MATH2004B — Test 1: 16:35–17:25, Sept 27

Surname ______________ **First Name** ______________**Student #** __________

Total: 15 points. No partial marks for Questions 1-4.
Closed book! Non-programmer calculators are allowed!

---

1. (1 point) Let $\vec{u} = (1, 2, -1)$ and $\vec{v} = (2, 1, k)$ be orthogonal. Find k.

   (a) 1 (b) 2 (c) 3 (d) 4 (e) -3

   Solution: (d)

   $\vec{u} \cdot \vec{v} = 0, \Rightarrow (1, 2, -1) \cdot (2, 1, k) = 0, \Rightarrow 4 - k = 0.$

2. (1 point) Given the linear equation of the plane Π: $2x + 3y + 4z - 4 = 0$ and the line $L : x = 1 + 3t, y = 2 + 2t, z = 3 - t,\ t \in \mathbb{R}$. Find the intersection between Π and L.

   (a) $(5, -2, 5)$ (b) $(-5, 2, 5)$ (c) $(-5, -2, -5)$ (d) $(5, 2, 5)$ (e) $(-5, -2, 5)$

   Solution: (e)

   The parametric equation of the line is:

   $$x = 1 + 3t, y = 2 + 2t, z = 3 - t.$$

   Substitute this into the plane we get

   $$2(1 + 3t) + 3(2 + 2t) + 4(3 - t) - 4 = 0, \Rightarrow t = -2.$$

   Thus the point is (-5,-2,5).

3. (1 point) Find the area of the triangle with vertices $P(-3, 2, 3)$, $Q(-3, 2, 1)$, and $R(-3, 1, 2)$.

   (a) 1 (b) 2 (c) 3 (d) 4 (e) 1/2

   Solution: (a). $\vec{PQ} = Q - P = (0, 0, -2)$, $\vec{PR} = R - P = (0, -1, -1)$. $A = \frac{1}{2}||\vec{PQ} \times \vec{PR}|| = \frac{1}{2}||(0, 0, -2)|| = 1.$

4. (1 point) Find the cosine of the angle between two planes $x-2y-2z=3$ and $3x+4y=1$.

   (a) $-\frac{1}{15}$ (b) $-\frac{1}{3}$ (c) $\frac{4}{5\sqrt{5}}$ (d) 1.2 (e) 0.7

   Solution: (b).
   $\cos\theta = \frac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}|\,|\vec{n_2}|} = \frac{(1,-2,-2)\cdot(3,4,0)}{|(1,-2,-2)|\,|(3,4,0)|} = \frac{-5}{3(5)}$.

5. (3 points) Determine whether the two lines are parallel/intersected/skewed: $L_1 : x = 1+t, y=-2+3t, z=4-t; L_2 : x=-s, y=1-s, z=-3-2s$.

   Solution: (1) The two direction vectors are $v_1 = (1,3,-1)$ and $v_2 = (-1,-1,-2)$. They are not parallel.

   (2) No intersection: From "$x=x$" and "$y=y$" we have $1+t=-s, -2+3t=1-s \Rightarrow s=-3, t=2$. By L_1, $z=2$; by L_2, $z=3$. Two z values are not equal.

   (3) Thus they are skewed.

6. (4 points) Find the equation of the plane Π containing the three points $P(1,2,-1)$, $Q(3,1,-1)$ and $R(1,1,0)$.

   Solution: We have
$$\vec{PQ}=(2,-1,0), \quad \vec{PR}=(0,-1,1).$$
   The normal vector of the plane is
$$\mathbf{n}=\vec{PQ}\times\vec{PR}=(-1,-2,-2).$$
   The equation of the plane is
$$(-1,-2,-2)\cdot((x,y,z)-(1,2,-1))=0, \Rightarrow -(x-1)-2(y-2)-2(z+1)=0,$$
   i.e.,
$$x+2y+2z-3=0.$$

7. (4 points) Find the tangent line to the parametric curve given by
$$x=t^3-t^2, y=3t^2, \ at\ (0,3).$$

Solution: At first we need the slope of the tangent line.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6t}{3t^2 - 2t} = \frac{6}{3t - 2}.$$

When $(x, y) = (0, 3)$, $t = 1$. The slope of the tangent line is 6.

The tangent line is: $y = 6x + 3$.