Concordia University
Department of Computer Science and Software Engineering

COMP232 Mathematics for Computer Science

Assignment 4, Fall 2016, Due December 1, 2016

1. Use mathematical induction to show that $f_{n-1} \cdot f_{n+1} - f_n^2 = (-1)^n$ for n in the set of positive integers.

2. The sequence of Fibonacci numbers is defined by

$$f_0 = 0, f_1 = 1, \text{ and } f_n = f_{n-1} + f_{n-2}, \forall n > 1.$$

The sequence of Lucas numbers is defined by

$$l_0 = 2, l_1 = 1, \text{ and } l_n = l_{n-1} + l_{n-2}, \forall n > 1.$$

Prove that $f_n + f_{n+2} = l_{n+1}$, whenever n is a positive integer, where f_i and l_i are the ith Fibonacci number and ith Lucas number, respectively.

3. For each of the following relations on the set $\mathbb{Z}$ of integers, determine if it is reflexive, symmetric, anti-symmetric, or transitive. On the basis of these properties, state whether or not it is an equivalence relation or a partial order.
   (a) $R = \{(a,b) \in \mathbb{Z}^2 : a^2 = b^2\}$.
   (b) $S = \{(a,b) \in \mathbb{Z}^2 : |a-b| \le 1\}$.

4. Prove that $\{(x,y) \in \mathbb{R}^2 : x - y \in \mathbb{Q}\}$ is an equivalence relation on the set of real numbers, where $\mathbb{Q}$ denotes the set of rational numbers.

5. Prove or disprove the following statements:
   (a) Let R be a relation on the set $\mathbb{Z}$ of integers such that xRy if and only if $xy \ge 1$. Then, R is irreflexive.
   (b) Let R be a relation on the set $\mathbb{Z}$ of integers such that xRy if and only if $x = y + 1$ or $x = y - 1$. Then, R is irreflexive.
   (c) Let R and S be reflexive relations on a set A. Then, $R - S$ is irreflexive.

6. Let R be the relation on $\mathbb{Z}^+$ defined by xRy if and only if $x < y$. Then, in the Set Builder Notation, $R = \{(x,y) : y - x > 0\}$. (a) Use the Set Builder Notation to express the transitive closure of R. (b) Use the Set Builder Notation to express the composite relation $\mathbb{R}^n$, where n is a positive integer.

7. Give the transitive closure of the relation $R = \{(a,c),(b,d),(c,a),(d,b),(e,d)\}$ on $\{a,b,c,d,e\}$.

8. Give an example to show that when the symmetric closure of the reflexive closure of the transitive closure of a relation is formed, the result is not necessarily an equivalence relation.