#1 Bertrand problem

Assume two firms (firm 1 and firm 2) with the following cost structure

$$C_1(Q_1) = 10Q_1$$
$$C_2(Q_2) = 20Q_2$$

that face the following demand

$$P = 100 - Q$$

(where Q is total sales of the homogeneous good)

Assume the firm competes on price and make their decision simultaneously

a) Calculate the equilibrium price
b) How much will each firm produce
c) Is your answer in a) a N.E. set of strategies

#2 Stackelberg problem (differentiated goods)

Assume two firms (firm 1 and firm 2) with the following cost structure

$$C_1(Q_1) = 4Q_1$$
$$C_2(Q_2) = 4Q_2$$

that face the following demands

$$P_1 = 20 - 2Q_1 - Q_2$$
$$P_2 = 20 - Q_1 - 2Q_2$$

Assume the firms compete on quantity in a sequential manner where firm 2 chooses its output first and then firm 1 makes its choice

a) calculate the equilibrium quantities of both firms

Answers

#1 $MC_1 = \frac{dC_1(Q_1)}{dQ_1} = 10$

$MC_2 = \frac{dC_2(Q_2)}{dQ_2} = 20$

a) $(P_1, P_2) = (19.99, 20)$

b) Firm 2 will have no sales, and firm 1 will produce

$Q_1 = 100 - P_1$

$= 100 - 19.99$

$Q_1 = 80.01$

c) Yes $(P_1, P_2) = (19.99, 20)$

Answer #2

Start with the firm that makes its output decision last

$\max_{Q_1}$ Profit @ firm 1

$\max_{Q_1} P_1Q_1 - C_1(Q_1)$

$\max_{Q_1} (20 - 2Q_1 - Q_2)Q_1 - 4Q_1$

$\max_{Q_1} 20Q_1 - 2Q_1^2 - Q_1Q_2 - 4Q_1$

$16 - 4Q_1 - Q_2 = 0$

$Q_1 = \frac{16 - Q_2}{4}$

$\max_{Q_2}$ Profits @ firm 2 s.t. $Q_1 = \frac{16 - Q_2}{4}$

$\max_{Q_2} P_2Q_2 - C_2(Q_2)$ s.t. $Q_1 = \frac{16 - Q_2}{4}$

$\max_{Q_2} (20 - 2Q_2 - Q_1)Q_2 - 4Q_2$ s.t. $Q_1 = \frac{16 - Q_2}{4}$

$\max_{Q_2} 20Q_2 - 2Q_2^2 - Q_1Q_2 - 4Q_2$ s.t. $Q_1 = \frac{16 - Q_2}{4}$

$$\max_{Q_2} \; 20Q_2 - 2Q_2^2 - \left(\frac{16 - Q_2}{4}\right)Q_2 - 4Q_2$$

$$20 - 4Q_2 - 4 + \frac{2Q_2}{4} - 4 = 0$$

$$Q_2 =$$

$$Q_2 = 12/3.5 = 3.4$$

$$Q_1 = \frac{16 - Q_2}{4}$$

$$Q_1 = \frac{16 - 3.4}{4}$$

$$Q_1 = 3.15$$