

ECON 301 - Section A

Assignment # 1

Due on Tuesday, Oct. 18 at 13:15

TRUE or FALSE. Explain whether each of the following statements is true or false, and why. (All credits are assigned to explanations) (5 marks each)

1. The utility function $U(x_1, x_2) = \ln(x_1 + 2x_2)$ corresponds to preferences for perfect substitutes. **True. It is a monotone transformation (ln function) of a perfect substitute.**
2. If a person's income increases, then his indifference curves shift up and to the right. **False. Change in income doesn't change the preferences or the indifference curves.**
3. If preferences are transitive, more is always preferred to less. **False. Transitive just says if $x \succ y$ and $y \succ z$, then $x \succ z$, for example if the commodities are bad then consumer will not prefer more, that means x has the lowest amount then y and z has the highest amount of the bads.**
4. If consumers consume only two goods, both of them can not be inferior. **True. You can show this with an income offer curve or IE in Slutsky equation.**
5. A consumer consumes only two goods. If her income is doubled and at the same time price of good one is tripled, without any further information about her preferences we can not say whether she is better off or not. **True. Plot the old and the new BL, since one part of the new is above the old and one part below we can't say whether she is better off without knowing the preferences.**
6. Maude thinks pink roses and white roses are perfect substitutes, one for one. Pink rose currently cost \$2 per unit and white rose cost \$3 per unit. If the price of white rose rises to \$7 per unit. There will be no substitution effect. **True, she only buys pink roses, before and after the price change $\Rightarrow TE=IE=SE=0$.**

Part two (2 questions) - You need to show your work. No credits without an explanation (70 marks).

1. Eva has \$48 which she decides to spend on x and y. Commodity x costs \$5 per unit and commodity y costs \$6 per unit. Her preferences over these commodities could be presented by the utility function $U(x, y) = \min \{5x, 2y\}$.

(a) Draw her indifference map (two sample indifference curves are enough) and her budget line. (5 marks) **The kink of the indifference curves happen when $5x = 2y$.**

(b) How many x and y she will choose? (10 marks) $x = 2.4$, $y = 6$

(c) Suppose price of y drops to \$4. What is her optimal consumption bundle after this change? Decompose Eva's changes in consumption into income and substitution effects for the two commodities. (10 marks) $x = 3.2$, $y = 8$. **Perfect complements \Rightarrow SE=0, TE=IE for x: IE=3.2-2.4=.8, for y: IE=2.**

2. A consumer consumes only food (x_1) and alcohol (x_2) and he has a utility function $U(x_1, x_2) = x_1^a + x_2^a$.

(a) Derive the ordinary demand functions for him. (15 marks)

- **If $a = 1 \Rightarrow$ perfect substitutes \Rightarrow he consumes only the cheaper commodity. So if $p_1 < p_2 \Rightarrow x_1^* = m/p_1$ and $x_2^* = 0$. If $p_1 > p_2$ then $x_1^* = 0$ and $x_2^* = m/p_2$.**
- **If $a > 1 \Rightarrow$ concave preferences and again he just consumes the cheaper commodity. So if $p_1 < p_2 \Rightarrow x_1^* = m/p_1$ and $x_2^* = 0$. If $p_1 > p_2$ then $x_1^* = 0$ and $x_2^* = m/p_2$.**
- **If $0 < a < 1$, then this is a well-behaved (strictly convex) and following the tangency condition we have**

$$\left(\frac{x_1}{x_2}\right)^{a-1} = \frac{P_1}{P_2} \Rightarrow x_1 = \left(\frac{P_1}{P_2}\right)^{\frac{1}{a-1}} x_2, \text{ plugging in the BL we have } x_1 = \frac{m}{p_2 + p_1 \left(\frac{P_1}{P_2}\right)^{\frac{1}{a-1}}}, x_2 = \frac{m}{p_1 + p_2 \left(\frac{P_2}{P_1}\right)^{\frac{1}{a-1}}}$$

(b) Now suppose, $a = \frac{1}{2}$, are these goods gross complement or gross substitute? (5 marks)

$$x_1 = \frac{mp_2}{p_1(p_1+p_2)}, x_2 = \frac{mp_1}{p_2(p_1+p_2)}$$

$$\frac{\partial x_i}{\partial p_j} > 0 \Rightarrow x_i \text{ is a gross substitute for } x_j. \text{ for example } \frac{\partial x_1}{\partial p_2} = \frac{mp_1(p_1+p_2)-mp_2p_1}{(p_1(p_1+p_2))^2} = \frac{m}{(p_1+p_2)^2} > 0$$

- For all remaining parts suppose $p_1 = \$2$ and $p_2 = \$3$, $m = \$90$, and $a = \frac{1}{2}$.

- (c) Write down the budget constraint and drive the consumer's optimal consumption bundle. (10 marks)

$$90 = 2x_1 + 3x_2 \text{ based on part a } x_1 = 27, x_2 = 12.$$

- (d) Now suppose the government collects a tax of \$1/unit of alcohol (good 2). What is the new budget constraint and how many of each product he will consume? (assume consumption in partial quantity is possible) (5 marks)

$$90 = x_1 + (3 + 1)x_2, x_1 = 30, x_2 = 7.5.$$

- (e) Find the substitution effect, income effect and total effect of this price change on alcohol (good 2) (10 marks) (hint: replace p_1 in the demand function for alcohol by its value, \$2)

$$\Delta m = x_2 \Delta p_2 = 12 * 1, m' = 102$$

$$x'_2 = \frac{2 * 102}{4(6)} = 8.5 \Rightarrow \text{TE} = -4.5, \text{SE} = 8.5 - 12 = -3.5 \text{ and IE} = -1.$$