Tutorial 1

Math Review

1. Calculate the partial derivatives of each of the following functions with respect to x_1 and x_2. (a, b, c, d are constant.)

   (a) $f(x_1, x_2) = cx_1^a x_2^b + d$

   (b) $f(x_1, x_2) = \ln(x_1 - ax_2x_1 + 1)^2$

   (c) $f(x_1, x_2) = e^{2x_1^{\frac{1}{2}}} + x_1x_2 + a$

   (d) $f(x_1, x_2) = \frac{ax_1 - 5}{-x_2^2 + 1}$

2. For which values of x_1 and x_2 the following function is minimum?

   (a) $f(x_1, x_2) = x_1^2 - 2x_1x_2 + 3x_2^2 + 2x_1 - 2x_2$

3. Consider the following function f defined over the interval $[-3, 3]$:

$$f(x) = \begin{cases} 4 - (x+2)^2 & \text{if } x \leq -1 \\ x^2(2-x) & \text{if } x \geq -1 \end{cases}$$

   (a) Sketch a rough graph of the function

   (b) Find all the critical points (i.e., points with zero derivative) of the function. Identify the interior local maxima and minima.

   (c) Is the function discontinuous anywhere? Is the function non-differentiable anywhere? If your answer to any of the questions is yes, is there a local maxima or minima at these points ?

   (d) Is there a local maxima or minima at the extremes ?

4. In each of the following , the values of x and y are non-negative numbers:

   (a) Maximize xy subject to $x + 2y = 12$ by substitution

   (b) Minimize $x + 2y$ subject to $xy = 18$ by substitution

Chapter 2

1. If you could exactly afford either 2 units of x and 11 units of y, or 4 units of x and 7 units of y, then if you spent all of your income on y, how many units of y could you buy?

2. Clara spends her entire budget and consumes 5 units of x and 13 units of y. The price of x is twice the price of y. Her income doubles and the price of y doubles, but the price of x stays the same. If she continues to buy 13 units of y, what is the largest number of units of x that she can afford?

3. Suppose a consumer consumes food (F) and other goods (G). Government pays a subsidy (s) on value of food and imposes an income tax (t) on income of this consumer, then present budget constraint for this consumer.

4. Suppose the consumer consumes two goods: CDs and pizzas. Her income is $m = \$200$. The price of CDs is $p_1 = \$20$ and $p_2 = \$10$.

   (a) Write down the budget constraint. Illustrate graphically the budget set. Calculate the slope of the budget line.

   (b) Now suppose the government levies a 25% tax on each CD the consumer purchases. What is the new budget constraint? Draw the new budget line on the same graph as in part a.

   (c) Without further information about the consumer's preferences, can you determine whether the consumer is better off? Explain.

5. In the world of Hogwarts, they have two kinds of money, blue and red. Every commodity has two prices — a red money price and a blue money price. Everybody at Hogwarts has two incomes, a red money income and a blue money income. In order to buy an object , a person at Hogwarts has to pay the object's red money price in red money, and it's blue money price in blue money. The Ministry of Magic that rules Hogwarts *forbids* the trade of one kind of money for the other.

   There are just two consumer goods at Hogwarts, firewhisky and pumpkin pasties. All of Hogwarts prefer more to less. The blue prices are 1 bcu (bcu stands for blue currency unit) per unit of firewhisky and 1 bcu per unit of pumpkin pasty.

The red prices are 2 rcu (rcu stands for red currency unit) per unit of firewhisky and 6 rcus per unit of pumpkin pasty.

(a) Harry Potter, who lives in Hogwarts, has blue income of 10 qand red income of 30. Draw in a graph Harry's red budget and blue budget. Shade the budget set containing all the commodity bundles that Harry can afford given his budget constraints.

(b) Hermione is a second resident at Hogwarts. She faces the same prices as Harry. She has the same red income as Harry. But Hermione has a blue income of 20. Explain how Hermione will not spend all her blue income no matter what her tastes may be

True/False Indicate whether the statement is true or false.

1. If all prices are doubled and money income is left the same, the budget set does not change because relative prices do not change.

2. If all prices double and income triples, then the budget line will become steeper.

3. If there are two goods and the prices of both goods rise, then the budget line must become steeper.

4. There are two goods. You know how much of good 1 a consumer can afford if she spends all of her income on good 1. If you knew the ratio of the prices of the two goods, then you could draw the consumer's budget line without any more information.