assume a cournot duopoly with the following cost structure

Firm 1 $C_1(Q_1) = 10Q_1$
Firm 2 $C_2(Q_2) = 20Q_1$

and with the following market demand

$P = 100 - Q$, where $Q \equiv Q_1 + Q_2$

Calculate the equilibrium output of each firm, and the equilibrium price, and consumer surplus

Max_{Q_1} Profits of 1

$\text{Max}_{Q_1} P(Q)Q_1 - C_1(Q_1)$

$\text{Max}_{Q_1} (100 - Q_1 - Q_2)Q_1 - 10Q_1$

$\text{Max}_{Q_1} 90Q_1 - Q_1^2 - Q_1Q_2$

$90 - 2Q_1 - Q_2 = 0$ (1)

Max_{Q_2} Profits of 2

$\text{Max}_{Q_2} P(Q)Q_2 - C_2(Q_2)$

$\text{Max}_{Q_2} (100 - Q_1 - Q_2)Q_2 - 20Q_2$

$\text{Max}_{Q_2} 80Q_2 - Q_2^2 - Q_1Q_2$

$80 - 2Q_2 - Q_1 = 0$ (2)

multiply equation (1) by -2, and add to equation (2)

$$-180 + 4Q_1 + 2Q_2 = 0$$
$$80 - Q_1 - 2Q_2 = 0$$
$$-100 + 3Q_1 = 0$$

$Q_1^* = \frac{100}{3}$ sub into eq(1) $90 - 2Q_1 - Q_2 = 0$

$90 - 2\left(\frac{100}{3}\right) - Q_2 = 0$

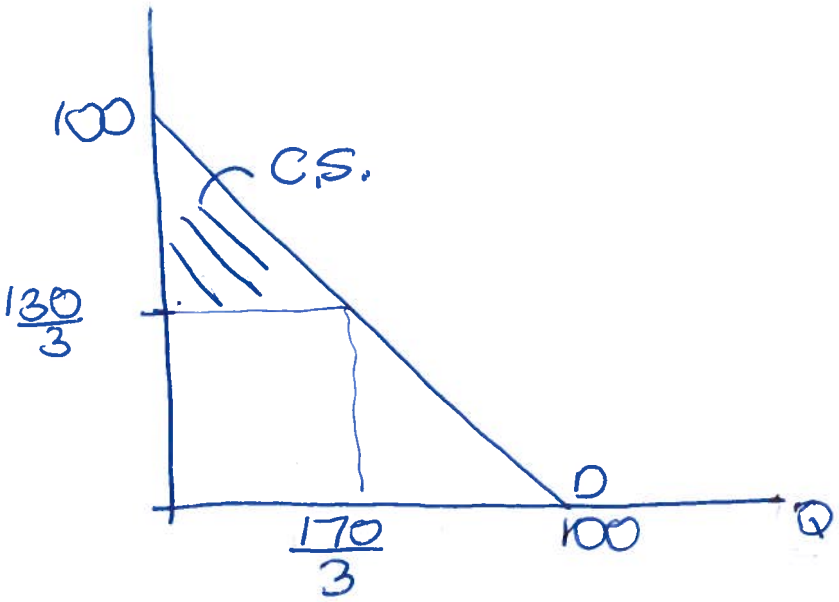
$Q_2^* = 70/3$

$P = 100 - Q_1 - Q_2$

$= 100 - \frac{100}{3} - \frac{70}{3}$

$P^* = 130/3$

$$Q = Q_1 + Q_2$$
$$Q = \frac{100}{3} + \frac{70}{3}$$
$$Q = \frac{170}{3}$$



$$C.S. = \frac{1}{2} \cdot \text{base} \cdot \text{height}$$
$$= \frac{1}{2}\left(\frac{170}{3} - 0\right)\left(100 - \frac{130}{3}\right)$$
$$= \frac{1}{2}\left(\frac{170}{3}\right)^2$$