dy = 5.$$

4. (4 points) Find $\int_C (x^3+3y+30z)ds$, where C is the curve $\vec{r}(t) = (2t, \frac{2}{3}t^3, \frac{1}{3}t^3)$, $0 \le t \le 1$.

Solution:

$$\int_C (x^3+3y+20z)ds = \int_0^1 (x^3+3y+30z)\sqrt{[x'(t)]^2+[y'(t)]^2+[z'(t)]^2}dt$$

1 point

$$= \int_0^1 20t^3\sqrt{4+5t^4}dt = \int_4^9 \sqrt{u}du = \frac{38}{3}.$$

3 points

5. (4 points) Find $\iint\limits_R (x+2y)dA$, where R is the region bounded by $y = x+1$ and $y = x^2+1$.

Solution: The intersection points of $y = x+1$ and $y = x^2+1$: $x+1 = x^2+1, \Rightarrow x = 0, 1$. Thus $R = \{(x,y) : 0 \le x \le 1, x^2+1 \le y \le x+1\}$.

1 point

$$\iint\limits_R (x+2y)dA = \int_0^1 \int_{x^2+1}^{x+1} (x+2y)dy\,dx = \int_0^1 (-x^4-x^3+2x)\,dx = -\frac{1}{5}-\frac{1}{4}+1 = \frac{11}{20} = 0.55.$$

3 points

6. (4 points) Evaluate the integral $\int_0^3 \int_{y^2}^9 4y\sin(x^2)\ dxdy$ by finding R_{xy} and R_{yx}.

Solution: $R_{xy} = \{(x,y) : y^2 \le x \le 9,\ 0 \le y \le 3\}$ and $R_{yx} = \{(x,y) : 0 \le y \le \sqrt{x},\ 0 \le y \le 9\}$.

1 point

$$\int_0^3 \int_{y^2}^9 4y\sin(x^2)\ dxdy = \int_0^9 \int_0^{\sqrt{x}} 4y\sin(x^2)\ dydx$$

1 point

$$= \int_0^9 \left(2y^2 \sin(x^2)\right) \Big|_{y=0}^{y=\sqrt{x}} \; dx = \int_0^9 \left(2x \sin(x^2)\right) \; dx$$

$$= -\cos(x^2)\Big|_0^9 = 1 - \cos 81.$$

2 points